

6.012 Problem Set #3 - Solutions

Problem #1: E3.12

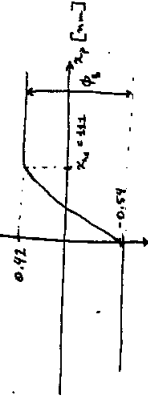
$$\boxed{N_A = 10^{19} \text{ cm}^{-3}, N_D = 10^{17} \text{ cm}^{-3}}$$

$$\phi_p = -570 \text{ mV}, \phi_n = 420 \text{ mV}$$

$$\phi_B = \phi_n - \phi_p = 0.96 \text{ V}$$

$$x_{p0} = \frac{N_A}{N_D} x_{n0} = 0.01 x_{n0}$$

$$(a) x_{n0} = \sqrt{\frac{2 \epsilon_s \phi_B}{q N_D} \left(\frac{N_A}{N_D + N_A} \right)} = 1.11 \times 10^{-5} \text{ cm} = \boxed{11.1 \text{ nm}}$$



$$(b) |E_c(x=0)| = \frac{q N_D}{\epsilon_s} x_{n0} = \boxed{1.7 \times 10^5 \text{ V/cm}}$$

$$(c) x_{n, \text{max}} = \frac{\epsilon_s E_{\text{max}}}{q N_D} = 3.26 \times 10^{-5} \text{ cm}$$

$$x_n(V_0) = x_{n0} \sqrt{1 - (V_0/\phi_B)}$$

$$V_{D, \text{max}} = -\phi_B \left(\left(\frac{x_{n, \text{max}}}{x_{n0}} \right)^2 - 1 \right)$$

$$\boxed{V_{D, \text{max}} = -7.32 \text{ Volts}}$$

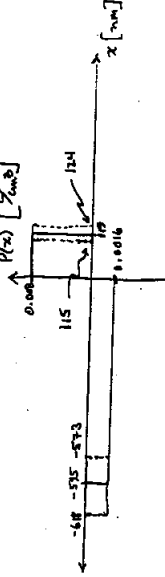
Problem #2: E3.14

$$(a) \phi_B = \phi_n - \phi_p = (60 \text{ mV}) \left(\log \left(\frac{10^{16}}{10^{10}} \right) + \log \left(\frac{10^{16}}{10^{10}} \right) \right) = 0.762 \text{ Volts}$$

$$x_{n0} = \sqrt{\frac{2 \epsilon_s \phi_B}{q N_D} \left(\frac{N_A}{N_D + N_A} \right)} = 5.76 \times 10^{-6} \text{ cm}$$

$$x_n(V_D) = -2.5 \text{ V} = x_{n0} \sqrt{1 - \left(\frac{V_D}{\phi_B} \right)} = 1.19 \times 10^{-5} \text{ cm} = 119 \text{ nm}$$

$$x_p = \frac{N_D}{N_A} x_n = 5 x_n = 5.95 \times 10^{-5} \text{ cm} = 595 \text{ nm}$$

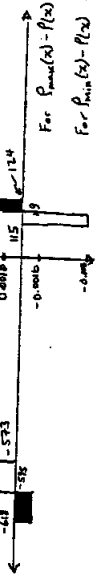


(b) Now Applying $v_2(t) = V_D + v_d(t)$ where $v_d(t) = (250 \text{ mV}) \cos(\omega t)$

Extremes are when: $v_d(t) = -2.75 \text{ Volts}$, $x_n = 1.24 \times 10^{-5} \text{ cm}$ & $x_p = 6.15 \times 10^{-5} \text{ cm}$

$v_d(t) = -2.25 \text{ Volts}$, $x_n = 1.15 \times 10^{-5} \text{ cm}$ & $x_p = 5.73 \times 10^{-5} \text{ cm}$

These are plotted on graph above.



These are not equal and opposite, but very close. It is not equal and opposite because x_n, x_p is not linear with V_D . At very small signals it seems linear because $x_n \propto \sqrt{1 - (V_D/\phi_B)} \approx 1 - \frac{1}{2} (V_D/\phi_B)$ for small V_D .

$$(d) C_{j0} = \frac{q N_D x_{n0}}{2 \phi_B} = 3.02 \times 10^{-8} \text{ cm}^{-2} \quad C_j(V_D) = \frac{C_{j0}}{\sqrt{1 - V_D/\phi_B}}$$

$$C_j(V_D = -2.5 \text{ Volts}) = 1.46 \times 10^{-8} \text{ cm}^{-2}$$

$$g_j = (49) (C_j(2.5)) = \boxed{3.65 \times 10^{-9} \text{ cm}^{-2}}$$

This is similar to the figures found in part (b).

Problem #3: E3.15

(a) Plot C_{ij} with N_A from 10^{13} cm^{-3} to 10^{19} cm^{-3} w/ $N_A = 10^{19} \text{ cm}^{-3}$

$$C_{ij} = \sqrt{\frac{E_i}{2 \cdot V_B}} \left(\frac{N_A N_D}{N_A + N_D} \right) \left[\log_2 \left(\frac{N_A}{N_D} \right) + \log_2 \left(\frac{N_A}{N_D} \right) \right]$$

Plotted on MS-Excel:

N_A	μN_D	C_{ij}	X_{no}	V_{max}	$C(V_{max})$	$C(V_{max})$
1.00E+13	7.29E-01	1.07743E-09	6.0000E-10	-9197.7	3.20E-12	3.38E-02
6.00E+13	7.42E-01	2.3419E-09	2.2306E-09	-16274.8	1.00E-11	1.48E-02
1.00E+14	7.49E-01	3.2744E-09	3.1916E-09	-19373.4	3.20E-11	1.02E-02
6.00E+14	8.22E-01	7.1303E-09	7.2067E-09	-46262.0	1.00E-10	3.11E-01
1.00E+15	8.40E-01	8.7477E-09	1.0472E-08	-61715.0	3.00E-10	3.11E-01
6.00E+15	8.62E-01	1.1891E-08	1.4294E-08	-91349.0	1.00E-09	3.38E-01
1.00E+16	8.81E-01	1.5903E-08	1.9818E-08	-126527.0	3.00E-09	3.81E-01
6.00E+16	9.06E-01	2.3298E-08	2.7230E-08	-181602.0	1.00E-08	4.41E-01
1.00E+17	9.28E-01	3.1994E-08	3.6442E-08	-247704.0	3.00E-08	5.11E-01
6.00E+17	9.47E-01	4.2420E-08	4.8770E-08	-334508.0	1.00E-07	6.07E-01
1.00E+18	9.61E-01	5.4942E-08	6.4371E-08	-451149.0	3.00E-07	7.17E-01
6.00E+18	9.71E-01	6.9770E-08	8.5709E-08	-607878.0	1.00E-06	8.47E-01
1.00E+19	9.78E-01	8.7174E-08	1.1141E-07	-817698.0	3.00E-06	9.91E-01
6.00E+19	9.84E-01	1.0746E-07	1.4048E-07	-1.1075E+06	1.00E-05	1.15E+00
1.00E+20	9.87E-01	1.3020E-07	1.7200E-07	-1.4918E+06	3.00E-05	1.33E+00
6.00E+20	9.89E-01	1.5603E-07	2.0445E-07	-2.0211E+06	1.00E-04	1.53E+00
1.00E+21	9.90E-01	1.8444E-07	2.4018E-07	-2.7009E+06	3.00E-04	1.75E+00
6.00E+21	9.91E-01	2.1594E-07	2.7939E-07	-3.6418E+06	1.00E-03	2.00E+00
1.00E+22	9.92E-01	2.5100E-07	3.3298E-07	-4.9149E+06	3.00E-03	2.28E+00
6.00E+22	9.93E-01	2.8917E-07	3.9107E-07	-6.5006E+06	1.00E-02	2.59E+00
1.00E+23	9.94E-01	3.3094E-07	4.6318E-07	-8.5006E+06	3.00E-02	2.93E+00
6.00E+23	9.95E-01	3.7694E-07	5.5006E-07	-1.1075E+07	1.00E-01	3.31E+00
1.00E+24	9.96E-01	4.2770E-07	6.5318E-07	-1.4918E+07	3.00E-01	3.72E+00
6.00E+24	9.97E-01	4.8371E-07	7.7200E-07	-1.9317E+07	1.00E+00	4.18E+00
1.00E+25	9.98E-01	5.4442E-07	9.0770E-07	-2.5419E+07	3.00E+00	4.69E+00
6.00E+25	9.99E-01	6.1074E-07	1.0603E-06	-3.3419E+07	1.00E+01	5.25E+00
1.00E+26	1.00E+00	6.8200E-07	1.2300E-06	-4.3419E+07	3.00E+01	5.85E+00
6.00E+26	1.00E+00	7.5903E-07	1.4200E-06	-5.6419E+07	1.00E+02	6.49E+00
1.00E+27	1.00E+00	8.4200E-07	1.6300E-06	-7.2419E+07	3.00E+02	7.17E+00
6.00E+27	1.00E+00	9.3100E-07	1.8600E-06	-9.3419E+07	1.00E+03	7.89E+00
1.00E+28	1.00E+00	1.0270E-06	2.1100E-06	-1.2042E+08	3.00E+03	8.65E+00
6.00E+28	1.00E+00	1.1300E-06	2.3800E-06	-1.5342E+08	1.00E+04	9.45E+00
1.00E+29	1.00E+00	1.2400E-06	2.6700E-06	-1.9342E+08	3.00E+04	1.029E+01
6.00E+29	1.00E+00	1.3500E-06	2.9800E-06	-2.5042E+08	1.00E+05	1.119E+01
1.00E+30	1.00E+00	1.4700E-06	3.3100E-06	-3.2542E+08	3.00E+05	1.215E+01
6.00E+30	1.00E+00	1.5900E-06	3.6600E-06	-4.2842E+08	1.00E+06	1.319E+01
1.00E+31	1.00E+00	1.7200E-06	4.0300E-06	-5.6042E+08	3.00E+06	1.441E+01
6.00E+31	1.00E+00	1.8600E-06	4.4200E-06	-7.2242E+08	1.00E+07	1.573E+01
1.00E+32	1.00E+00	2.0100E-06	4.8300E-06	-9.3442E+08	3.00E+07	1.717E+01
6.00E+32	1.00E+00	2.1700E-06	5.2600E-06	-1.2042E+09	1.00E+08	1.873E+01
1.00E+33	1.00E+00	2.3400E-06	5.7100E-06	-1.5342E+09	3.00E+08	2.041E+01
6.00E+33	1.00E+00	2.5200E-06	6.1800E-06	-1.9342E+09	1.00E+09	2.221E+01
1.00E+34	1.00E+00	2.7100E-06	6.6700E-06	-2.5042E+09	3.00E+09	2.413E+01
6.00E+34	1.00E+00	2.9100E-06	7.1800E-06	-3.2542E+09	1.00E+10	2.617E+01
1.00E+35	1.00E+00	3.1200E-06	7.7100E-06	-4.2842E+09	3.00E+10	2.843E+01
6.00E+35	1.00E+00	3.3400E-06	8.2600E-06	-5.6042E+09	1.00E+11	3.081E+01
1.00E+36	1.00E+00	3.5700E-06	8.8300E-06	-7.2242E+09	3.00E+11	3.331E+01
6.00E+36	1.00E+00	3.8100E-06	9.4200E-06	-9.3442E+09	1.00E+12	3.593E+01
1.00E+37	1.00E+00	4.0600E-06	1.0030E-05	-1.2042E+10	3.00E+12	3.867E+01
6.00E+37	1.00E+00	4.3200E-06	1.0660E-05	-1.5342E+10	1.00E+13	4.163E+01
1.00E+38	1.00E+00	4.5900E-06	1.1310E-05	-1.9342E+10	3.00E+13	4.481E+01
6.00E+38	1.00E+00	4.8700E-06	1.2000E-05	-2.5042E+10	1.00E+14	4.821E+01
1.00E+39	1.00E+00	5.1600E-06	1.2710E-05	-3.2542E+10	3.00E+14	5.183E+01
6.00E+39	1.00E+00	5.4600E-06	1.3450E-05	-4.2842E+10	1.00E+15	5.567E+01
1.00E+40	1.00E+00	5.7700E-06	1.4220E-05	-5.6042E+10	3.00E+15	5.973E+01
6.00E+40	1.00E+00	6.0900E-06	1.5020E-05	-7.2242E+10	1.00E+16	6.401E+01
1.00E+41	1.00E+00	6.4200E-06	1.5850E-05	-9.3442E+10	3.00E+16	6.851E+01
6.00E+41	1.00E+00	6.7600E-06	1.6710E-05	-1.2042E+11	1.00E+17	7.323E+01
1.00E+42	1.00E+00	7.1100E-06	1.7600E-05	-1.5342E+11	3.00E+17	7.827E+01
6.00E+42	1.00E+00	7.4700E-06	1.8520E-05	-1.9342E+11	1.00E+18	8.363E+01
1.00E+43	1.00E+00	7.8400E-06	1.9470E-05	-2.5042E+11	3.00E+18	8.931E+01
6.00E+43	1.00E+00	8.2200E-06	2.0450E-05	-3.2542E+11	1.00E+19	9.533E+01
1.00E+44	1.00E+00	8.6100E-06	2.1460E-05	-4.2842E+11	3.00E+19	1.0169E+02
6.00E+44	1.00E+00	9.0100E-06	2.2500E-05	-5.6042E+11	1.00E+20	1.0839E+02
1.00E+45	1.00E+00	9.4200E-06	2.3570E-05	-7.2242E+11	3.00E+20	1.1543E+02
6.00E+45	1.00E+00	9.8400E-06	2.4670E-05	-9.3442E+11	1.00E+21	1.2281E+02
1.00E+46	1.00E+00	1.0270E-05	2.5800E-05	-1.2042E+12	3.00E+21	1.3053E+02
6.00E+46	1.00E+00	1.0710E-05	2.6960E-05	-1.5342E+12	1.00E+22	1.3869E+02
1.00E+47	1.00E+00	1.1160E-05	2.8150E-05	-1.9342E+12	3.00E+22	1.4721E+02
6.00E+47	1.00E+00	1.1620E-05	2.9370E-05	-2.5042E+12	1.00E+23	1.5611E+02
1.00E+48	1.00E+00	1.2090E-05	3.0620E-05	-3.2542E+12	3.00E+23	1.6539E+02
6.00E+48	1.00E+00	1.2570E-05	3.1900E-05	-4.2842E+12	1.00E+24	1.7515E+02
1.00E+49	1.00E+00	1.3060E-05	3.3210E-05	-5.6042E+12	3.00E+24	1.8539E+02
6.00E+49	1.00E+00	1.3560E-05	3.4550E-05	-7.2242E+12	1.00E+25	1.9611E+02
1.00E+50	1.00E+00	1.4070E-05	3.5920E-05	-9.3442E+12	3.00E+25	2.0731E+02
6.00E+50	1.00E+00	1.4590E-05	3.7320E-05	-1.2042E+13	1.00E+26	2.1899E+02
1.00E+51	1.00E+00	1.5120E-05	3.8750E-05	-1.5342E+13	3.00E+26	2.3115E+02
6.00E+51	1.00E+00	1.5660E-05	4.0210E-05	-1.9342E+13	1.00E+27	2.4379E+02
1.00E+52	1.00E+00	1.6210E-05	4.1700E-05	-2.5042E+13	3.00E+27	2.5691E+02
6.00E+52	1.00E+00	1.6770E-05	4.3220E-05	-3.2542E+13	1.00E+28	2.7051E+02
1.00E+53	1.00E+00	1.7340E-05	4.4770E-05	-4.2842E+13	3.00E+28	2.8461E+02
6.00E+53	1.00E+00	1.7920E-05	4.6350E-05	-5.6042E+13	1.00E+29	2.9921E+02
1.00E+54	1.00E+00	1.8510E-05	4.7960E-05	-7.2242E+13	3.00E+29	3.1431E+02
6.00E+54	1.00E+00	1.9110E-05	4.9600E-05	-9.3442E+13	1.00E+30	3.3001E+02
1.00E+55	1.00E+00	1.9720E-05	5.1270E-05	-1.2042E+14	3.00E+30	3.4631E+02
6.00E+55	1.00E+00	2.0340E-05	5.2970E-05	-1.5342E+14	1.00E+31	3.6321E+02
1.00E+56	1.00E+00	2.1070E-05	5.4700E-05	-1.9342E+14	3.00E+31	3.8071E+02
6.00E+56	1.00E+00	2.1810E-05	5.6460E-05	-2.5042E+14	1.00E+32	3.9881E+02
1.00E+57	1.00E+00	2.2560E-05	5.8250E-05	-3.2542E+14	3.00E+32	4.1751E+02
6.00E+57	1.00E+00	2.3320E-05	6.0070E-05	-4.2842E+14	1.00E+33	4.3691E+02
1.00E+58	1.00E+00	2.4090E-05	6.1920E-05	-5.6042E+14	3.00E+33	4.5701E+02
6.00E+58	1.00E+00	2.4870E-05	6.3800E-05	-7.2242E+14	1.00E+34	4.7781E+02
1.00E+59	1.00E+00	2.5660E-05	6.5710E-05	-9.3442E+14	3.00E+34	4.9931E+02
6.00E+59	1.00E+00	2.6460E-05	6.7650E-05	-1.2042E+15	1.00E+35	5.2151E+02
1.00E+60	1.00E+00	2.7270E-05	6.9620E-05	-1.5342E+15	3.00E+35	5.4441E+02
6.00E+60	1.00E+00	2.8090E-05	7.1620E-05	-1.9342E+15	1.00E+36	5.6801E+02
1.00E+61	1.00E+00	2.8920E-05	7.3650E-05	-2.5042E+15	3.00E+36	5.9231E+02
6.00E+61	1.00E+00	2.9760E-05	7.5710E-05	-3.2542E+15	1.00E+37	6.1731E+02
1.00E+62	1.00E+00	3.0610E-05	7.7800E-05	-4.2842E+15	3.00E+37	6.4301E+02
6.00E+62	1.00E+00	3.1470E-05	7.9920E-05	-5.6042E+15	1.00E+38	6.6941E+02
1.00E+63	1.00E+00	3.2340E-05	8.2070E-05	-7.2242E+15	3.00E+38	6.9651E+02
6.00E+63	1.00E+00	3.3220E-05	8.4250E-05	-9.3442E+15	1.00E+39	7.2431E+02
1.00E+64	1.00E+00	3.4110E-05	8.6460E-05	-1.2042E+16	3.00E+39	7.5281E+02
6.00E+64	1.00E+00	3.5010E-05	8.8690E-05	-1.5342E+16	1.00E+40	7.8201E+02
1.00E+65	1.00E+00	3.5920E-05	9.0940E-05	-1.9342E+16	3.00E+40	8.1191E+02
6.00E+65	1.00E+00	3.6840E-05	9.3210E-05	-2.5042E+16	1.00E+41	8.4251E+02
1.00E+66	1.00E+00	3.7770E-05	9.5500E-05	-3.2542E+16	3.00E+41	8.7381E+02
6.00E+66	1.00E+00	3.8710E-05	9.7810E-05	-4.2842E+16	1.00E+42	9.0581E+02
1.00E+67	1.00E+00	3.9660E-05	1.0014E-04	-5.6042E+16	3.00E+42	9.3851E+02
6.00E+67	1.00E+00	4.0620E-05	1.0250E-04	-7.2242E+16	1.00E+43	9.7191E+02
1.00E+68	1.00E+00	4.1590E-05	1.0487E-04	-9.3442E+16	3.00E+43	1.0060E+03
6.00E+68	1.00E+00	4.2570E-05	1.0727E-04	-1.2042E+17	1.00E+44	1.0415E+03
1.00E+69	1.00E+00	4.3560E-05	1.0968E-04	-1.5342E+17	3.00E+44	1.0786E+03
6.00E+69	1.00E+00	4.4560E-05	1.1211E-04	-1.9342E+17	1.00E+45	1.1163E+03
1.00E+70	1.00E+00	4.5570E-05	1.1456E-04	-2.5042E+17	3.00E+45	1.1546E+03
6.00E+70	1.00E+00	4.6590E-05	1.1703E-04	-3.2542E+17</		

Problem #4: P3.6

$$(a) \chi_{no} = \sqrt{\frac{2\epsilon_s \phi_0}{q N_A} \left(\frac{N_A}{N_A + N_D} \right)}$$

$$\phi_0 = (60 \text{ mV}) \left(\log \left(\frac{10^{16}}{10^{10}} \right) \right) + \log \left(\frac{5 \cdot 10^{16}}{10^{10}} \right)$$

$$\phi_0 = 0.762$$

$$\chi_{no} = 2.88 \times 10^{-5} \text{ cm} = 0.288 \mu\text{m}$$

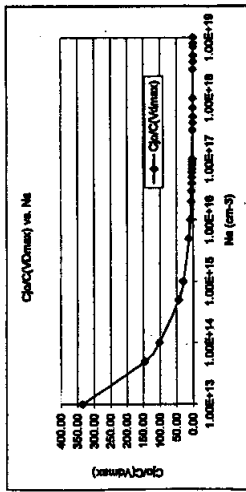
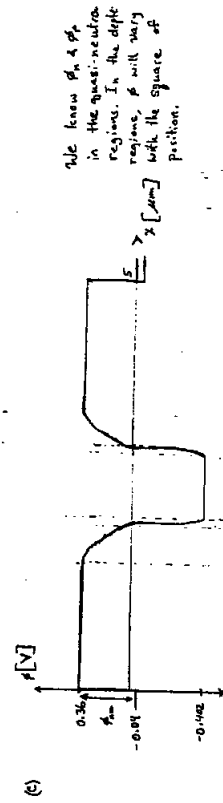
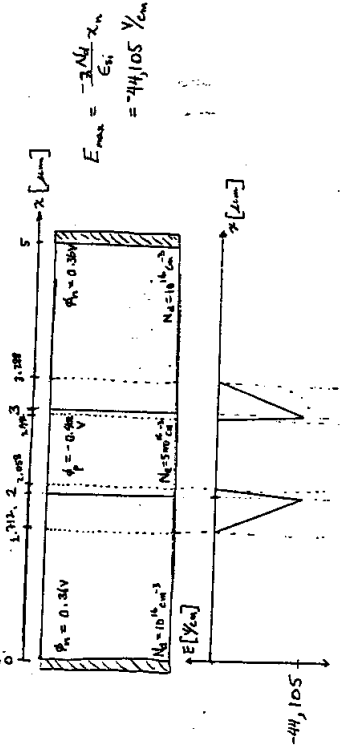
$$\chi_{po} = \frac{N_D}{N_A} \chi_{no} = 5.76 \times 10^{-6} \text{ cm} = 0.058 \mu\text{m}$$

$$W = 0.288 + 0.058$$

$$W = 0.346 \mu\text{m}$$

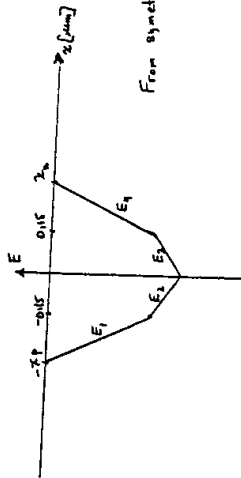
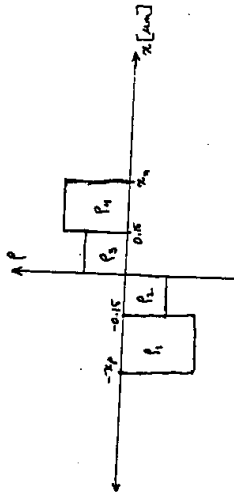
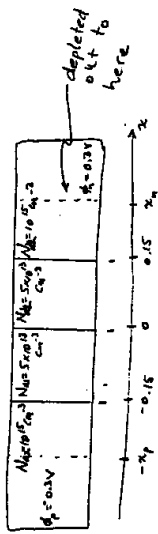
for both junctions

(b) There are two p-n junctions. Charge in depletion region is constant, so E-field is linear. Maximum |E| is at the p-n junction.

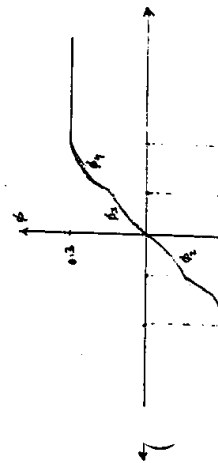


Problem #5: P3.8

The depletion region extends to the higher doped regions. The low doped regions are fully depleted.



From symmetry $x_n = x_p$



Problem #5: P3.8 (cont.)

$$\frac{dE}{dx} = -\rho$$

$$E_1 = \frac{qN_A}{\epsilon_s \epsilon_0} (x + x_p)$$

$$E_2(0) = -\frac{qN_A}{\epsilon_s \epsilon_0} (x_p - 0.15)$$

$$E_2 = -\frac{qN_A}{\epsilon_s \epsilon_0} (x + 0.15)$$

$$E_2(0) = -\frac{qN_A}{\epsilon_s \epsilon_0} (0.15) - \frac{qN_A}{\epsilon_s \epsilon_0} (x_p - 0.15)$$

$$\frac{d\phi}{dx} = -E(x)$$

$$\phi_1 = \frac{qN_A}{2\epsilon_s \epsilon_0} (x + x_p)^2 + \phi_p$$

$$\phi_1(-0.15) = -\frac{qN_A}{2\epsilon_s \epsilon_0} (x_p - 0.15)^2 + \phi_p$$

$$\phi_2 = \frac{qN_A}{2\epsilon_s \epsilon_0} (x + 0.15)^2 + \frac{qN_A}{2\epsilon_s \epsilon_0} (x_p - 0.15)^2 + \phi_p$$

$$\phi_2(0) = \frac{qN_A}{2\epsilon_s \epsilon_0} (0.15)^2 + \frac{qN_A}{2\epsilon_s \epsilon_0} (x_p - 0.15)^2 + \phi_p$$

By symmetry, we know that $\phi_1(0) = \phi_2(0)$;
convert to cm

$$\frac{qN_A}{2\epsilon_s \epsilon_0} (0.15)^2 + \frac{qN_A}{2\epsilon_s \epsilon_0} (x_p - 0.15)^2 - 0.3 = 0$$

$$x_p = 7.76 \times 10^{-5} \text{ cm. } \leftarrow \text{positive answer.}$$

$$x_n = 7.76 \times 10^{-5} \text{ cm}$$

$$W = 1.55 \times 10^{-4} \text{ cm}$$

(b) Previous pictures w/ $x_n = x_p$ & $-x_p = -x_n$.

$$E_{max} = E_2(0) = -13132 \text{ V/cm}$$

$$E_1(-0.15) = -13017 \text{ V/cm}$$

$$(c) \phi_2(0) = \frac{qN_A}{2\epsilon_s \epsilon_0} (0.15 \mu\text{m})^2 + \frac{qN_A}{2\epsilon_s \epsilon_0} (0.85 \times 10^{-4})^2 + (\phi_p + V_0) = 0$$

$$V_0 = -0.258 \text{ volts}$$

Problem #6:

(a) Resonant frequency: $f_r = \frac{1}{\sqrt{LC}} = 2.4 \text{ GHz w/ Lnt} \Rightarrow C_r = 4.40 \text{ pF}$

$$C_{j0} = \frac{qN_A X_{po}}{2\phi_B} = \frac{qE_s (N_A N_D)}{2\phi_B (N_A + N_D)} = 3.05 \times 10^{-8} \text{ F/cm}^2$$

$$\phi_B = \phi_n - \phi_p = 0.54 + 0.36 = 0.9 \text{ volts}$$

$$C_r = A C_{j0} \Rightarrow A = 1.44 \times 10^{-4} \text{ cm}^2$$

$$(b) X_{n\max} = \frac{E_s E_{\max}}{qN_A} = 3.26 \times 10^{-7} \text{ cm} \quad X_{no} = \sqrt{\frac{2E_s \phi_B}{8N_A} \left(\frac{N_A}{N_A + N_D} \right)}$$

$$X_{no} = 3.428 \times 10^{-2}$$

$$X_n(V_b) = X_{no} \sqrt{1 - \left(\frac{V_b}{\phi_B} \right)}$$

$$V_{b\max} = -\phi_B \left[\left(\frac{X_{n\max}}{X_{no}} \right)^2 - 1 \right] = -80.7 \text{ volts}$$

$$(c) C_j(V_b) = \frac{C_{j0}}{\sqrt{1 - V_b/\phi_B}} \Rightarrow C_j(0) = C_{j0} = 3.05 \times 10^{-8} \text{ F/cm}^2$$

$$C_j(V_{b\max}) = 3.20 \times 10^{-9} \text{ F/cm}^2$$

$$C_r(0) = 4.40 \text{ pF} \quad C_r(V_{b\max}) = 0.461 \text{ pF}$$

$$f_{r\min} = 2.4 \text{ GHz} \quad f_{r\max} = 7.41 \text{ GHz}$$

Plot increases as $\left(1 - \frac{V_b}{\phi_B}\right)^{1/4}$

