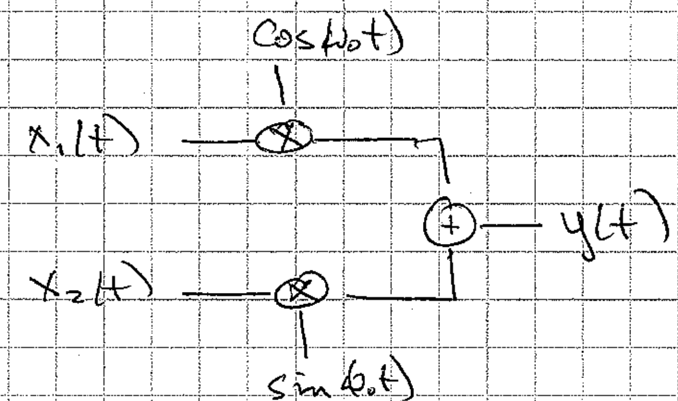
6003R May 5, 2010

Example Quadrature modulation

Basic idea is that cosine and sine functions are orthogonal, so that perhaps we might encode two signals instead of one.

Modulation scheme

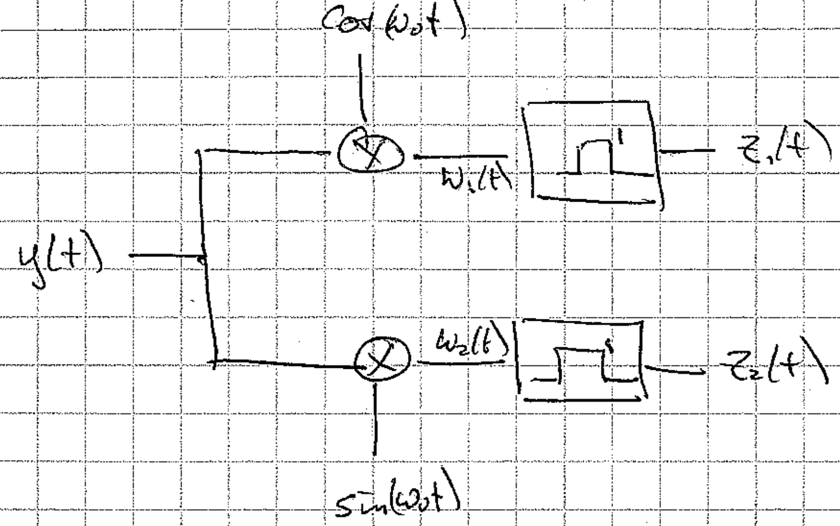


$$y(t) = x_1(t)\cos(\omega_0 t) + x_2(t)\sin(\omega_0 t)$$

$$Y(j\omega) = \frac{1}{2}\left[X_1(j(\omega+\omega_0)) + X_1(j(\omega-\omega_0))\right] + \frac{1}{2j}\left[X_2(j(\omega+\omega_0)) - X_2(j(\omega-\omega_0))\right]$$

Q1

Demodulation



$$w_1(t) = \cos(\omega_0 t)\left[x_1(t)\cos(\omega_0 t) + x_2(t)\sin(\omega_0 t)\right]$$

$$= \left[\frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}\right]\left[\frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}\right] x_1(t)$$

$$+ \left[\frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}\right]\left[\frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j}\right] x_2(t)$$

$$= \frac{1}{4}\left[e^{2j\omega_0 t} + 2 + e^{-2j\omega_0 t}\right] x_1(t)$$

$$+ \frac{1}{4j}\left[e^{2j\omega_0 t} - e^{-2j\omega_0 t}\right] x_2(t)$$

So low pass filter of $w_1(t) \Rightarrow \frac{1}{2}x_1(t)$

$$w_2(t) = \sin(\omega_0 t)\left[x_1(t)\cos\omega_0 t + x_2(t)\sin(\omega_0 t)\right]$$

$$= \left[\frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j}\right]\left[\frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}\right] x_1(t)$$

$$+ \left[\frac{e^{+j\omega_0 t} - e^{-j\omega_0 t}}{2j}\right]\left[\frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j}\right] x_2(t)$$

$$= \frac{1}{4j}\left[e^{2j\omega_0 t} - e^{-2j\omega_0 t}\right] x_1(t)$$

$$- \frac{1}{4}\left[e^{2j\omega_0 t} - 2 + e^{-2j\omega_0 t}\right] x_2(t)$$

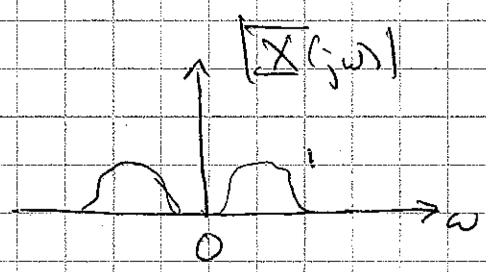
Low pass filtering $w_2(t) \Rightarrow \frac{1}{2} x_2(t)$

So, the scheme seems to work!

(Q3)

Example: Single-sideband modulation and demodulation

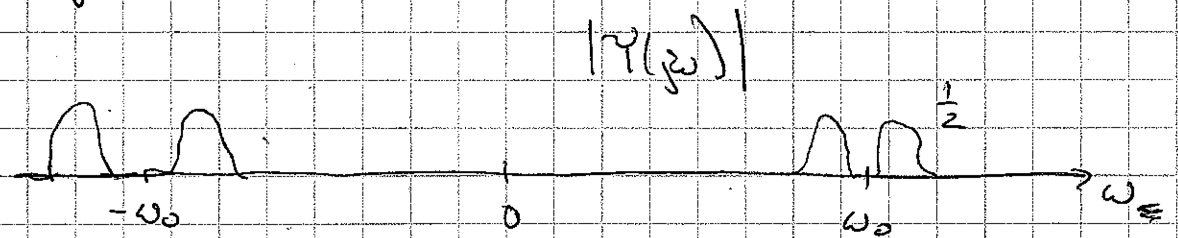
Thinking:



Transform of $x(t)$ has both positive and negative frequency components. We need both to construct the signal.

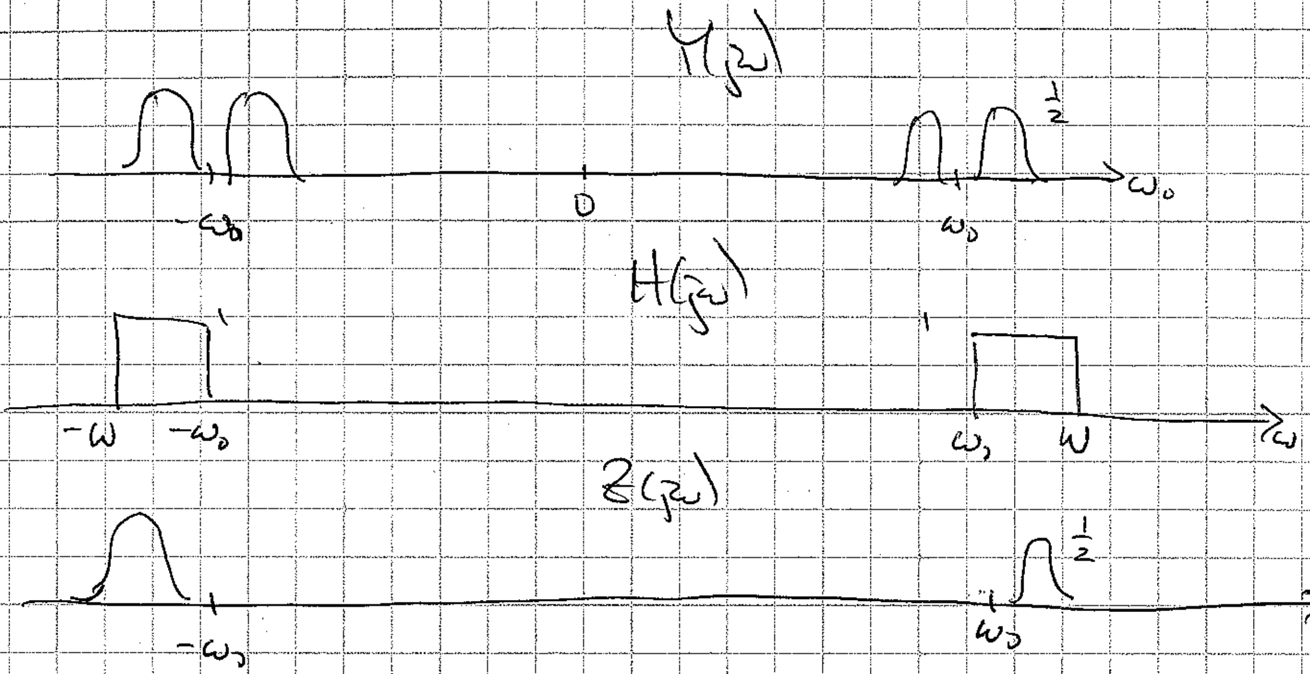
More thinking:

Consider transform of modulated $\cos(\omega_0 t)\, x(t) = y(t)$ signal.

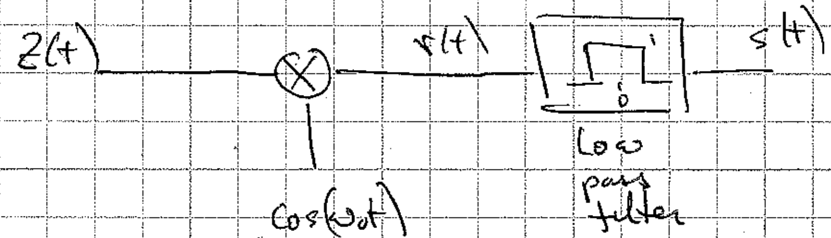


This one has 2 copies, each with + and - frequency components. It is possible to eliminate half of the modulated signal!

Use band-pass filter

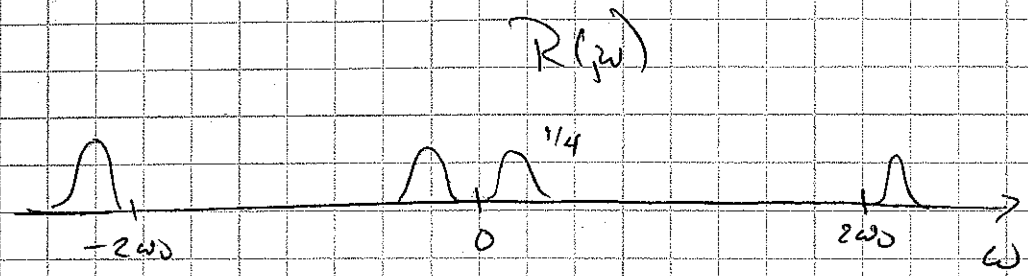


Demodulation

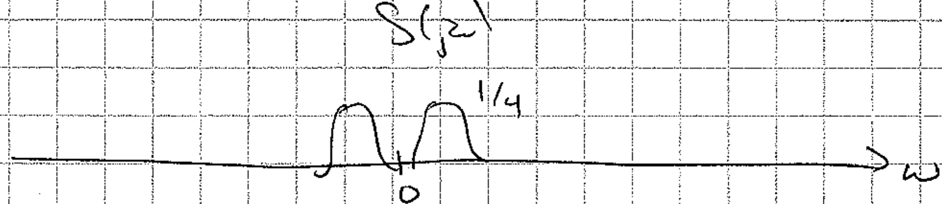


$$r(t) = z(t)\cos(\omega_0 t)$$

$$R(j\omega) = \frac{1}{2\pi} Z(j\omega) * \frac{1}{2}\left[2\pi\delta(\omega-\omega_0) + 2\pi\delta(\omega+\omega_0)\right]$$



Next, low-pass filter to recover $s(t)$



So, it looks like $s(t) = \frac{1}{4}x(t)$, and the demodulation is successful.

What is $z(t)$?

$$z(t) = \int_{-\infty}^{\infty} Z(j\omega) e^{j\omega t} \frac{d\omega}{2\pi}$$

$$= \int_{\omega_0}^{W} X(j(\omega-\omega_0)) e^{j\omega t} \frac{d\omega}{2\pi}$$

$$+ \int_{-W}^{-\omega_0} X(j(\omega+\omega_0)) e^{j\omega t} \frac{d\omega}{2\pi}$$

$$X(j(\omega-\omega_0)) = \int_{-\infty}^{\infty} e^{-j(\omega-\omega_0)t} x(t)\, dt$$

$$X(j(\omega+\omega_0)) = \int_{-\infty}^{\infty} e^{-j(\omega+\omega_0)t} x(t)\, dt$$

$$z(t) = \int_{\omega_0}^{W} \left[\int_{-\infty}^{\infty} dt' e^{-j(\omega-\omega_0)t'} x(t') \right] e^{j\omega t} \frac{d\omega}{2\pi}$$

$$+ \int_{-W}^{-\omega_0} \left[\int_{-\infty}^{\infty} dt' e^{j(\omega+\omega_0)t'} x(t') \right] e^{j\omega t} \frac{d\omega}{2\pi}$$

$$= \int_{-\infty}^{\infty} x(t') e^{+j\omega_0 t'} \left[\int_{\omega_0}^{W} e^{j\omega(t-t')} \frac{d\omega}{2\pi} \right] dt'$$

$$+ \int_{-\infty}^{\infty} x(t') e^{-j\omega_0 t'} \left[\int_{-W}^{\omega_0} e^{j\omega(t-t')} \frac{d\omega}{2\pi} \right. dt'$$

$$\int_{\omega_0}^{W} e^{j\omega(t-t')}\frac{d\omega}{2\pi} = \left.\frac{e^{j\omega(t-t')}}{2\pi j(t-t')}\right|_{\omega_0}^{W}$$

$$= \frac{e^{jW(t-t')} - e^{j\omega_0(t-t')}}{2\pi j(t-t')}$$

$$\int_{-W}^{-\omega_0} e^{j\omega(t-t')}\frac{d\omega}{2\pi} = \left.\frac{e^{j\omega(t-t')}}{2\pi j(t-t')}\right|_{-W}^{-\omega_0}$$

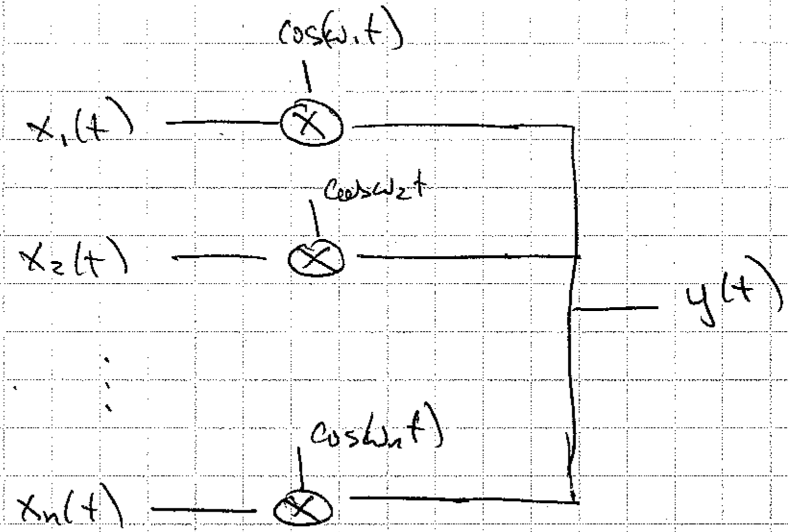
$$= \frac{e^{-j\omega_0(t-t')} - e^{-jW(t-t')}}{2\pi j(t-t')}$$

$$z(t) = \int_{-\infty}^{\infty} x(t')\, e^{j\omega_0 t}\left[\frac{e^{jW(t-t')} - e^{j\omega_0(t-t')}}{2\pi j(t-t')}\right] dt'$$

$$+ \int_{-\infty}^{\infty} x(t')\, e^{-j\omega_0 t}\left[\frac{e^{-j\omega_0(t-t')} - e^{-jW(t-t')}}{2\pi j(t-t')}\right] dt'$$

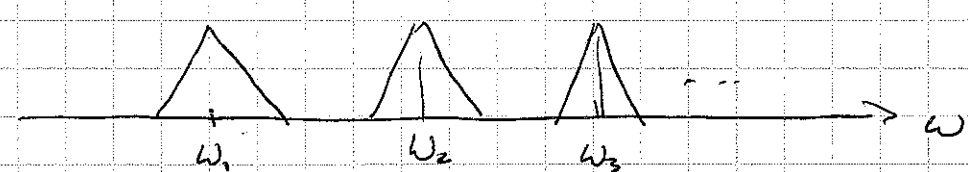
Example: Multiplexing

First try:

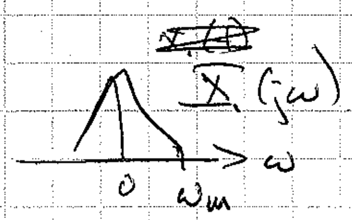


What do things look like in frequency?

$Y(j\omega)$



Thinking about frequency separation

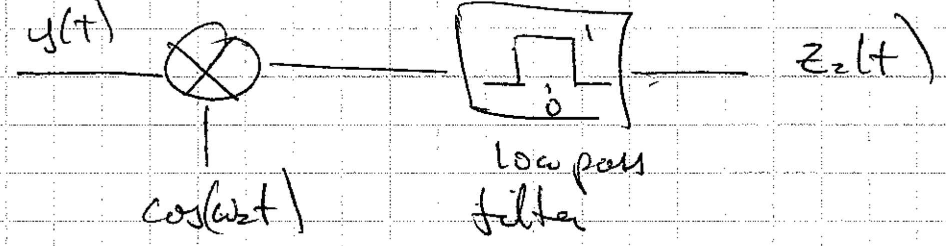


need $\omega_2 - \omega_1 > \omega_{m_2} + \omega_{m_1}$

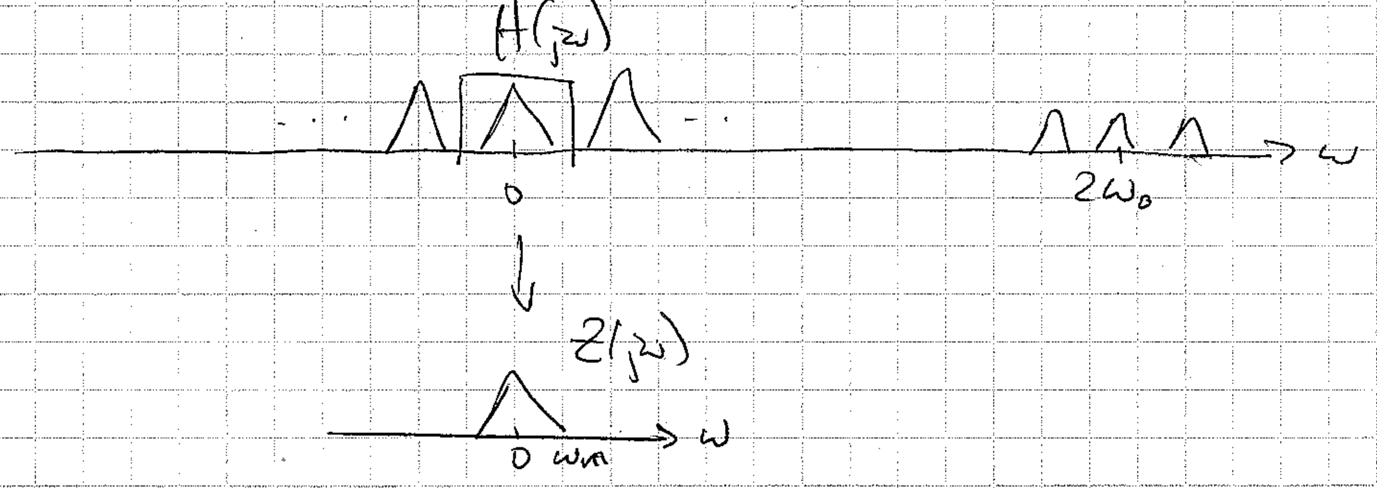
Maximum frequency

M1

<u>What about de-modulation?</u>



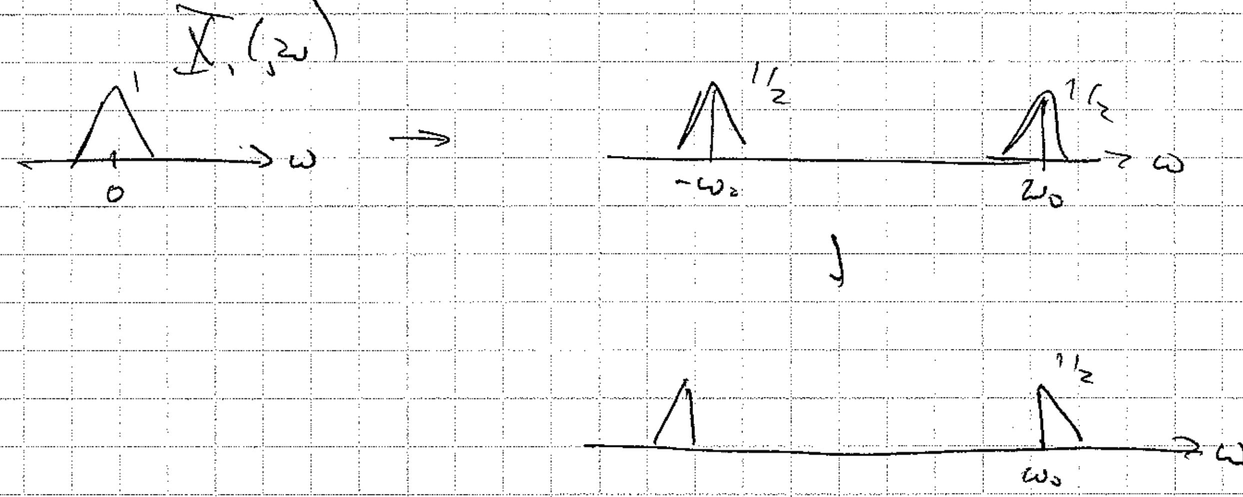
Look at it in frequency



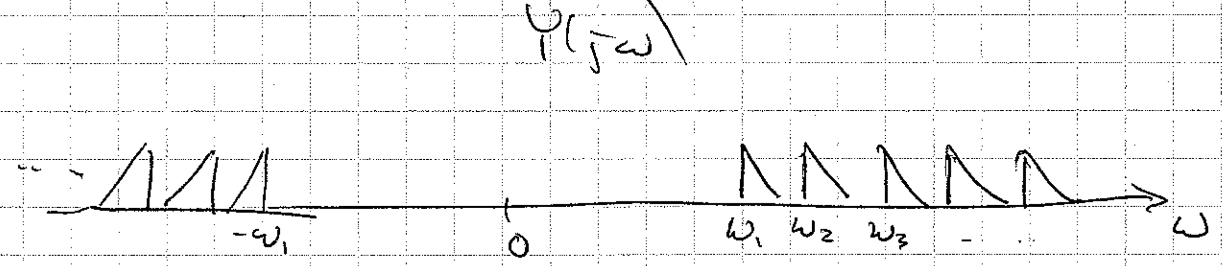
Simple ~~general~~ demodulation scheme that we have been using seems ~~can~~ to work ok here.

Can we fit in more signals into available bandwidth?

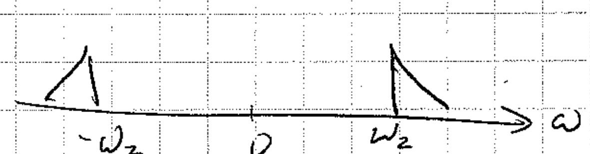
Make use of single side-band modulation



Then add to other single side band signals



Demodulation - first filter at high frequency to get



then modulate with $\cos(\omega_2 t)$ and low-pass filter.