6.003 Recitation, March 12 2010, Sections 3 & 4.

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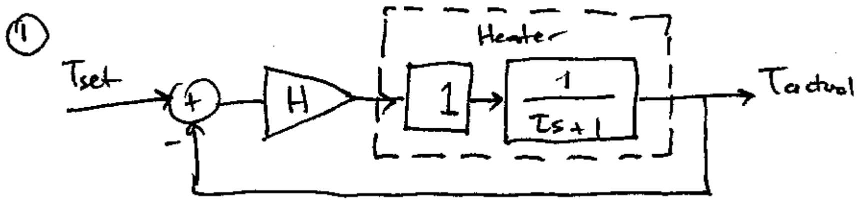
Today

① CT thermostat control

② DT thermostat control

③ Relationship between the two.

①

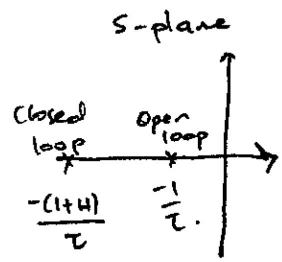


$$\frac{T_{actual}(s)}{T_{set}(s)} = \frac{\frac{H}{\tau s+1}}{1+\frac{H}{\tau s+1}} = \frac{H}{\tau s+1+H} = \frac{H/(1+H)}{\tau s/(1+H)+1}$$

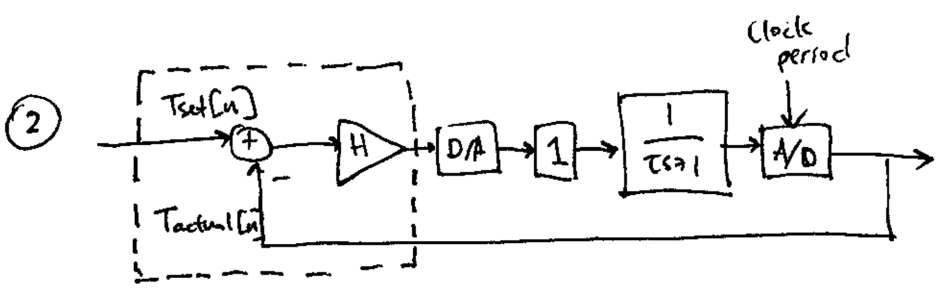
As $H \to \infty$, $\frac{H}{1+H} \to 1$, so we have speedup. If H is finite, time const. speeds up by $(1+H)$, i.e., $\tau \to \frac{\tau}{1+H}$

The steady-state response, set $s=j\omega=0$.

$$\Rightarrow \frac{T_{actual}(0)}{T_{set}(0)} = \frac{H}{1+H}.$$



②

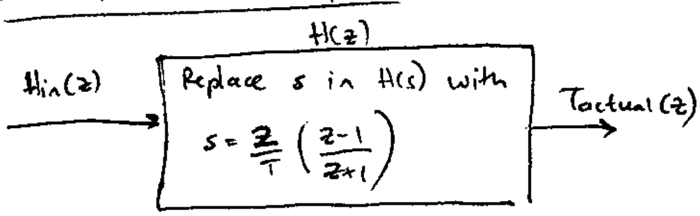


Aside: Trapezoidal approx.:

$$s = \frac{2}{T}\left(\frac{z-1}{z+1}\right)$$

$$z = \frac{1+\frac{sT}{2}}{1-\frac{sT}{2}}$$

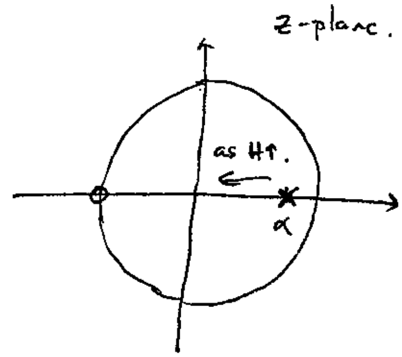
Z-transform of Heater Box:



$$\frac{T_{actual}(z)}{T_{set}(z)} = \frac{H}{\frac{\tau 2}{T}\left(\frac{z-1}{z+1}\right)+1+H} = \frac{\frac{T}{2\tau}H(1+z)}{(z-1)+(z+1)\frac{T}{2\tau}(1+H)}$$

$$= \frac{\frac{T}{2\tau}H(1+z^{-1})}{\cancel{z}\left(1+\frac{T}{2\tau}(1+H)\right)-\left(1-\frac{\tau}{2T}(1+H)\right)z^{-1}}$$

$$= \left\{\frac{\frac{T}{2\tau}H}{1+\frac{T}{2\tau}(1+H)}\right\}\frac{1+z^{-1}}{1-\underbrace{\left[\frac{1-\left(1-\frac{T}{2\tau}\right)(1+H)}{\left(\frac{T}{2\tau}\right)(1+H)+1}\right]}_{\alpha}z^{-1}}$$



As $H \to \infty$, $\alpha \to -1$.

Note that the impulse response has oscillations for $\alpha < 0$. If $\alpha = 0$, when $\frac{T}{2\tau}(1+H) = 1$,

i.e. $\frac{\tau}{1+H} = \frac{T}{2}$.

CT speedup!