(1) Given the following difference equation:

$$y[n] = y[n-1] + 2x[n]$$

1. Draw a block diagram for this system.

2. Will this system converge, diverge, or neither?

3. Derive the operator representation for this system. What are the poles?

4. If $y[-1] = 41$, and $x[n] = n$, what are the first five outputs of this system?

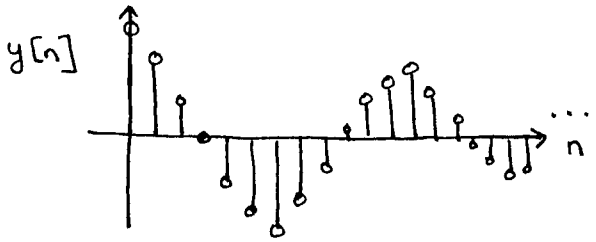
5. Write code to generate the system function for this system.

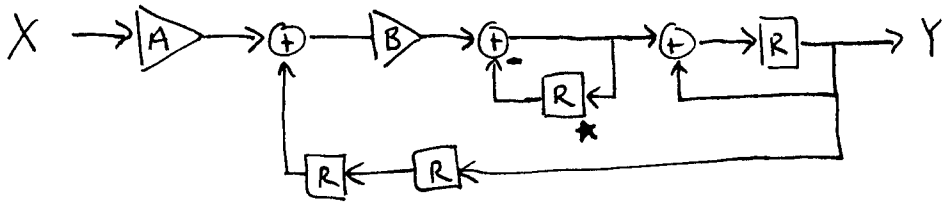
6. Write code to implement this system as a state machine.

(2) How many poles does each of the following systems have? Are the systems stable?

1. $\frac{Y}{X} = \frac{-2 - R + 3R^2}{2R^2 - R}$

2. $y[n] = y[n-1] + y[n-3] + x[n-2] - 2x[n-4]$

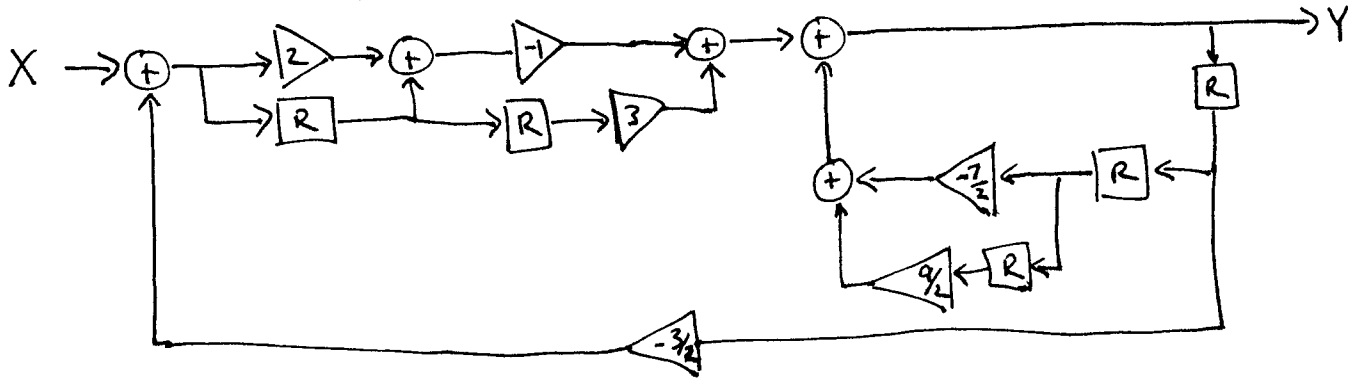
3. 

4. 

4a. How would the answer change if we moved the delay marked by ★ into the forward path?

4b. For the original system, how many poles can be real?

(3) Simplify this block diagram:



(4) Thermoelectric (a.k.a. Peltier) devices can transfer heat in one of two directions based on the direction of current flow into/out-of the device. FridgOven Inc., an MIT-based startup, wants to use these devices to make a kitchen appliance that will keep a pre-made dinner cool during the day, then heat it up automatically in the evening. As a recent 6.01 graduate, you've been hired to design the control system for this appliance. You are told the following:

1. The input is a desired temperature $T_d[n]$.
2. The rate of temperature change in the appliance is proportional to the rate of heat transfer, which in turn is proportional to the current supplied to the Peltier.
3. A thermometer reports the average temperature in the appliance over the previous minute (i.e., approximately the temperature 30 seconds ago).

1. Draw a block diagram for this system.

2. Derive $\frac{T_o}{T_d}$, where T_o is the temperature of the appliance.

3. What are the poles of this system? Plot them as a function of gain.

4. For what gains is this system stable? For what gains are the poles real?

5. Plot the time response of this system for a step increase in Td for the largest gain for which poles are real.

6. Plot the time response to the same input for twice the gain used in part 5.