6.003R

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Control of a 1st-order system

Peter Hagelstein

We know how this system works...

$$\underbrace{\frac{d}{dt}y(t) + \frac{y(t)}{\tau}}_{\text{Relaxation}} = \underbrace{\frac{x(t)}{\tau}}_{\text{source}} \qquad y(t) \text{ is height}$$

$$X \longrightarrow \boxed{\frac{1/\tau}{s+1/\tau}} \longrightarrow Y$$

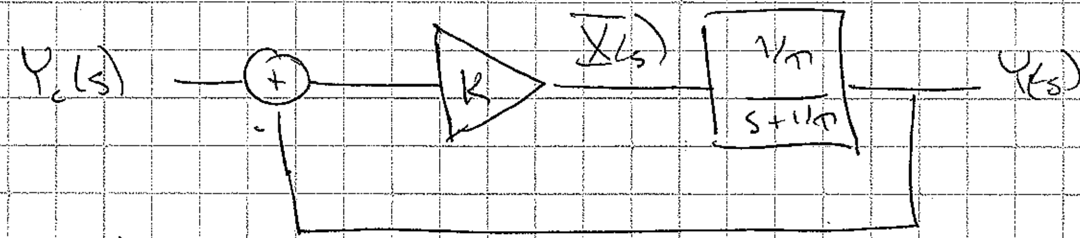
OK, suppose we want to control the water level, how might it work?

Define $y_c(t)$ to be desired height.

Maybe use difference between $y(t)$ and $y_c(t)$ to control the source...

let $\quad x(t) = K\left[y_c(t) - y(t)\right]$

Simplest approach



Lets see if it works, and how it works

$$Y(s) = \frac{1/\tau}{s + 1/\tau} K\left[Y_c(s) - Y(s) \right]$$

$$\left[1 + \frac{K/\tau}{s + 1/\tau} \right] Y(s) = \frac{K/\tau}{s + 1/\tau} Y_c(s)$$

$$\frac{Y(s)}{Y_c(s)} = \frac{\frac{K/\tau}{s + 1/\tau}}{1 + \frac{K/\tau}{s+1/\tau}} = \frac{\frac{K}{\tau}}{s + \frac{(K+1)}{\tau}}$$

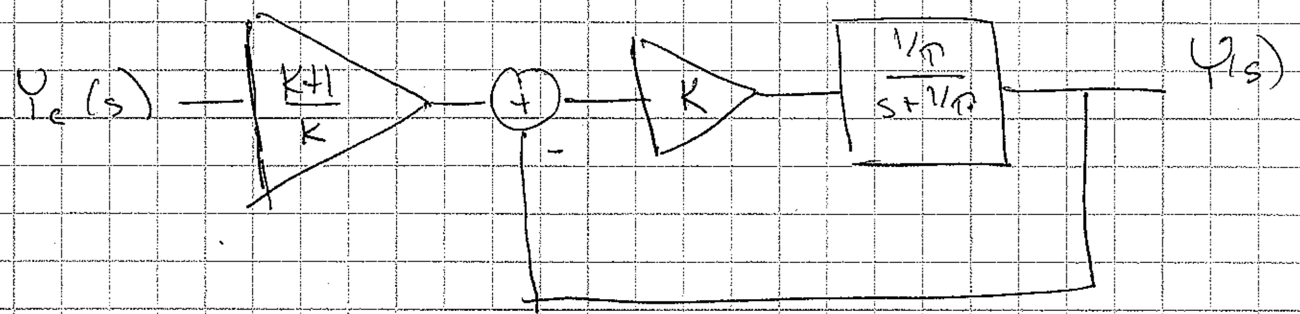
Two thoughts...

(1) if K is large, and s small, then $Y(s) \to \frac{K}{K+1} Y_c(s)$

and we can get a pretty good system

(2) Pole seems to have moved from $-\frac{1}{\tau}$ to $-\frac{K+1}{\tau}$

Can fix things to correct amplitude



$$Y(s) = \frac{\frac{1}{\tau}}{s+1/\tau} K \left[\frac{K+1}{K} Y_c(s) - Y(s) \right]$$

$$\left[1 + \frac{K/\tau}{s+1/\tau} \right] Y(s) = \frac{\frac{K+1}{\tau}}{s+1/\tau} Y_c(s)$$

$$Y(s) = \frac{\frac{\frac{K+1}{\tau}}{s+1/\tau} Y_c(s)}{1 + \frac{K/\tau}{s+1/\tau}} = \frac{\frac{K+1}{\tau}}{s + \frac{K+1}{\tau}} Y_c(s)$$

$$H(s) = \frac{\frac{K+1}{\tau}}{s + \frac{K+1}{\tau}}$$

OK - it is better for small s

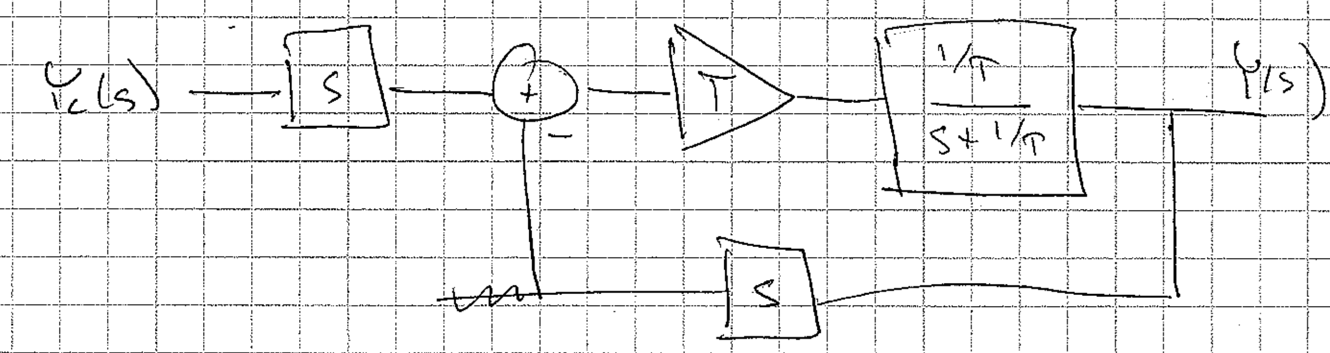
Modified control system

Suppose we have a sensor that measures $\frac{dy}{dt}$ = Rate of change of water level. Can we still control the water level?

thought: let $x(t) = T\left(\frac{dy_c}{dt} - \frac{dy}{dt}\right)$

since the control is based on the rate, we don't have feedback for the height. Probably we need to make sure it starts at the right place.

Feedback system?



$$Y(s) = \frac{1/\tau}{s+1/\tau}\, T \left[s Y_c(s) - s Y(s) \right]$$

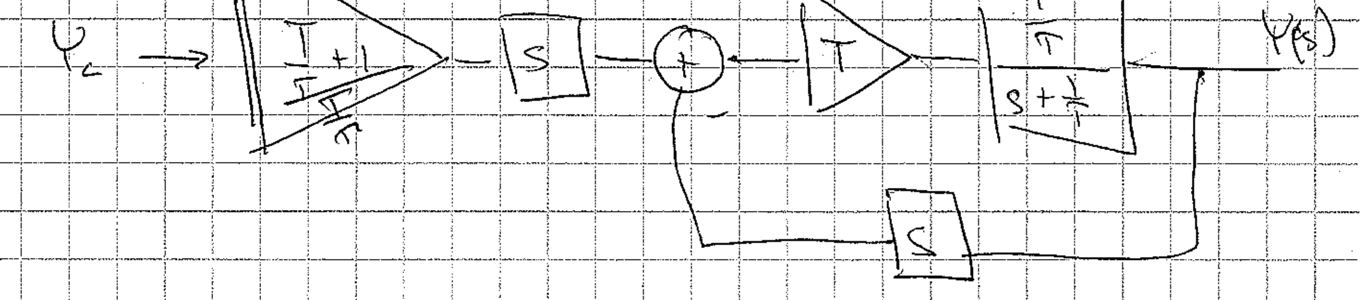
$$\left[1 + \frac{(T/\tau)\, s}{s + 1/\tau} \right] Y(s) = \frac{(T/\tau)\, s}{s + 1/\tau}\, Y_c(s)$$

$$\frac{Y(s)}{Y_c(s)} = \frac{\dfrac{\left(\frac{T}{\tau}\right) s}{s + 1/\tau}}{1 + \dfrac{\left(\frac{T}{\tau}\right) s}{s + \frac{1}{\tau}}} = \frac{\left(\frac{T}{\tau}\right) s}{s + \frac{1}{\tau} + \left(\frac{T}{\tau}\right) s}$$

Does it work? It works differently!

$$H(s) \longrightarrow \frac{\frac{T}{\tau}}{\frac{T}{\tau} + 1} = \frac{T}{T+\tau}$$ for large T and/or large s

This one works as a high frequency controller. We can correct the amplitude using



For this one we can write

$$\frac{Y(s)}{Y_c(s)} = \frac{\left(\frac{T}{\tau}+1\right)s}{s\left(\frac{T}{\tau}+1\right)+\frac{1}{\tau}}$$

Compare corrected versions

correction based on $y(t)$:

feedback based on $\frac{dy}{dt}$