6.003 Recitation, Sections 3 & 4, Friday Apr. 2, 2010.

Today .:

① Properties of Fourier Series
② Example prob. 1.
③ " " 2; half-wave rectification.

$x(t) = x(t-T)$

①

Property	Signal Input	Fourier Series
Linearity	$\alpha x(t) + \beta y(t)$	$\alpha a_k + \beta b_k$
Differentiation	dx/dt	$j\frac{2\pi k}{T} a_k$
Integration	$\int_{-\infty}^{t} x(\tau)d\tau$	$\frac{1}{j\frac{2\pi k}{T}} a_k$
Periodic Convolution	$\int_T x(\tau)y(t-\tau)d\tau$	$T a_k b_k$
Shift	$x(t-t_0)$	$e^{-j\frac{2\pi k}{T}t_0} a_k$.
Scaling	$x(at)$, $a>0$	a_k ← Period changes to T/a but coeff r charge.
Time reversal	$x(-t)$	a_{-k}
Complex Conjugation	$x^*(t)$	a^*_{-k}
Averaging	$\frac{1}{T}\int_T x(\tau)d\tau$	a_0
Parseval's Thm	$\frac{1}{T}\int_T \lvert x(t)\rvert^2 dt$	$\sum_k \lvert a_k\rvert^2$.
Even-odd Symmetry	$x(t) = x(-t)$	$a_k = a_{-k}$, both real.
	$x(t) = -x(-t)$	$a_k = -a_{-k}$, both imaginary.

② Find the Fourier series corresponding to $\left[x(t) \xleftrightarrow{F} \{a_k\}\right]$

(a) $x(t-t_0) + x(t+t_0)$

$\longleftrightarrow e^{-j\frac{2\pi k}{T}t_0} a_k + e^{j\frac{2\pi k}{T}t_0} a_k = 2\cos\left(\frac{2\pi k t_0}{T}\right) a_k.$

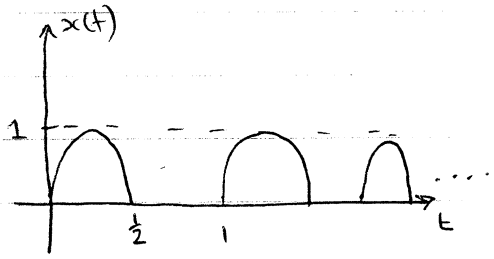
(b) $Ev\{x(t)\} = \frac{x(t)+x(-t)}{2} \longleftrightarrow \frac{1}{2}(a_k + a_{-k})$

(c) $Re\{x(t)\} = \frac{x(t)+x^*(t)}{2}$ Note: $Im\{x(t)\} = \frac{x(t)-x^*(t)}{2}$

$\longleftrightarrow \frac{1}{2}(a_k + a_{-k}^*)$.

(d) $x(3t-1) = x(3(t-\frac{1}{3})) \longleftrightarrow e^{-j\frac{2\pi}{T}(\frac{1}{3})k}\, a_k$

(3)



Half-wave rectified version of $\sin(\frac{2\pi t}{T_1})$

$x(t) - x(t-\frac{1}{2}) = \sin(2\pi t)$ —— (A)

$\mathcal{F}(x(t)) = \sum_k a_k e^{j2\pi kt}$

$\Rightarrow \mathcal{F}(A) = \sum_k \left(a_k e^{j2\pi kt} - a_k e^{-j2\pi kt}\right) = \frac{1}{2j}e^{j2\pi t} - \frac{1}{2j}e^{-j2\pi t}$

$\Rightarrow \sum_k a_k(1-e^{-jk\pi}) = \ldots$

$\Rightarrow a_1(1-(-1)) = \frac{1}{2j}$

$a_{-1}(1-(-1)) = \frac{-1}{2j}$

$\Rightarrow a_1 = \frac{1}{4j}$

$a_{-1} = \frac{-1}{4j}$

$a_k = 0$, $k \notin \{1,-1\}$.

What about even k:

$\sum_k a_k(1-1) = 0 \Rightarrow a_k = \frac{0}{0}$ ... indeterminate, $\Rightarrow$ solve using analysis formula.

$$a_k = \frac{1}{1}\int_0^{1/2}\left(\frac{e^{j2\pi t} - e^{-j2\pi t}}{2j}\right) e^{-2\pi k t}\, dt$$

$$= \frac{1}{2j}\int_0^{1/2}\left(e^{-j2\pi(k-1)} - e^{-j2\pi(k+1)}\right) dt$$

$$= \frac{1}{2j}\left[\ \cdots\ \right]\Big|_0^{1/2} = \frac{1}{2j}\left[\frac{-1-1}{-j(2\pi)(k-1)} - \frac{-1-1}{-j(2\pi)(k+1)}\right]$$

$$= \frac{-1}{2\pi}\left[\frac{1}{k-1} - \frac{1}{k+1}\right] = \boxed{\frac{-1}{\pi(k^2-1)} = a_k, \quad k \text{ even}.}$$

$$\Rightarrow \text{HWR}(\sin 2\pi t) = \frac{1}{2}\sin(2\pi t) + \frac{1}{\pi} - \sum_{\substack{k \text{ even},\\ k>0}} \frac{2}{\pi(k^2-1)}\cos(2\pi k t)$$