6.003 Recitation, Sections 3 & 4, Wednesday March 31.

Today: ① Quick revision of Fourier series (from lecture)
② Parseval's Thm for Fourier series
③ Computing π with Fourier series (Parseval)
④ Total harmonic distortion & linearity.

① $x(t) = x(t-T)$, (T: period), $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j\frac{2\pi k}{T}t}$ ← synthesis equation.

$$a_k = \frac{1}{T}\int_T x(t) e^{-j\frac{2\pi k}{T}t} dt$$ ← analysis equation

② $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j\frac{2\pi k}{T}t}$, $x^*(t) = \sum a_k^* e^{-j\frac{2\pi k}{T}t}$

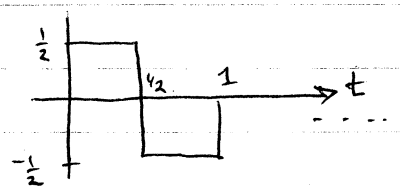
Now multiply & average over a period of T.

$$\int_T x(t)x^*(t)dt = \sum_{k=-\infty}^{\infty} (a_k a_k^*) T$$

$$\Rightarrow \frac{1}{T}\int_T |x(t)|^2 dt = \sum_k |a_k|^2 \Rightarrow \boxed{\frac{1}{T}\int x^2(t)\,dt = \sum |a_k|^2}$$

⇒ Parseval's thm: average energy in $x(t)$ = (sum of) average energy in each harmonic.

③



Fourier coefficients from analysis formula are $\begin{cases} \frac{1}{jk\pi}; & k \text{ odd} \\ 0; & k \text{ even} \end{cases}$

The average energy in the square wave is $(\frac{1}{2})^2$

Sum of energies in each harmonic is $\sum_{\substack{k=-\infty \\ k\,\text{odd}}}^{\infty} \frac{1}{k^2\pi^2} = \frac{2}{\pi^2}\left(\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots\right)$

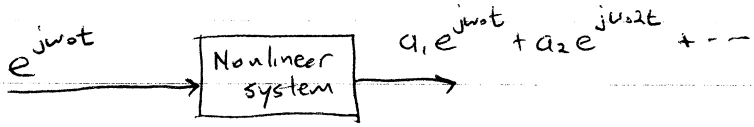
Therefore, $\frac{1}{4} = \frac{2}{\pi^2}\left(1 + \frac{1}{3^2} + \frac{1}{5^2} + \cdots\right)$

$\Rightarrow \pi = 2\sqrt{2}\sqrt{1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots}$

m tmtH poncho will my love pick up my new mower ginger...

$\pi = 3.14159265358979\ldots$

(4) Total Harmonic Distortion. (THD)



$$THD \triangleq \frac{\sqrt{a_2^2 + a_3^2 + a_4^2 + \cdots}}{a_1}$$

← energy in all non

← energy in harmonics.

For the square wave, $THD = \frac{\sqrt{\frac{\pi^2}{8} - 1^2}}{1} = 0.48.$

Note

Triangle wave has much lower THD, Hi-Fi in audio implies THD < 1%.

Mnemonics to memorize P_i:

s, z	0
t d	1
n	2.
m	3
r	4
l	5
j, ch, sh	6
k, g	7
f, v	8
p, b	9