6-003 Recitation, Sections 3 & 4, Wed. Apr. 21 2010

Today:

1. Basics of windowing in CT & DT
2. Rectangular windows
3. Triangular windows
4. Generalization to fourier-transform tradeoffs.

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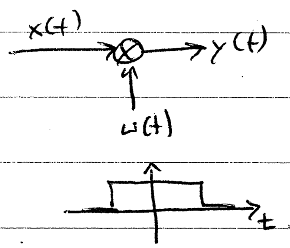
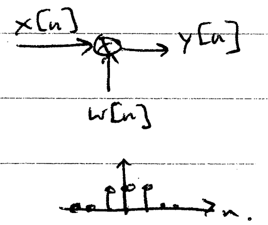
(1)

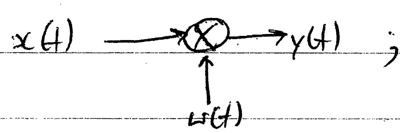




(a) in radio Tx, you gate transmission; (b) in lenses, the aperature control in the lens controls the depth of focus & spatial resolution. } CT

(c) Construct a FIR kernel from an IRR (infinite impulse response)
(d) windowing a time series before an FFT. } DT.

(2)



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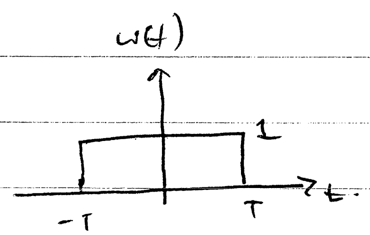
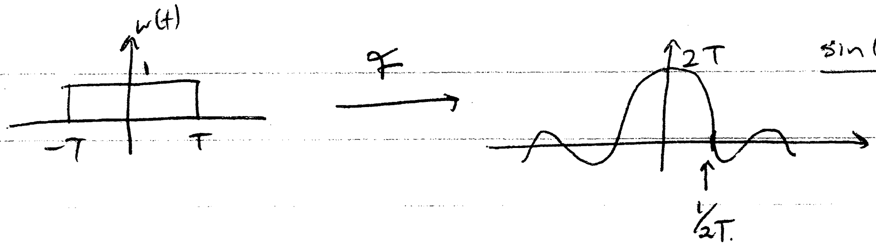


<u>Design w(t) s.t. $Y(f) \approx X(f)$.</u> The problem is trivial if $T \to \infty$.
$\Rightarrow$ the challenge is to design a finite time window.

Since $y(t) = x(t)\,w(t)$, $\Rightarrow$ $Y(f) = (X * W)(f)$

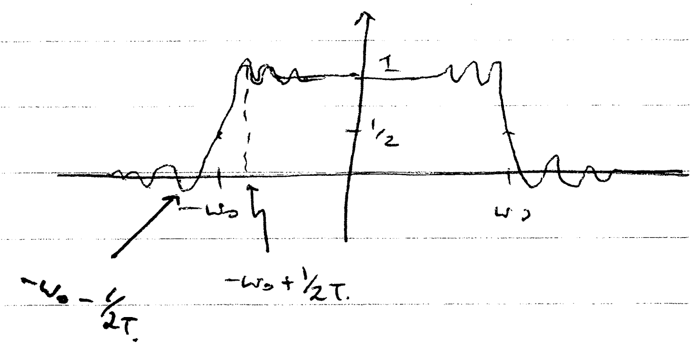
2.



$\frac{\sin(2\pi f T)}{\pi f}$

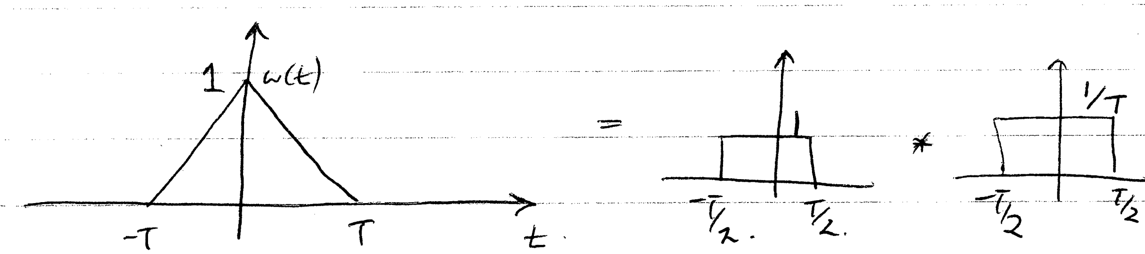
$Y(f) = (X * W)(f)$

Assume $X(f)$ is some bandlimited signal.

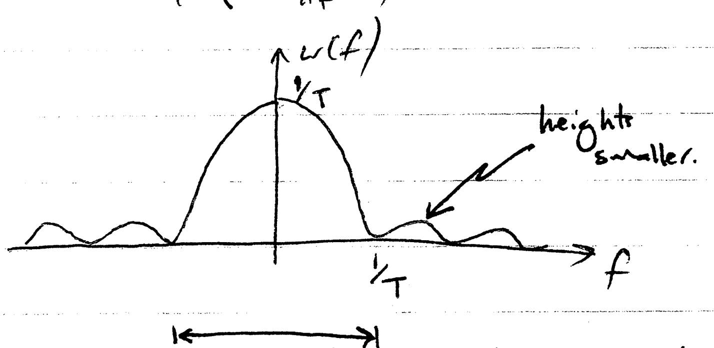


This is Gibbs phenomenon!

③

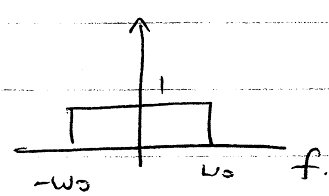


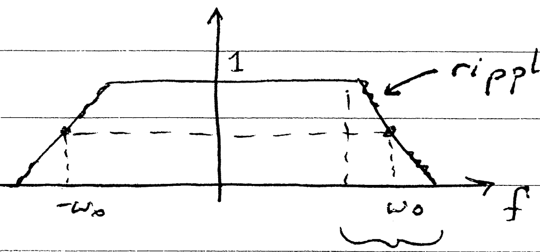
$\Rightarrow W(f) = \frac{1}{T}\left(\frac{\sin \pi f T}{\pi f}\right)^2$



width of main lobe wider than sinc of rectangular window.

then assume $X(f)$





There is no Gibbs phenomenon! (since the sinc. is positive)

(4) A freq. used window that represents a good tradeoff between having low side-lobe energy & not too much bluffing at the freq edges is the 'Hamming window' or ~~'cosine~~ 'raised cosine'

$\Rightarrow w(t) = \frac{1}{2}\left(1 + \cos\frac{\pi t}{T}\right)$

	Temporal width	Freq. width	~~Factor~~	Fall off
Rectangle	Wide	Narrowest	Fn discontinuous	$1/f$
Triangle	Medium	Medium	f' discont.	$1/f^2$
Raised cosine	Narrowest	Widest	f'' discont.	$1/f^3$.

$\Delta t \; \Delta f \geq \pi.$

Δt: width of $h(t)$ in TD

Δf: width of $h(t)$ in FD.