6.003, Recitation, Sections 3 & 4, Friday Apr. 16

Today:

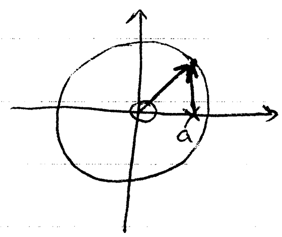
1. Low pass
2. 'Pure' differentiator
3. DT Highpass

---

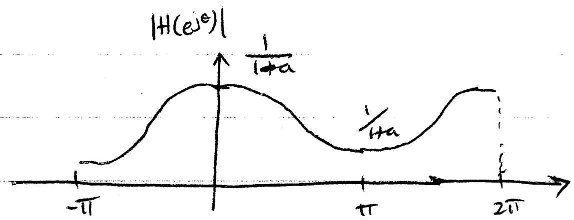
① CT case: $H(s) = \frac{1}{\tau s+1} \Rightarrow |H(j\omega)| = \frac{1}{\sqrt{1+\omega^2\tau^2}}$ $\measuredangle H(j\omega) = \tan^{-1}(\omega\tau)$

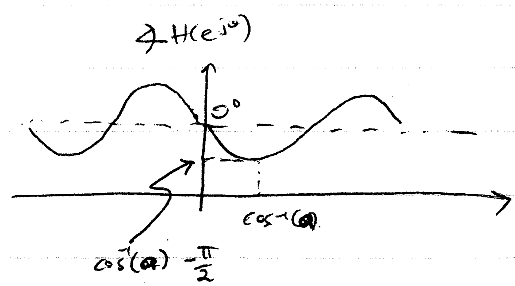
DT case: $H(z) = \frac{1}{1-az^{-1}} = \frac{z}{z-a}$; $z = e^{j\theta}$ & $H(e^{j\theta}) = \frac{e^{j\theta}}{e^{j\theta}-a}$.

$$|H(e^{j\theta})| = \frac{1}{\sqrt{(\cos\theta - a)^2 + \sin^2\theta}} \quad ; \quad \measuredangle H(e^{j\theta}) = \theta - \tan^{-1}\left(\frac{\sin\theta}{\cos\theta - a}\right)$$



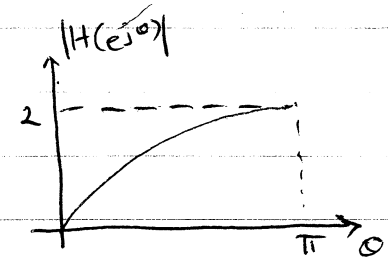
$\Rightarrow$

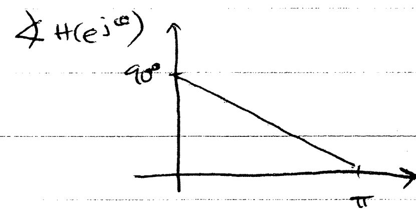




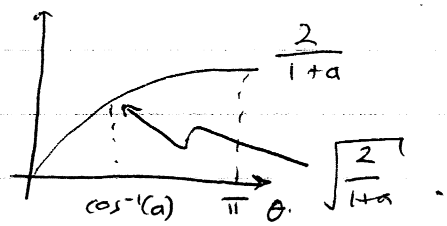
② $h[n] = x[n] - x[n-1]$, $H(e^{j\theta}) = 1 - e^{-j\theta} = e^{-j\theta/2}(2j)\sin\theta/2 = 2e^{j(\frac{\pi}{2}-\frac{\theta}{2})}\sin\theta/2$.

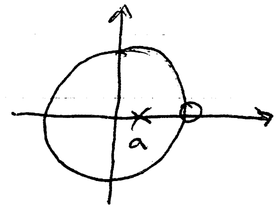
$\Rightarrow |H(e^{j\theta})| = 2\sin\theta/2$, $\measuredangle H(e^{j\theta}) = \frac{\pi}{2} - \frac{\theta}{2}$.



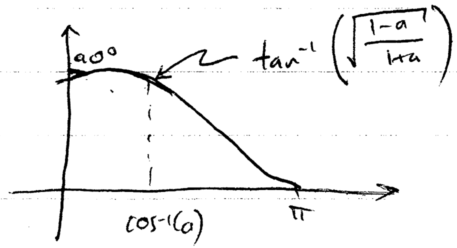


(3) $|H(e^{j\theta})|$





$\measuredangle H(e^{j\theta})$



As $a \to 0$, you get pure differentiator. As $a \to 0$, you get the identity. //