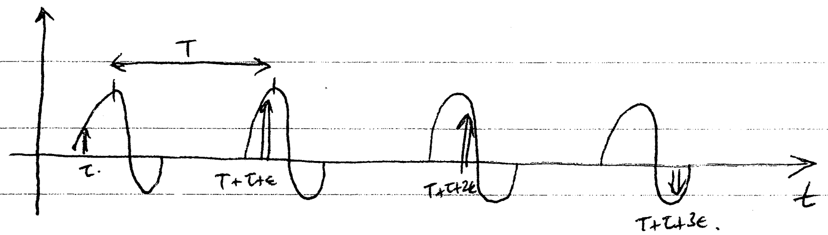
6.003 Recitation, Sections 3 & 4, Friday Apr. 30, 2010

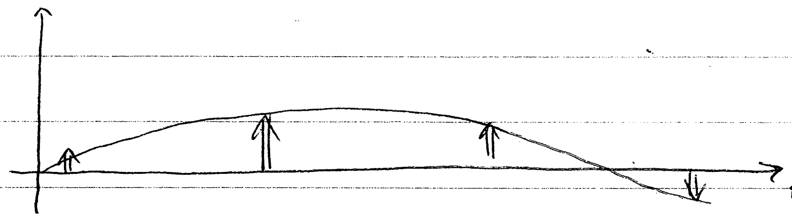
Today: ① Basic ideas: High speed in spite of 'low speed' sampling

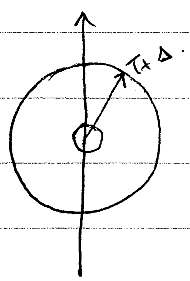
② Fourier analysis

③ Limits of operation based on nyquist & sampling.

①







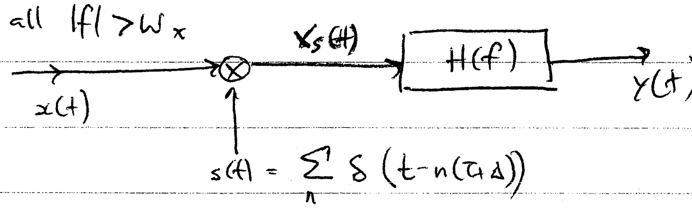
$\Delta \to T+\Delta$. i.e., there's a stretching factor. $x(t) \to x(at)$

$a = \frac{\Delta}{T+\Delta}$. $y(t) \sim x(at)$

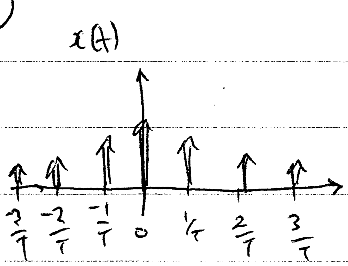
$|X(f)| = \phi$ for all $|f| > W_x$

$\forall |f| > W_Y$
$|H(f)| = \phi$

$W_Y < W_x$



②



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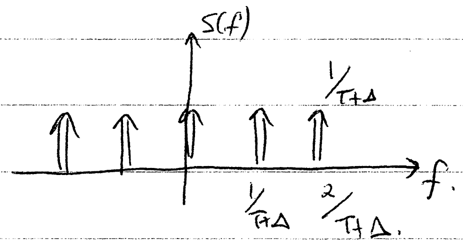
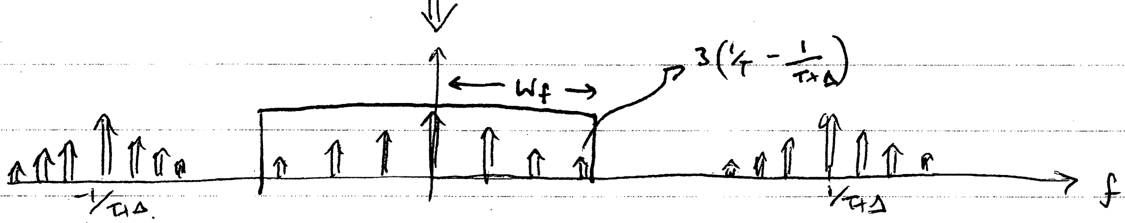


$\Downarrow$



Pf. $\delta\left(f-\frac{m}{T}\right) * \delta\left(f+\frac{n}{T+\Delta}\right) = \delta\left(f-\left(\frac{m}{T}-\frac{n}{T+\Delta}\right)\right)$

$$= \delta\left(f - \underbrace{\frac{m-n}{T+\Delta}}_{\text{center point with } n.} + \underbrace{\frac{m}{T}\left(\frac{\Delta}{T+\Delta}\right)}_{\text{stretched (version) function}}\right)$$

(3) $W_x \frac{\Delta}{T+\Delta} < W_y$ ← perfect reconstruction.

$W_y < \frac{1}{2(T+\Delta)}$ ← Prevent aliasing by Nyquist.

From both constraints, we find $W_x\left(\frac{\Delta}{T+\Delta}\right) < \frac{1}{2(T+\Delta)}$

$\Rightarrow \boxed{\Delta < \frac{1}{2W_x}}$ ← @least two sampling points for the highest harmonic.

The period T has to be large enough s.t. $W_x\left(\frac{\Delta/T}{1+\Delta/T}\right) < W_y$ $\boxed{\Rightarrow T > \Delta\left(\frac{W_x - W_y}{W_y}\right)}$