

Table 1: Properties of the Laplace Transform

Property	Signal	Transform	ROC
	$x(t)$	$X(s)$	R
	$x_1(t)$	$X_1(s)$	R_1
	$x_2(t)$	$X_2(s)$	R_2
Linearity	$ax_1(t) + bx_2(t)$	$aX_1(s) + bX_2(s)$	At least $R_1 \cap R_2$
Time shifting	$x(t - t_0)$	$e^{-st_0} X(s)$	R
Shifting in the s -Domain	$e^{s_0 t} x(t)$	$X(s - s_0)$	Shifted version of R [i.e., s is in the ROC if $(s - s_0)$ is in R]
Time scaling	$x(at)$	$\frac{1}{ a } X\left(\frac{s}{a}\right)$	“Scaled” ROC (i.e., s is in the ROC if (s/a) is in R)
Conjugation	$x^*(t)$	$X^*(s^*)$	R
Convolution	$x_1(t) * x_2(t)$	$X_1(s)X_2(s)$	At least $R_1 \cap R_2$
Differentiation in the Time Domain	$\frac{d}{dt} x(t)$	$sX(s)$	At least R
Differentiation in the s -Domain	$-tx(t)$	$\frac{d}{ds} X(s)$	R
Integration in the Time Domain	$\int_{-\infty}^t x(\tau) d(\tau)$	$\frac{1}{s} X(s)$	At least $R \cap \{\Re\{s\} > 0\}$

Initial- and Final Value Theorems

If $x(t) = 0$ for $t < 0$ and $x(t)$ contains no impulses or higher-order singularities at $t = 0$, then

$$x(0^+) = \lim_{s \rightarrow \infty} sX(s)$$

If $x(t) = 0$ for $t < 0$ and $x(t)$ has a finite limit as $t \rightarrow \infty$, then

$$\lim_{t \rightarrow \infty} x(t) = \lim_{s \rightarrow 0} sX(s)$$

Table 2: Laplace Transforms of Elementary Functions

Signal	Transform	ROC
1. $\delta(t)$	1	All s
2. $u(t)$	$\frac{1}{s}$	$\Re\{s\} > 0$
3. $-u(-t)$	$\frac{1}{s}$	$\Re\{s\} < 0$
4. $\frac{t^{n-1}}{(n-1)!}u(t)$	$\frac{1}{s^n}$	$\Re\{s\} > 0$
5. $-\frac{t^{n-1}}{(n-1)!}u(-t)$	$\frac{1}{s^n}$	$\Re\{s\} < 0$
6. $e^{-\alpha t}u(t)$	$\frac{1}{s + \alpha}$	$\Re\{s\} > -\Re\{\alpha\}$
7. $-e^{-\alpha t}u(-t)$	$\frac{1}{s + \alpha}$	$\Re\{s\} < -\Re\{\alpha\}$
8. $\frac{t^{n-1}}{(n-1)!}e^{-\alpha t}u(t)$	$\frac{1}{(s + \alpha)^n}$	$\Re\{s\} > -\Re\{\alpha\}$
9. $-\frac{t^{n-1}}{(n-1)!}e^{-\alpha t}u(-t)$	$\frac{1}{(s + \alpha)^n}$	$\Re\{s\} < -\Re\{\alpha\}$
10. $\delta(t - T)$	e^{-sT}	All s
11. $[\cos \omega_0 t]u(t)$	$\frac{s}{s^2 + \omega_0^2}$	$\Re\{s\} > 0$
12. $[\sin \omega_0 t]u(t)$	$\frac{\omega_0}{s^2 + \omega_0^2}$	$\Re\{s\} > 0$
13. $[e^{-\alpha t} \cos \omega_0 t]u(t)$	$\frac{s + \alpha}{(s + \alpha)^2 + \omega_0^2}$	$\Re\{s\} > -\Re\{\alpha\}$
14. $[e^{-\alpha t} \sin \omega_0 t]u(t)$	$\frac{\omega_0}{(s + \alpha)^2 + \omega_0^2}$	$\Re\{s\} > -\Re\{\alpha\}$
15. $u_n(t) = \frac{d^n \delta(t)}{dt^n}$	s^n	All s
16. $u_{-n}(t) = \underbrace{u(t) * \dots * u(t)}_{n \text{ times}}$	$\frac{1}{s^n}$	$\Re\{s\} > 0$

Table 3: Properties of the z -Transform

Property	Sequence	Transform	ROC
	$x[n]$	$X(z)$	R
	$x_1[n]$	$X_1(z)$	R_1
	$x_2[n]$	$X_2(z)$	R_2
Linearity	$ax_1[n] + bx_2[n]$	$aX_1(z) + bX_2(z)$	At least the intersection of R_1 and R_2
Time shifting	$x[n - n_0]$	$z^{-n_0}X(z)$	R except for the possible addition or deletion of the origin
Scaling in the z -Domain	$e^{j\omega_0 n}x[n]$	$X(e^{-j\omega_0}z)$	R
	$z_0^n x[n]$	$X\left(\frac{z}{z_0}\right)$	$z_0 R$
	$a^n x[n]$	$X(a^{-1}z)$	Scaled version of R (i.e., $ a R =$ the set of points $\{ a z\}$ for z in R)
Time reversal	$x[-n]$	$X(z^{-1})$	Inverted R (i.e., $R^{-1} =$ the set of points z^{-1} where z is in R)
Time expansion	$x_{(k)}[n] = \begin{cases} x[r], & n = rk \\ 0, & n \neq rk \end{cases}$ for some integer r	$X(z^k)$	$R^{1/k}$ (i.e., the set of points $z^{1/k}$ where z is in R)
Conjugation	$x^*[n]$	$X^*(z^*)$	R
Convolution	$x_1[n] * x_2[n]$	$X_1(z)X_2(z)$	At least the intersection of R_1 and R_2
First difference	$x[n] - x[n - 1]$	$(1 - z^{-1})X(z)$	At least the intersection of R and $ z > 0$
Accumulation	$\sum_{k=-\infty}^n x[k]$	$\frac{1}{1-z^{-1}}X(z)$	At least the intersection of R and $ z > 1$
Differentiation in the z -Domain	$nx[n]$	$-z\frac{dX(z)}{dz}$	R

Initial Value Theorem
 If $x[n] = 0$ for $n < 0$, then
 $x[0] = \lim_{z \rightarrow \infty} X(z)$

Table 4: **Some Common z -Transform Pairs**

Signal	Transform	ROC
1. $\delta[n]$	1	All z
2. $u[n]$	$\frac{1}{1-z^{-1}}$	$ z > 1$
3. $-u[-n-1]$	$\frac{1}{1-z^{-1}}$	$ z < 1$
4. $\delta[n-m]$	z^{-m}	All z except 0 (if $m > 0$) or ∞ (if $m < 0$)
5. $\alpha^n u[n]$	$\frac{1}{1-\alpha z^{-1}}$	$ z > \alpha $
6. $-\alpha^n u[-n-1]$	$\frac{1}{1-\alpha z^{-1}}$	$ z < \alpha $
7. $n\alpha^n u[n]$	$\frac{\alpha z^{-1}}{(1-\alpha z^{-1})^2}$	$ z > \alpha $
8. $-n\alpha^n u[-n-1]$	$\frac{\alpha z^{-1}}{(1-\alpha z^{-1})^2}$	$ z < \alpha $
9. $[\cos \omega_0 n]u[n]$	$\frac{1 - [\cos \omega_0]z^{-1}}{1 - [2 \cos \omega_0]z^{-1} + z^{-2}}$	$ z > 1$
10. $[\sin \omega_0 n]u[n]$	$\frac{[\sin \omega_0]z^{-1}}{1 - [2 \cos \omega_0]z^{-1} + z^{-2}}$	$ z > 1$
11. $[r^n \cos \omega_0 n]u[n]$	$\frac{1 - [r \cos \omega_0]z^{-1}}{1 - [2r \cos \omega_0]z^{-1} + r^2 z^{-2}}$	$ z > r$
12. $[r^n \sin \omega_0 n]u[n]$	$\frac{[r \sin \omega_0]z^{-1}}{1 - [2r \cos \omega_0]z^{-1} + r^2 z^{-2}}$	$ z > r$