Lecture 21

Frequency Response of Amplifiers (I)

COMMON-SOURCE AMPLIFIER

Outline

1. Intrinsic Frequency Response of MOSFETs
2. Frequency Response of Common-Source Amplifier
3. Miller Effect

Reading Assignment:
Howe and Sodini, Chapter 10, Sections 10.1-10.4

Summary of Key Concepts

- f_T (***short-circuit current-gain cut-off frequency***)
  - figure of merit to assess intrinsic frequency response of transistors
- In MOSFET, to first order

$$\mathbf{f_T} = \frac{1}{2\pi\tau_T}$$

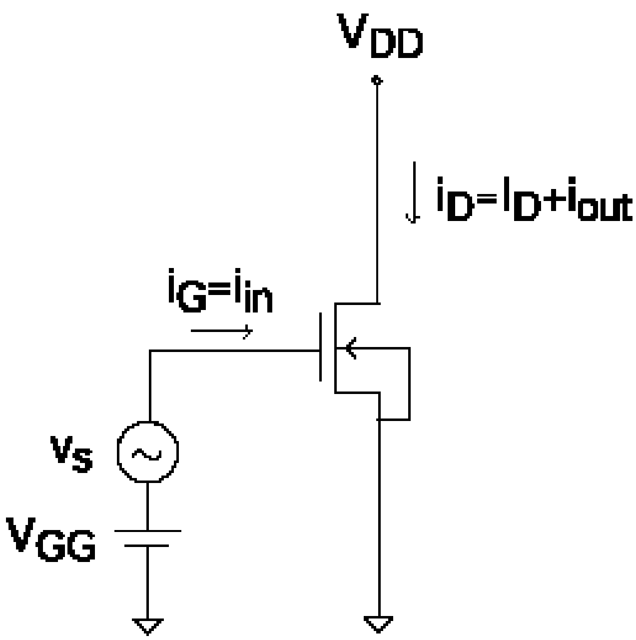
  - where τ_T is the ***transit time*** of electrons through the channel
- In common-source amplifier, voltage gain rolls off at high frequency because C_{gs} and C_{gd} short circuit the input
- In common-source amplifier, effect of C_{gd} on bandwidth is amplified by amplifier voltage gain.
- ***Miller Effect*** is the effect of capacitance across voltage gain nodes magnified by the voltage gain
  - ***trade-off between gain and bandwidth***

1. Intrinsic Frequency Response of MOSFET

How does one assess the intrinsic frequency response of a transistor?

$f_\tau \equiv$ short-circuit current-gain cut-off frequency [GHz]

Consider a MOSFET biased in saturation regime with small-signal source applied to gate:

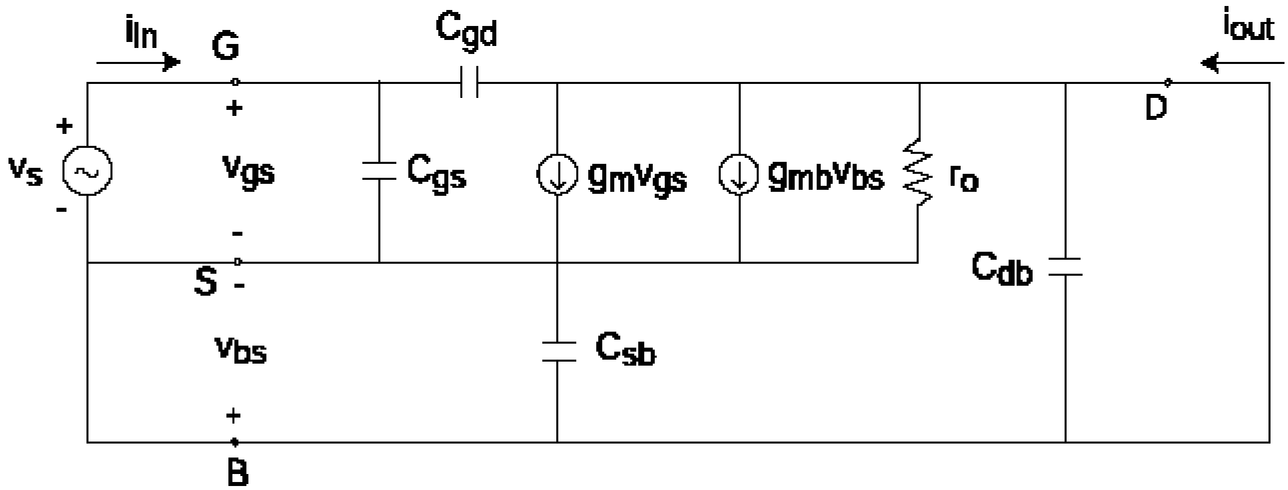


v_s at input $\Rightarrow i_{out}$ at output : transistor effect
$\Rightarrow i_{in}$ at input : due to gate capacitance

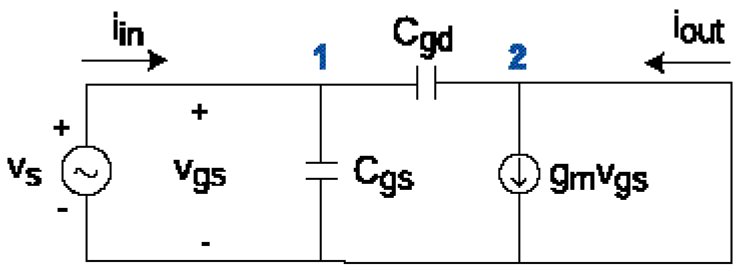
Frequency dependence : $\mathbf{f} \uparrow \Rightarrow \mathbf{i_{in}} \uparrow \Rightarrow \left| \frac{\mathbf{i_{out}}}{\mathbf{i_{in}}} \right| \downarrow$

$$\mathbf{f_T} \equiv \text{frequency at which} \left| \frac{i_{out}}{i_{in}} \right| = 1$$

Complete small-signal model in saturation



$v_{bs}=0$



Node 1:

$$\mathbf{i_{in} - j\omega C_{gs} v_{gs} - j\omega C_{gd} v_{gs} = 0}$$

$$\Rightarrow \quad \mathbf{i_{in} = j\omega (C_{gs} + C_{gd}) v_{gs}}$$

Node 2:

$$\mathbf{i_{out} - g_m v_{gs} + j\omega C_{gd} v_{gs} = 0}$$

$$\Rightarrow \quad \mathbf{i_{out} = (g_m - j\omega C_{gd}) v_{gs}}$$

Current Gain

$$\mathbf{h}_{21} = \frac{\mathbf{i_{out}}}{\mathbf{i_{in}}} = \frac{\mathbf{g_m} - \mathbf{j}\omega\mathbf{C_{gd}}}{\mathbf{j}\omega\left(\mathbf{C_{gs}} + \mathbf{C_{gd}}\right)}$$

Magnitude of h_{21}:

$$\left|\mathbf{h}_{21}\right| = \frac{\sqrt{\mathbf{g_m}^2 + \omega^2\mathbf{C_{gd}}^2}}{\omega\left(\mathbf{C_{gs}} + \mathbf{C_{gd}}\right)}$$

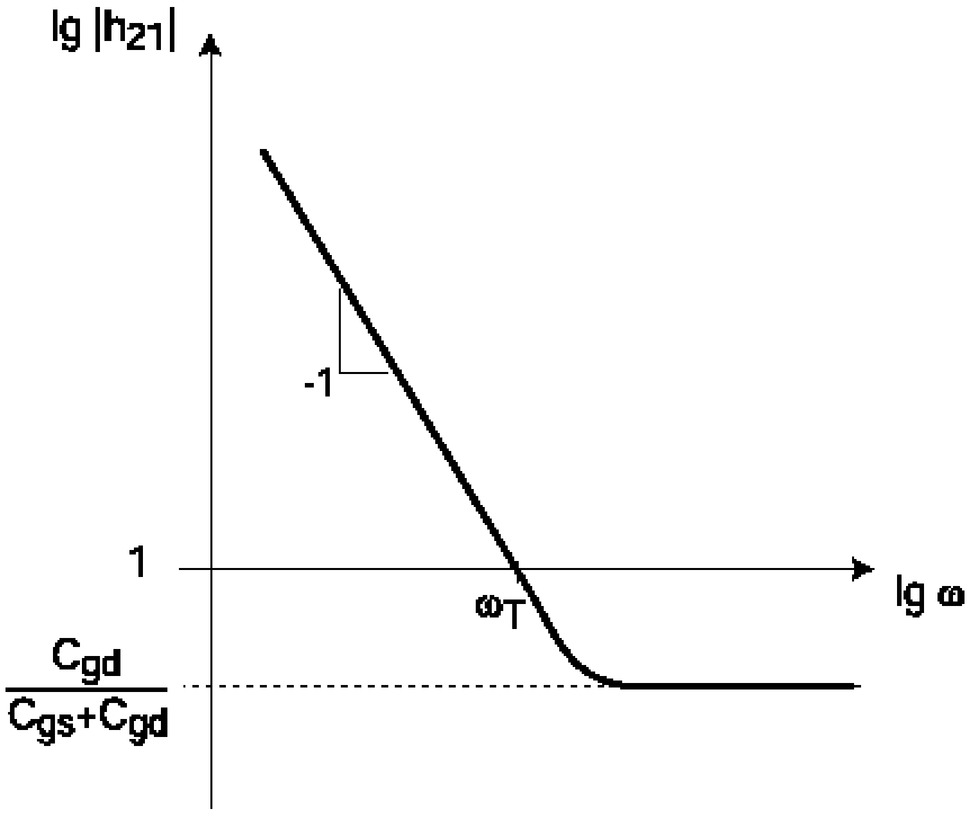
Low Frequency, $\omega << \frac{\mathbf{g_m}}{\mathbf{C_{gd}}}$

$$\left|\mathbf{h}_{21}\right| \approx \frac{\mathbf{g_m}}{\omega\left(\mathbf{C_{gs}} + \mathbf{C_{gd}}\right)}$$

High Frequency, $\omega >> \frac{\mathbf{g_m}}{\mathbf{C_{gd}}}$

$$\left|\mathbf{h}_{21}\right| \approx \frac{\mathbf{C_{gd}}}{\mathbf{C_{gs}} + \mathbf{C_{gd}}}$$

Current Gain (contd.)



$|h_{21}|$ becomes unity at:

$$\omega_T = 2\pi \mathbf{f}_T = \frac{\mathbf{g_m}}{\mathbf{C_{gs}} + \mathbf{C_{gd}}}$$

Then:

$$\mathbf{f}_T = \frac{\mathbf{g_m}}{2\pi(\mathbf{C_{gs}} + \mathbf{C_{gd}})}$$

Current Gain (contd…)

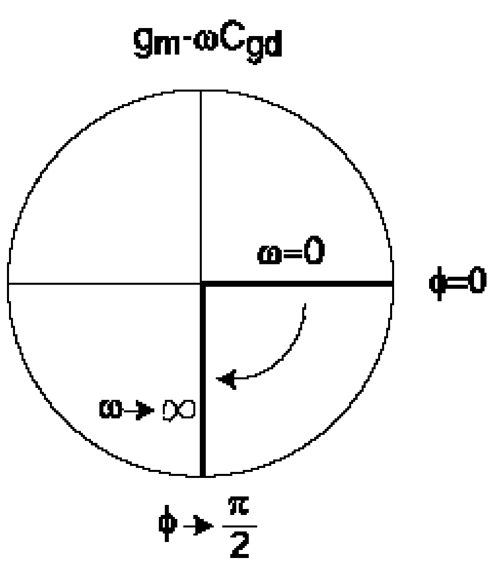
Phase of h_{21}:

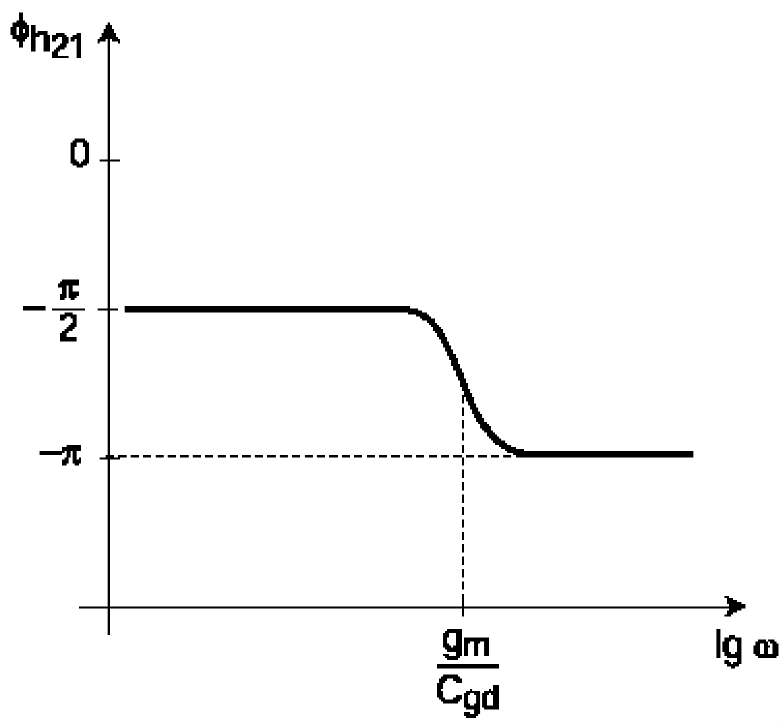
Low Frequency, $\omega \ll \frac{g_m}{C_{gd}}$

$$\phi_{h_{21}} \approx -\frac{\pi}{2}$$

High Frequency, $\omega \gg \frac{g_m}{C_{gd}}$

$$\phi_{h_{21}} \approx -\pi$$





Physical Interpretation of f_T:

Consider:

$$\frac{1}{2\pi \mathbf{f_T}} = \frac{\mathbf{C_{gs}} + \mathbf{C_{gd}}}{\mathbf{g_m}} \approx \frac{\mathbf{C_{gs}}}{\mathbf{g_m}}$$

Plug in device physics expressions for C_{gs} and g_m:

$$\frac{1}{2\pi \mathbf{f_T}} \approx \frac{\mathbf{C_{gs}}}{\mathbf{g_m}} = \frac{\frac{2}{3}\mathbf{LWC_{ox}}}{\frac{\mathbf{W}}{\mathbf{L}}\mu \mathbf{C_{ox}}\left(\mathbf{V_{GS}} - \mathbf{V_T}\right)} = \frac{\mathbf{L}}{\mu \frac{3}{2} \frac{\mathbf{V_{GS}} - \mathbf{V_T}}{\mathbf{L}}}$$

or

$$\frac{1}{2\pi \mathbf{f_T}} \approx \frac{\mathbf{L}}{\mu \langle \mathbf{E_{channel}} \rangle} = \frac{\mathbf{L}}{\langle \mathbf{v_{channel}} \rangle} = \tau_{\mathbf{T}}$$

$\tau_T \equiv$ ***transit time*** from source to drain [s]

Then:

$$\mathbf{f_T} \approx \frac{1}{2\pi\tau_{\mathbf{T}}}$$

f_T gives an idea of the ***intrinsic delay*** of the transistor: Good first order figure of merit fro frequency response

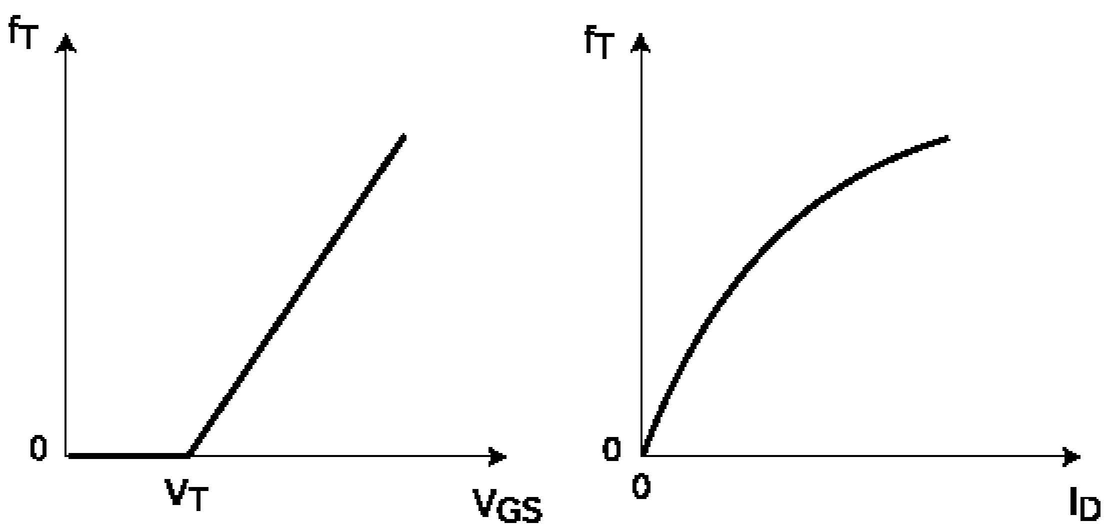
Frequency Response of MOSFET

How do we reduce τ_T and increase f_T?

- $L\downarrow$: trade-off cost
- $(V_{GS} - V_T)\uparrow \Rightarrow I_D\uparrow$: trade-off power
- $\mu\uparrow$: hard to do
- Note: f_T is independent of W

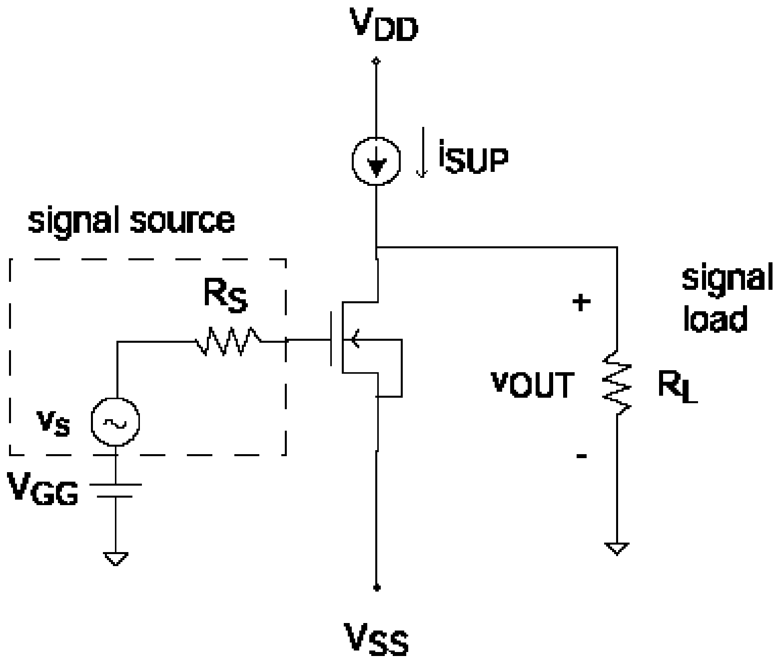
Impact of bias point on f_T:

$$\mathbf{f_T} = \frac{\mathbf{g_m}}{2\pi(\mathbf{C_{gs}} + \mathbf{C_{gd}})} = \frac{\frac{\mathbf{W}}{\mathbf{L}}\mu\mathbf{C_{ox}}(\mathbf{V_{GS}} - \mathbf{V_T})}{2\pi(\mathbf{C_{gs}} + \mathbf{C_{gd}})} = \frac{\sqrt{2\frac{\mathbf{W}}{\mathbf{L}}\mu\mathbf{C_{ox}}\mathbf{I_D}}}{2\pi(\mathbf{C_{gs}} + \mathbf{C_{gd}})}$$

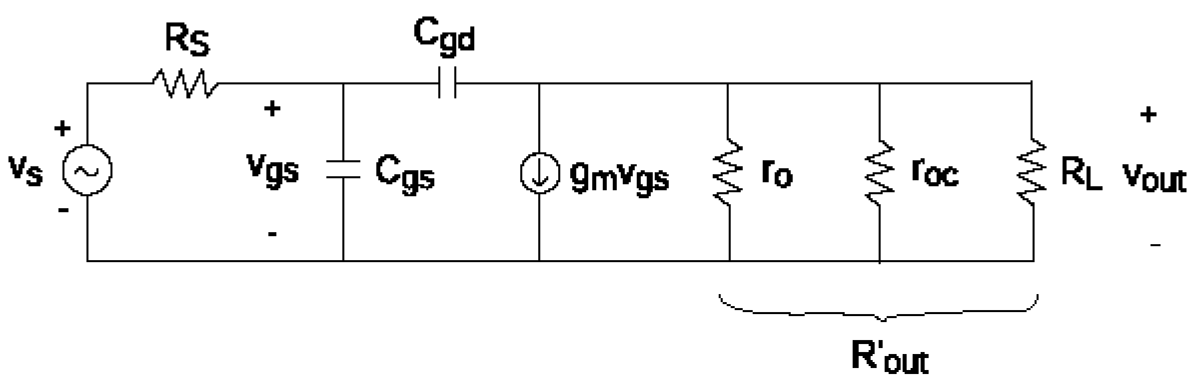


In a typical MOSFET at typical bias points: $f_T \approx$1-25 GHz

2. Frequency Response of the Common-Source Amplifier

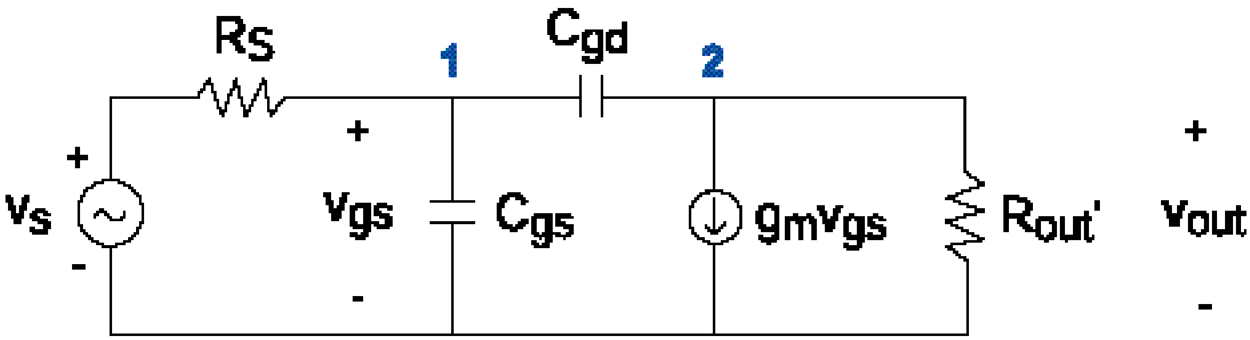


Small-signal equivalent circuit model:



Low-frequency voltage gain:

$$\mathbf{A_{v,LF}} = \frac{\mathbf{v_{out}}}{\mathbf{v_s}} = -\mathbf{g_m}\left(\mathbf{r_o} // \mathbf{r_{oc}} // \mathbf{R_L}\right) = -\mathbf{g_m}\mathbf{R'_{out}}$$



Node 1: $$\frac{v_s - v_{gs}}{R_S} - j\omega C_{gs} v_{gs} - j\omega C_{gd}\left(v_{gs} - v_{out}\right) = 0$$

Node 2: $$-g_m v_{gs} + j\omega C_{gd}\left(v_{gs} - v_{out}\right) - \frac{v_{out}}{R'_{out}} = 0$$

Solve for v_{gs} in **2**:

$$v_{gs} = v_{out}\frac{j\omega C_{gd} + \frac{1}{R'_{out}}}{j\omega C_{gd} - g_m}$$

Plug v_{gs} in **1** and solve for v_{out}/v_s:

$$A_v = \frac{-\left(g_m - j\omega C_{gd}\right)R'_{out}}{1 + j\omega R_S\left\{C_{gs} + C_{gd}\left[1 + R'_{out}\left(\frac{1}{R_S} + g_m\right)\right]\right\} - \omega^2 R_S R'_{out} C_{gs} C_{gd}}$$

Check that for $\omega = 0$, $A_{v,LF} = -g_m R'_{out}$

Simplify

1. Operate at $\omega << \omega_T = \dfrac{g_m}{C_{gs} + C_{gd}}$

$$\Rightarrow \quad g_m >> \omega\left(C_{gs} + C_{gd}\right) > \omega C_{gs},\ \omega C_{gd}$$

2. Assume g_m high enough so that

$$\frac{1}{R_s} + g_m \approx g_m$$

3. Compare ω^2 term in the denominator of A_v with a portion of the ω term:

$$\frac{\omega^2 R_S R'_{out} C_{gs} C_{gd}}{\omega R_S C_{gd} R'_{out} g_m} = \frac{\omega C_{gs}}{g_m} << 1$$

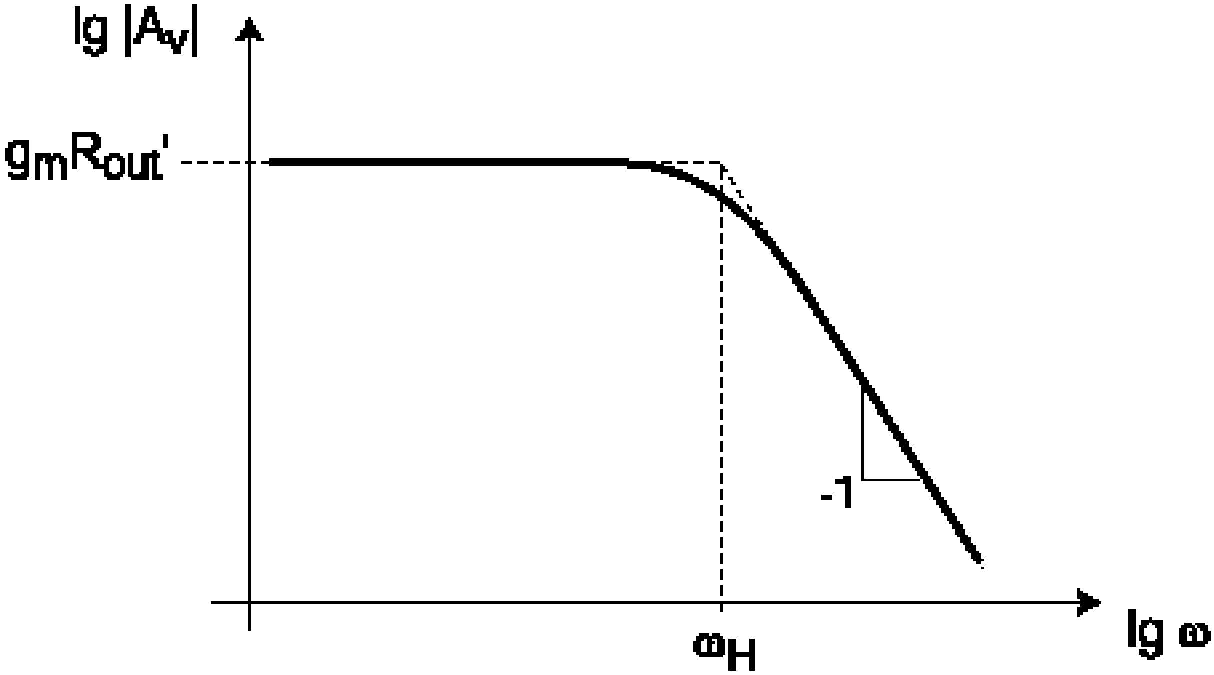
Then:

$$A_v = \frac{-g_m R'_{out}}{1 + j\omega R_S\left\{C_{gs} + C_{gd}\left[1 + g_m R'_{out}\right]\right\}}$$

This has the form:

$$A_v(\omega) = \frac{A_{v,LF}}{1 + j\dfrac{w}{w_H}}$$

Amplifier Frequency Response



At $\omega = \omega_H$:

$$|\mathbf{A_v}(\omega_H)| = \frac{\mathbf{A_{v,LF}}}{\sqrt{2}}$$

ω_H gives an idea of frequency beyond which $|A_v|$ starts rolling off quickly ⇒ ***bandwidth***

For the common source amplifier

$$\omega_H = \frac{1}{\mathbf{R_S}\left[\mathbf{C_{gs}} + \mathbf{C_{gd}}\left(1 + \mathbf{g_m R'_{out}}\right)\right]}$$

Frequency response of common-source amplifier limited by C_{gs} and C_{gd} shorting out the input.

Amplifier Frequency Response (Contd.)

We can re-write as:

$$\omega_H = \frac{1}{R_S\left[C_{gs} + C_{gd}\left(1 + \left|A_{v,LF}\right|\right)\right]}$$

To improve bandwidth,

- C_{gs}, C_{gd} $\downarrow \Rightarrow$ small transistor with low parasitics
- $|A_{v,LF}| \downarrow \Rightarrow$ do not use more gain than necessary

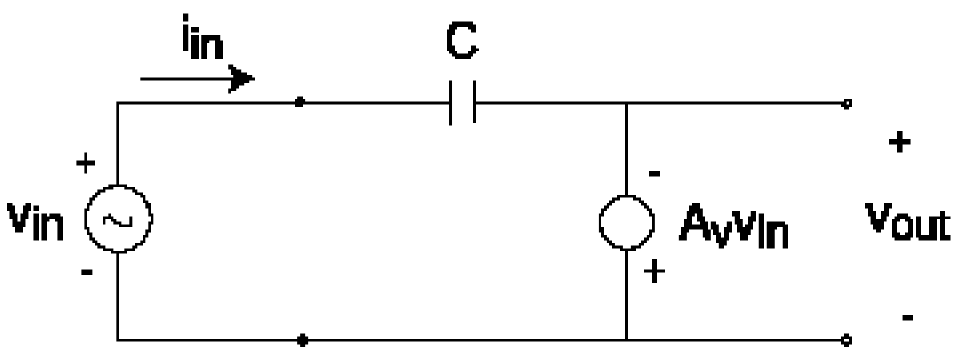
But…
Effect of C_{gd} on ω_H is being magnified by 1+ | $|A_{v,LF}|$

Why???

3. Miller Effect

In common-source amplifier, C_{gd} looks much bigger than it really is.

Consider a simple voltage-gain stage:



What is the input impedance?

$$\mathbf{i_{in}} = (\mathbf{v_{in}} - \mathbf{v_{out}})\mathbf{j\omega C}$$

But

$$\mathbf{v_{out}} = -\mathbf{A_v v_{in}}$$

Then:

$$\mathbf{i_{in}} = \mathbf{j\omega C}(1 + \mathbf{A_v})\mathbf{v_{in}}$$

Miller Effect (contd.)

or

$$\mathbf{Z_{in}} = \frac{\mathbf{v_{in}}}{\mathbf{i_{in}}} = \frac{1}{\mathbf{j\omega C}(1 + \mathbf{A_v})}$$

Looking in from the input, C appears bigger than it really is. This is called ***Miller Effect***.

When a capacitor is located across nodes where there is a voltage gain, its effect on bandwidth is amplified by the voltage gain ⇒ ***Miller Capacitance***

Why?

$$\mathbf{v_{in}} \uparrow \Rightarrow \mathbf{v_{out}} = -\mathbf{A_v v_{in}} \uparrow\uparrow \Rightarrow (\mathbf{v_{in}} - \mathbf{v_{out}}) \uparrow\uparrow \Rightarrow \mathbf{i_{in}} \uparrow\uparrow$$

In amplifier stages with voltage gain, it is critical to have small capacitance across nodes that have voltage gain.

As a result of the Miller effect, there is a fundamental ***gain-bandwidth trade-off*** in amplifiers.

What did we learn today?

Summary of Key Concepts

- f_T (***short-circuit current-gain cut-off frequency***)
  - frequency at which the short circuit current gain becomes 1.
  - figure of merit to assess intrinsic frequency response of transistors
- In MOSFET, to first order

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- In common-source amplifier, effect of C_{gd} on bandwidth is amplified by amplifier voltage gain.
- ***Miller Effect*** is the effect of capacitance across voltage gain nodes magnified by the voltage gain
  - ***trade-off between gain and bandwidth***