April 14, 1999 - Quiz #2

Name: Solutions

Recitation: ____________________

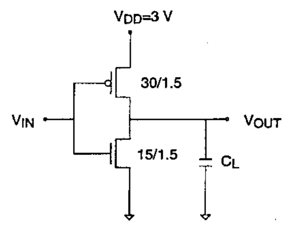
| problem | grade |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| total | |

**General guidelines** (please read carefully before starting):

- Make sure to write your name on the space designated above.
- **Open book**: you can use any material you wish.
- All answers should be given in the space provided. Please do not turn in any extra material. If you need more space, use the back page.
- You have **120 minutes** to complete your quiz.
- Make reasonable approximations and *state them*, i.e. quasi-neutrality, depletion approximation, etc.
- Partial credit will be given for setting up problems without calculations. NO credit will be given for answers without reasons.
- Use the symbols utilized in class for the various physical parameters, i.e. $\mu_n$, $I_D$, $\beta_F$, etc.
- Every numerical answer must have the proper units next to it. Points will be subtracted for answers without units or with wrong units.
- Use the following fundamental constants and physical parameters for silicon and silicon dioxide at room temperature:

$n_i = 1 \times 10^{10}\ cm^{-3}$
$kT/q = 0.026\ V$
$q = 1.60 \times 10^{-19}\ C$
$\epsilon_s = 1.05 \times 10^{-12}\ F/cm$
$\epsilon_{ox} = 3.45 \times 10^{-13}\ F/cm$

1. *(25 points)* Below is a CMOS inverter with the following device data: $t_{ox} = 15\ nm$, $\mu_n = 300\ cm^2/V \cdot s$, $\mu_p = 100\ cm^2/V \cdot s$, $V_{Tp} = -1\ V$, $\lambda_n = \lambda_p = 0\ V^{-1}$. The device dimensions are $W_p = 30\ \mu m$, $L_p = 1.5\ \mu m$, $W_n = 15\ \mu m$, $L_n = 1.5\ \mu m$.




1a) *(5 points)* Calculate $V_{Tn}$ so that $V_M = 1.5\ V$.

$$V_M = \frac{V_{TN} + \sqrt{\frac{K_p}{K_n}}\,(V_{DD} + V_{TP})}{1 + \sqrt{\frac{K_p}{K_n}}}$$

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = 2.3 \times 10^{-7}\ F/cm^2$$

$$K_n = \left(\frac{W}{L}\right)_n \mu_n C_{ox} = 6.9 \times 10^{-4}\ F/V \cdot s$$

$$K_p = \left(\frac{W}{L}\right)_p \mu_p C_{ox} = 4.6 \times 10^{-4}\ F/V \cdot s$$

$$V_{TN} = V_M\left(1 + \sqrt{\frac{K_p}{K_n}}\right) - \sqrt{\frac{K_p}{K_n}}\,(V_{DD} + V_{TP})$$

$$V_{TN} = 1.092\ V$$

$$\frac{S \quad \frac{F}{V \cdot S} \quad V^2}{V}$$

Assume $V_{Tn} = 1\ V$ for parts 1b-1e.

1b) *(5 points)* What is the maximum value for $C_L$ and still have both $t_{PLH}$ and $t_{PHL}$ less than 1 $ns$? (Neglect $C_{dB}$ for this calculation.)

$$t_{PHL} = \frac{C_L \cdot V_{OH}/2}{\frac{K_n}{2}\left(V_{OH} - V_{TN}\right)^2}$$

$$t_{PLH} = \frac{C_L \cdot V_{OH}/2}{K_P/2\left(V_{OH} + V_{TP}\right)^2}$$

$$V_{OH} = V_{DD}$$

$$C_L = \frac{t_{PHL}\frac{K_n}{2}\left(V_{DD} - V_{TN}\right)^2}{V_{DD}/2} = 9.2 \times 10^{-13}\ F$$

$$C_L = \frac{t_{PLH}\frac{K_P}{2}\left(V_{DD} + V_{TP}\right)^2}{V_{DD}/2}\quad 6.133 \times 10^{-13}\ F$$

$$C_{L\,max} = 6.133 \times 10^{-13}\ F \qquad .6133\ pF$$

1c) *(5 points)* A square wave signal from 0 to $V_{DD}$ is applied to the input of this CMOS inverter with a frequency of 10 MHz. Calculate the power dissipated in the inverter. For this part, assume $C_L = 1\ pF$ and neglect any other parasitic capacitance.

$$P_D = C_L V_{DD}^2 f = 1 \times 10^{-12} (3)^2 (10 \times 10^6)$$

$$= 90 \mu W$$

1d) *(5 points)* What is the value of $V_{DD}$ required to reduce the power dissipation in the inverter by a factor of 4?

$$\frac{P_D}{4} = C_L \left(\frac{V_{DD}}{2}\right)^2 f$$

$$V_{DD} = 1.5 V$$

1e) *(5 points)* If $V_{DD}$ is reduced to 1.5 $V$, sketch the voltage transfer characteristics, $V_{OUT}$ vs. $V_{IN}$, of the inverter.

$V_{OH} = V_{DD} \qquad V_{OL} = 0$

$V_{IL} = V_m + \frac{V_{DD} - V_m}{A_V} \qquad V_{IH} = V_m - \frac{V_m}{A_V}$

$$V_m = \frac{V_{TN} + \sqrt{\frac{K_P}{K_N}}(V_{DD} + V_{TP})}{1 + \sqrt{\frac{K_P}{K_N}}}$$

$= .775V$

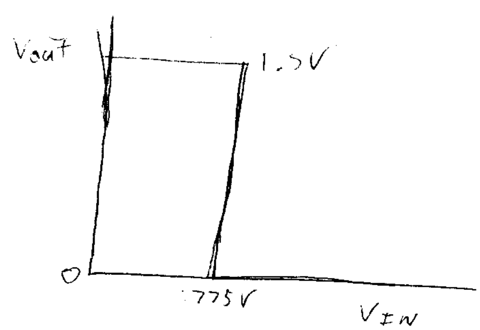
$A_V = -(g_{mn} + g_{mp})(r_{on} // r_{op})$

$r_{on} = \frac{1}{\lambda_n I_D} = \frac{1}{0} = \infty$

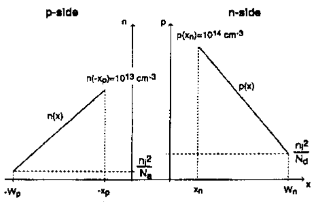
$r_{op} = \frac{1}{\lambda_p I_D} = \frac{1}{0} = \infty$

$A_V = \infty$

$V_{IL} = V_m \qquad V_{IH} = V_m$



2. *(40 points)* Below is a sketch *not to scale* of the minority carrier distribution across the quasi-neutral regions of a forward-biased p-n diode. For this diode, $W_p - x_p = 4\ \mu m$, $W_n - x_n = 3\ \mu m$, $D_n = 25\ cm^2/V \cdot s$, and $D_p = 10\ cm^2/V \cdot s$. The area of the junction is $10\ \mu m^2$.




2a) *(5 points)* Calculate the hole current injected into the n-side of the diode.

$$I_p = A_E\, J_{diff}^p \qquad J_{diff}^p = -q D_p \frac{\partial p}{\partial x}$$

$$\frac{\partial p}{\partial x} = -\left(\frac{p(x_n) - 0}{W_n - x_n}\right) \qquad \frac{n_i^2}{N_d} \ll p(x_n)$$

$$I_p = A_E\, q D_p \left(\frac{p(x_n) - 0}{W_n - x_n}\right)$$

$$I_p = 10(10^{-4})^2 q (10)\left(\frac{10^{14}}{3 \times 10^{-4}}\right) = 53.3\ nA$$

2b) *(5 points)* Calculate the electron current injected into the p-side of the diode.

$$I_n = A_E\, J_{diff}^n \qquad J_{diff}^n = q D_n \frac{\partial n}{\partial x}$$

$$\frac{\partial n}{\partial x} = \frac{n(-x_p) - 0}{-x_p + W_p} \qquad \frac{n_i^2}{N_a} \ll n(-x_p)$$

$$I_n = A_E\, q D_n \left(\frac{n(-x_p) - 0}{W_p - x_p}\right)$$

$$I_n = 10(10^{-4})^2 q\, 25 \left(\frac{10^{13}}{4 \times 10^{-4}}\right) = 10\ nA$$

2c) *(5 points)* Calculate the diffusion capacitance associated with carrier storage on the n-side of the diode.

$$C_d = \frac{q A_E}{2 V_{th}} (W_n - x_n) \underbrace{p_{no} e^{V_D/V_{th}}}_{p(x_n)}$$

$$= \frac{q \, 10(10^{-4})^2}{2 \; .026} (3\times10^{-4})(10^{14}) = 9.23 \text{ fF}$$

2d) *(5 points)* Calculate the diffusion capacitance associated with carrier storage on the p-side of the diode.

$$C_d = \frac{q A_E}{2 V_{th}} (W_p - x_p) \underbrace{n_{po} e^{V_D/V_{th}}}_{n(-x_p)}$$

$$= \frac{q \, 10(10^{-4})^2}{2 \; .026} (4\times10^{-4})(10^{13}) = 1.23 \text{ fF}$$

2e) *(5 points)* How much should the voltage across the junction increase if we wish to double the total current through the diode?

$$I = I_0 e^{qV_D/KT}$$

$$I_{new} = I_0 e^{qV_{Dnew}/KT}$$

$$I_{new} = 2I$$

$$\frac{I_{new}}{I} = \frac{I_0 e^{qV_{Dnew}/KT}}{I_0 e^{qV_D/KT}}$$

$$\frac{I_{new}}{I} = e^{q(V_{Dnew}-V_D)/KT}$$

$$V_{Dnew} - V_D = \frac{KT}{q}\ln\left(\frac{I_{new}}{I}\right)$$

$$V_{Dnew} - V_D = \frac{KT}{q}\ln(2) = 0.18\,V$$

2f) *(5 points)* If we increase the voltage in the manner suggested in the previous question, what happens to the total diffusion capacitance of the diode?

$$C_d = \frac{qA}{2V_{th}}\left((W_p - x_p)\frac{n_i^2}{N_a} + (W_n - x_n)\frac{n_i^2}{N_d}\right)e^{V_D/V_{th}}$$

$$C_{dnew} = \frac{qA}{2V_{th}}\left((W_p - x_p)\frac{n_i^2}{N_a} + (W_n - x_n)\frac{n_i^2}{N_d}\right)e^{V_{Dnew}/V_{th}}$$

$$\frac{C_{dnew}}{C_d} = \frac{\frac{qA}{2V_{th}}\left((W_p - x_p)\frac{n_i^2}{N_a} + (W_n - x_n)\frac{n_i^2}{N_d}\right)e^{V_{Dnew}/V_{th}}}{\frac{qA}{2V_{th}}\left((W_p - x_p)\frac{n_i^2}{N_a} + (W_n - x_n)\frac{n_i^2}{N_d}\right)e^{V_D/V_{th}}}$$

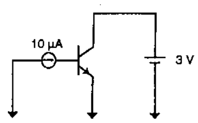
$$\frac{C_{dnew}}{C_d} = \frac{e^{V_{Dnew}/V_{th}}}{e^{V_D/V_{th}}} = e^{(V_{Dnew}-V_D)/V_{th})}$$

$$= e^{V_{th}\ln(2)/V_{th}} = e^{\ln(2)} = 2$$

$$\frac{C_{dnew}}{C_d} = 2 \qquad C_{dnew} = C_d * 2$$

**3.** *(35 points)* Consider a bipolar transistor with the following large-signal equivalent circuit model parameters $I_S = 10^{-16}$ $A$, $\beta_F = 100$, and $\beta_R = 1$. To answer some of the following questions, use the non-linear hybrid-$\pi$ model of the transistor presented in class.

☐ Answer the following questions when the device is biased as sketched in the diagram below:




3a) *(5 points)* In what regime is the device operating? Explain.

Fd. Active. $I_B > 0$ $V_{CE} = 3V$ $V_{BE} \approx .7V$

$V_{BC} < 0$

3b) *(5 points)* Calculate the collector current, $I_C$.

$I_C = \beta I_B = 100 \quad 10\mu A = 1mA$

3c) *(5 points)* Calculate the base-emitter voltage $V_{BE}$.

$I_C = I_S \, e^{\frac{V_{BE}}{V_{th}}}$

$V_{BE} = V_{th} \ln\left(\frac{I_C}{I_S}\right) = .026 \ln\left(\frac{10^{-3}}{10^{-16}}\right) = .778V$

2g) *(5 points)* What is the ratio of the doping levels across the junction: $N_a/N_d$?

$$\frac{p(x_n)}{n(-x_p)} = \frac{\frac{n_i^2}{n_d} e^{qV_D/KT}}{\frac{n_i^2}{N_a} e^{qV_D/KT}} = \frac{N_a}{N_d}$$

$$\frac{p(x_n)}{n(-x_p)} = \frac{10^{14}}{10^{13}} = 10 \qquad \frac{N_a}{N_d} = 10$$

2h) *(5 points)* In what direction should $N_a/N_d$ change if we wish to redesign the diode so as to get less diffusion capacitance <u>at the same current level</u>? (Assume that in redesigning the diode $D_n$, $D_p$, $W_n - x_n$, and $W_p - x_p$ do not change).

Choose one: *$N_a/N_d$ must increase*, *$N_a/N_d$ must decrease*. Explain (*no explanation, no points*).

$$C_d = \frac{qA}{2V_{th}}\left((w_p - x_p)\frac{n_i^2}{N_a}e^{V_D/V_{th}} + (w_n - x_n)\frac{n_i^2}{N_d}e^{V_D/V_{th}}\right)$$

$$C_d = \frac{qA}{2V_{th}}\left((w_p - x_p)\, n(-x_p) + (w_n - x_n)\, p(x_n)\right)$$

$C_d$ is proportional to $n(-x_p)$ and $p(x_n)$. $I = I_p + I_n$

$$I_p = q D_p A E \frac{p(x_n) - 0}{w_n - x_n}$$

$$I_n = q D_n A E \frac{n(-x_p) - 0}{w_p - x_p}$$

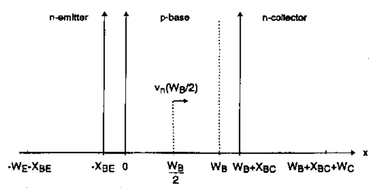
We can increase $I_n$ and decrease $I_p$ and maintain $I$. Since $\frac{D_n}{w_p - x_p} > \frac{D_p}{w_n - x_n}$ we can decrease $p(x_n)$ more than we increase $n(-x_p)$. In this way we get the same current with a lower $n(-x_p) + p(x_n)$. Since $C_d \propto n(-x_p) + p(x_n)$ $C_d$ will be lower too. $\frac{p(x_n)}{n(-x_p)} = \frac{N_a}{N_d}$.

We must lower $\frac{N_a}{N_d}$.

for $\frac{N_a}{N_d} = 10$ $C_d = 10.46\text{ fF}$

for $\frac{N_a}{N_d} = .35$ $C_d = 7.23\text{ fF}$

3d) *(5 points)* If the doping level in the base is $N_{aB} = 10^{17}\ cm^{-3}$ and the area of the base-emitter junction is $A_E = 40\ \mu m^2$, calculate the electron velocity in the middle of the quasi-neutral base (see sketch below).




$$I_C = A_E J$$

$$J = q\, n_p V_n$$

$$V_n = \frac{I_C}{A_E\, q\, n_p(W_B/2)}$$

$$n_p(x) = \frac{-\frac{n_i^2}{N_{aB}} e^{qV_{BE}/KT} - 0}{W_B} X + \frac{n_i^2}{N_{aB}} e^{qV_{BE}/KT}$$

$$n_p\left(\frac{W_B}{2}\right) = \frac{1}{2}\frac{n_i^2}{N_{aB}} e^{qV_{BE}/KT} = 4.95 \times 10^{15}\ cm^{-3}$$

$$V_n = \frac{I_C}{A_E\, q\, n_p(W_B/2)} = \frac{10^{-3}}{10(10^{-4})^2 q\,(4.95\times10^{15})}$$

$$V_n = 1.26 \times 10^{7}\ cm/s \qquad 1.26 \times 10^{5}\ m/s$$

3g) *(5 points)* Calculate the collector current, $I_C$.

$$I_C = I_S\left(e^{qV_{BE}/KT} - e^{qV_{BC}/KT}\right) - \frac{I_S}{B_R}\left(e^{q\frac{V_{BC}}{KT}} - 1\right)$$

$$V_{BE} = V_{BC}$$

$$I_c = \frac{-I_S}{B_R}\left(e^{q\frac{V_{BC}}{KT}} - 1\right)$$

$$-10^{-16}\left(e^{\frac{.7785}{.026}} - 1\right) = -1.01\,mA$$

1.01 mA out of the collector