6.003 Recitation, Friday April 9, 2010 Sections 3&4

Today

1. Fourier transform analysis & synthesis
2. Sinc. functions, duality
3. Lenses & sincs
4. Gibbs phenomenon & sincs.

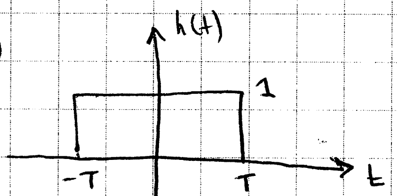
① $2\pi f = \omega$, $df = \frac{d\omega}{2\pi}$.

$\mathcal{F} \rightarrow X(f) = \int_{-\infty}^{+\infty} x(t)\, e^{-j2\pi ft}\, dt$ ← "analysis" formula

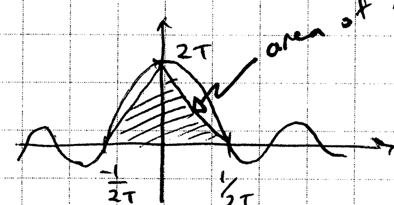
$\mathcal{F}^{-1} \rightarrow x(t) = \int_{-\infty}^{+\infty} X(f)\, e^{j2\pi ft}\, df$ ← "synthesis" formula.

Asympotic property: $\int_{-\infty}^{+\infty} x(t)\,dt = X(0)$, $\int_{-\infty}^{+\infty} X(f)\,df = x(0)$.

②



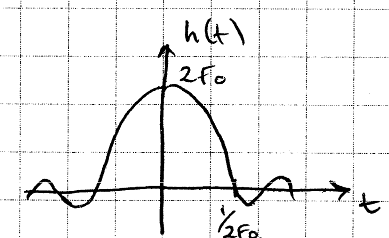
$\xrightarrow{\mathcal{F}}$



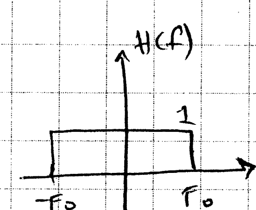
area of Δ = area of sinc. function.

$\text{sinc}(f) = \frac{\sin(2\pi fT)}{\pi f}$

$\lim_{f \to 0} \text{sinc}(f) = 2T$.



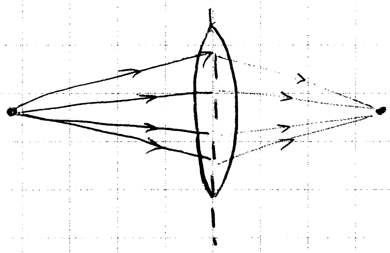
$\xrightarrow{\mathcal{F}}$

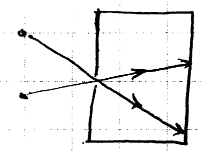


Duality:

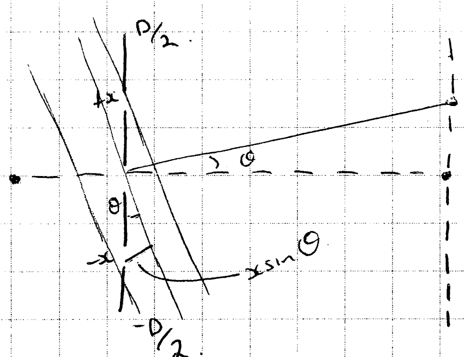
$$\begin{tikzcd}
 & x(t) \arrow[dr, "\mathcal{F}"] & \\
X(\cancel{f}) \arrow[ur, "\mathcal{F}"] & & X(f) \arrow[dl, "\mathcal{F}"] \\
 & x(-t) \arrow[ul, "\mathcal{F}"] &
\end{tikzcd}$$

(3)





Lens diffraction

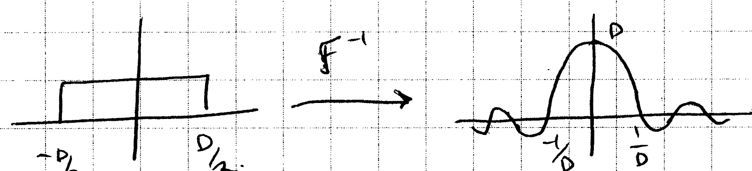


At the point on the image that corresponds to $+\Theta$, the image intensity is $\propto$ to

$$I_{out} \int_{-D/2}^{D/2} e^{+j 2\pi x \sin\Theta} \, dx$$

$$I_{in} \int_{-D/2}^{D/2} e^{+j \frac{2\pi \sin\Theta}{\lambda}} \, dx$$

Assume $\sin\Theta \approx \Theta$, if Θ is small.



$$\frac{\sin\left(2\pi \frac{\Theta}{\lambda} \frac{D}{2}\right)}{\pi\left(\frac{\Theta}{\lambda}\right)}$$

So sinc function is $\dfrac{\sin\left(\pi \frac{\Theta}{\lambda} D\right)}{\pi \, \Theta/\lambda}$, the real image is formed by convolving the sinc function with the ideal unfiltered image.

The first zero of the sinc is a good approx. to how much light is 'spreading'.
It occurs when $\dfrac{\Theta}{\lambda} = \dfrac{1}{D} \Rightarrow \Theta = \dfrac{\lambda}{D}$.

In a real lens, Θ corresponds to radial blur. And a point 'spreads' intensity in the imag to a radius around the geometric-optics ideal point of $\dfrac{\lambda}{D} f$.