March 10, 1999 - Quiz #1

Name: Solutions

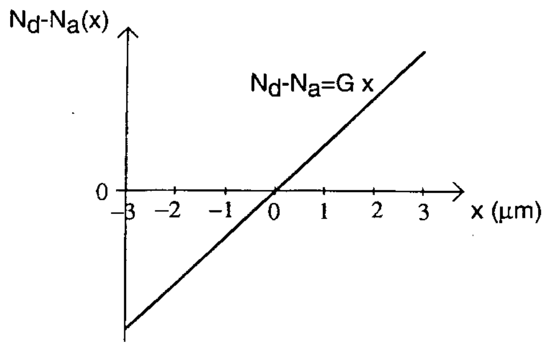
Recitation: ______________________

General guidelines (please read carefully before starting):

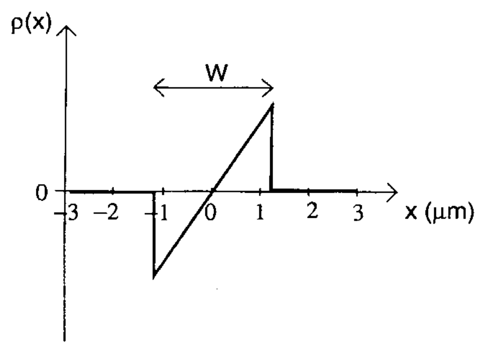
- Make sure to write your name on the space designated above.
- **Open book**: you can use any material you wish.
- All answers should be given in the space provided. Please do not turn in any extra material. If you need mo space, use the back page.
- You have **120 minutes** to complete your quiz.
- Make reasonable approximations and *state them*, i.e. quasi-neutrality, depletion approximation, etc.
- Partial credit will be given for setting up problems without calculations. **NO** credit will be given for answe without reasons.
- Use the symbols utilized in class for the various physical parameters, i.e. μ_n, I_D, E, etc.
- Every numerical answer must have the proper units next to it. Points will be subtracted for answers withou units or with wrong units.
- Use the following fundamental constants and physical parameters for silicon and silicon dioxide at roo temperature:

$n_i = 1 \times 10^{10}\ cm^{-3}$
$kT/q = 0.026\ V$
$q = 1.60 \times 10^{-19}\ C$
$\epsilon_s = 1.05 \times 10^{-12}\ F/cm$
$\epsilon_{ox} = 3.45 \times 10^{-13}\ F/cm$

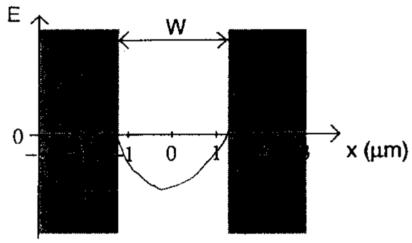
1. *(30 points)* You are given a semiconductor with a total length of 6 μm. It is doped with both donors and accept with a resulting net doping profile that has a linear distribution in space, as shown in the figure below. G is gradient of the net doping concentration. Assume that the semiconductor is in thermal equilibrium.



This doping distribution results in a space charge layer (a depletion layer) of width W that separates two quasi-neut regions. The resulting charge density distribution is sketched below.



1a) *(5 points)* Sketch the electric field in the depletion region. Derive an expression for the value of the electric fie at its peak magnitude. Use the depletion approximation and express your answer in terms of W and G.

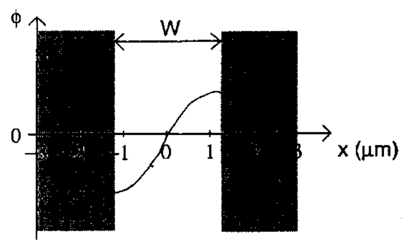


Integrate from $-W/2$ to 0 to find Q

$Q = \frac{1}{2}\left(\frac{W}{2}\right) G \frac{W}{2}$ $\qquad E_{max} = \frac{Q}{\epsilon_s}$

$|E_{max}|= \frac{W^2 G}{8 \epsilon_s}$

1b) *(5 points)* Sketch the electrostatic potential in the depletion region. Provide an expression for the value of tl electrostatic potential at $x = 0$. Use the depletion approximation and express your answer in terms of W and G.



$\phi(x = 0)=$ 0

1c) *(5 points)* Using $G = 10^{17}\ cm^{-3}/\mu m$, compute the electron concentration in the quasi-neutral region at $x = 2\ \mu$ and $x = -2\ \mu m$ *(numerical answer expected)*.

$$n_o(2\mu m) = N_d - N_a = G\,x = 10^{17}\frac{cm^{-3}}{\mu m} \times 2\mu m$$
$$= 2\times10^{17} cm^{-3}$$
$$p_o(-2\mu m) = N_a - N_d = -G\,x = -10^{17}\frac{cm^{-3}}{\mu m} \times 2\mu m$$
$$= 2\times10^{17} cm^{-3}$$
$$n_o(-2\mu m) = \frac{n_i^2}{p} = \frac{10^{20}}{2\times10^{17}} = 500 cm^{-3}$$

1d) *(5 points)* Using $G = 10^{17}\ cm^{-3}/\mu m$, compute the electrostatic potential ϕ at $x = 2\ \mu m$ and $x = -2\ \mu$ *(numerical answer expected)*.

$$\phi_{n\ 2\mu m} = 60mV \cdot \log\left(\frac{n}{n_i}\right) = 60mV \cdot \log\left(\frac{2\times10^{17}}{10^{10}}\right)$$
$$= 438mV$$
$$\phi_{p\ -2\mu m} = -60mV\ \log\left(\frac{p}{n_i}\right) = -60mV\ \log\left(\frac{2\times10^{17}}{10^{10}}\right)$$
$$= -438mV$$

1e) *(5 points)* Using $G = 10^{17}\ cm^{-3}/\mu m$ and $\mu_n = 1000\ cm^2/V \cdot s$, calculate the electron diffusion current J_n^{diff} at $x = 2\ \mu m$ and $x = -2\ \mu m$ *(numerical answer expected)*.

$$J_n^{diff} = q D_n \frac{\partial n}{\partial x} \qquad D_n = \mu_n \frac{kT}{q}$$

$$n = Gx \qquad \frac{\partial n}{\partial x} = G$$

$$J_n^{diff}(2\mu m) = q \frac{kT}{q} \mu_n G$$

$$(1.6\times10^{-19} C)(.025 V)(1000\ cm^2/V.s)(10^{17} cm^{-3}/\mu m)\left(\frac{10^4 \mu m}{cm}\right)$$

$$J_n^{diff}(2\mu m) = 4000\ A/cm^2$$

same at $-2\mu m$

1f) *(5 points)* Using the knowledge that you are in thermal equilibrium and the results of *1e)*, compute the electric field at $x = 2\ \mu m$ *(numerical answer expected)*.

$$J_n^{diff} = -J_n^{dr}$$

$$J_n^{dr} = -q\, n\, \mu_n E$$

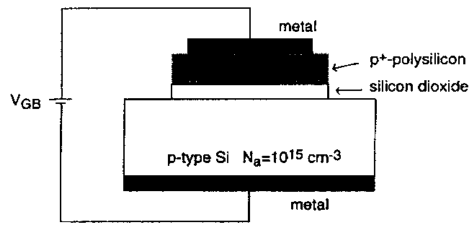
$$E = -\frac{J_n^{dr}}{q n \mu_n}$$

$$n = Gx$$

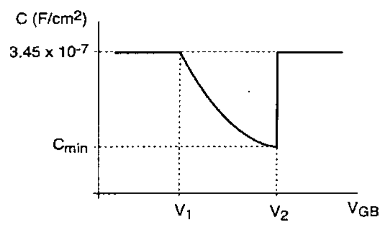
$$E = -\frac{4000\ A/cm^2}{(1.6\times10^{-19})(10^{17} cm^{-3}/\mu m)(2\mu m)(1000 cm^2/V.s)}$$

$$= -125\ V/cm$$

2. *(30 points)* You are given an MOS capacitor fabricated with a p^+ polysilicon gate and a p-type substrate with doping concentration of $N_a = 10^{15}\ cm^{-3}$, as sketched below.



The capacitance-voltage curve for this device is shown below:



2a) *(5 points)* Calculate $V_{GB} = V_1$.

$$V_1 = V_{FB} = -(\phi_{p+} - \phi_p)$$

$$= -(550mV + 300mV) = 250mV$$

2b) *(5 points)* Calculate the oxide thickness.

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}}$$

$$t_{ox} = \frac{\epsilon_{ox}}{C_{ox}}$$

$$= \frac{3.45\times10^{-13}\ F/cm}{3.45\times10^{-7}\ F/cm^2} = 10^{-6}\ cm \quad = 100\,A^{o}$$

2c) *(5 points)* Calculate $V_{GB} = V_2$.

$$V_2 = V_{TN} = V_{FB} - 2\phi_p + \frac{1}{C_{ox}}\sqrt{2q\,\epsilon_s\,N_a(-2\phi_p)}$$

$$= .25 - 2(-.3) + \frac{1}{3.45\times10^{-7}}\sqrt{2(1.6\times10^{-19})(1.035\times10^{-12})(10^{15})(-2)(-.3)}$$

$$= .89\ V$$

2d) *(5 points)* Calculate C_{min}. $\frac{1}{C} = \frac{1}{Cox} + \frac{1}{Cb}$

$+ V_{TN}$

$$\frac{1}{C_{min}} = \frac{1}{C_{ox}} + \frac{1}{C_{bmin}}$$

$$C_{bmin} = \frac{\epsilon_s}{X_{dmax}} \qquad X_{dmax} = \sqrt{\frac{2\epsilon_s}{qN_a}(-2\phi_p)}$$

$$X_{dmax} = \sqrt{\frac{2(1.035\times10^{-12})(-2)(-.3)}{(1.6\times10^{-19})(10^{15})}} = 8.8\times10^{-5}\,cm$$

$$C_{b_{min}} = 1.17\times10^{-8}\,F/cm^2$$

$$C_{min} = 1.14\times10^{-8}\,F/cm^2$$

2e) *(5 points)* Calculate the electric field in the oxide when $V_{GB} = V_2 + 1\ V$.

$$V_{GB} = V_2 + 1 = .89 + 1 = 1.89$$ inversion ~~saturation~~

$$E_{ox} = \frac{Q_G}{\epsilon_{ox}}$$

$$Q_G = -Q_N - Q_{Bmax}$$

$$Q_{Bmax} = -qN_a X_{dmax} = (1.6\times10^{-19})(10^{15})(.88\mu m) = 1.41\times10^{-8}\,C/cm^2$$

$$Q_n = -C_{ox}(V_{GB} - V_{TN}) = -3.45\times10^{-7}(1) = 3.45\times10^{-7}\,C/cm^2$$

$$Q_G = 3.59\times10^{-7}\,C/cm^2$$

$$E_{ox} = \frac{Q_G}{\epsilon_{ox}} = 10.4\times10^5\,V/cm$$

2f) *(5 points)* Calculate the electric field in the oxide when $V_{GB} = V_1 - 1\ V$.

$V_{GB} = .25 - 1 = -.75$ accumulation

$E_{ox} = \frac{Q_G}{\epsilon_{ox}}$

$Q_G = C_{ox}(V_{GB} - V_{FB}$

$= (3.45\times10^{-7})(-.75 - .25) = 3.45\times10^{-7}\ C/cm^2$

$E_{ox} = \frac{Q_G}{\epsilon_{ox}} = 1\times10^{6}\ \frac{V}{cm}$

3. *(40 points)* Consider an n-channel MOSFET with gate length $L = 1\ \mu m$, gate width $W = 10\ \mu m$, inversion lay electron mobility $\mu_n = 300\ cm^2/V \cdot s$, threshold voltage $V_T = 1\ V$, oxide thickness $t_{ox} = 15\ nm$, body paramet $\gamma = 0.5\ V^{1/2}$, and potential in substrate $\phi_p = -0.4\ V$.

□ Answer the following questions when the device is biased with $V_{GS} = 3\ V$, $V_{DS} = 0.1\ V$ and $V_{BS} = 0\ V$.

3a) *(5 points)* In what regime is the device operating? Explain.

$V_{GS} > V_T \qquad V_{DS} < V_{GS} - V_T \qquad$ triode

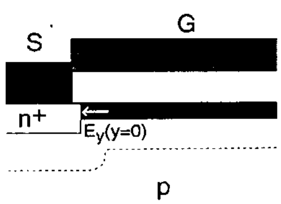
3b) *(5 points)* Calculate the drain current, I_D.

$$I_D = \mu_n C_{ox} \frac{W}{L}\left[V_{GS} - V_T - \frac{V_{DS}}{2}\right] V_{DS}$$

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = 2.3 \times 10^{-7}\ F/cm^2$$

$$I_D = 1.35 \times 10^{-4}\ A$$

3c) *(5 points)* Calculate the *lateral* electric field at the source end of the channel, $E_y(y = 0)$ (this is the field in t channel in the direction of the channel, as sketched below).



$L = 1\ \mu m = 10^{-4}\ cm$

$$E_y = \frac{V_{DS}}{L} = \frac{.1}{10^{-4}\ cm} = 1000\ V/cm$$

□ Answer the following questions when the device is biased with $V_{GS} = 3\ V$, $V_{DS} = 3\ V$ and $V_{BS} = 0\ V$.

3d) *(5 points)* In what regime is the device operating? Explain.

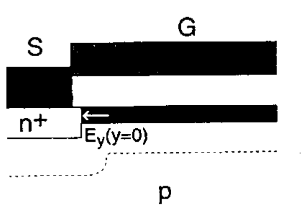
$V_{GS} > V_T \quad V_{DS} > V_{GS} - V_T \quad$ saturation

3e) *(5 points)* Calculate the drain current, I_D.

$$I_D = \frac{1}{2}\frac{W}{L}\mu_n C_{ox}(V_{GS} - V_T)^2$$

$$= 1.38\,mA$$

3f) *(5 points)* Calculate the *lateral* electric field at the source end of the channel, $E_y(y=0)$ (this is the field in t channel in the direction of the channel, as sketched below).



$$E_y = \frac{V_{DS}}{L} = \frac{3}{10^{-4}} = 3\times10^4\ V/cm$$

□ Answer the following questions when the device is biased with $V_{GS} = 1.5\ V$, $V_{DS} = 3\ V$ and $V_{BS} = -4\ V$.

3g) *(5 points)* In what regime is the device operating? Explain.

$$V_{TN} = V_{TO} + \gamma_n\left(\sqrt{-2\phi_p - V_{BS}} - \sqrt{-2\phi_p}\right)$$

$$1 + .5\left(\sqrt{-2(-.4) - (-4)} - \sqrt{(-2)(-.4)}\right)$$

$$V_{TN} = 1.648$$

$$V_{GS} < V_T$$

cutoff

3h) *(5 points)* Calculate the drain current, I_D.

$$I_D = 0$$