6.012 Problem Set #3 – Solutions

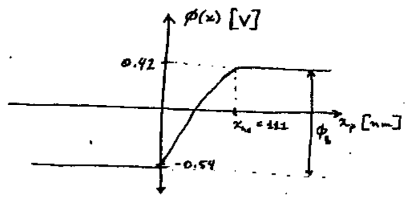
Problem #1: E3.12

$N_a = 10^{19}\,\text{cm}^{-3}$	$N_d = 10^{17}\,\text{cm}^{-3}$
$\phi_p = -540\,\text{mV}$	$\phi_n = 420\,\text{mV}$

$\phi_B = \phi_n - \phi_p = 0.96\,\text{V}$

$x_{po} = \frac{N_d}{N_a} x_{no} = 0.01\, x_{no}$

(a) $x_{no} = \sqrt{\left(\frac{2\epsilon_{si}\phi_B}{q N_d}\right)\left(\frac{N_a}{N_d + N_a}\right)} = 1.11 \times 10^{-5}\,\text{cm} = \boxed{111\ \text{nm}}$



(b) $|E_o(x=0)| = \frac{q N_d}{\epsilon_{si}} x_{no} = \boxed{1.7 \times 10^5\ \text{V/cm}}$

(c) $x_{n\,max} = \frac{\epsilon_{si} E_{max}}{q N_d} = 3.26 \times 10^{-5}\ \text{cm}$

$x_n(V_D) = x_{no}\sqrt{1 - (V_D/\phi_B)}$

$V_{D\,max} = -\phi_B\left(\left(\frac{x_{n\,max}}{x_{no}}\right)^2 - 1\right)$

$\boxed{V_{D\,max} = -7.32\ \text{Volts}}$

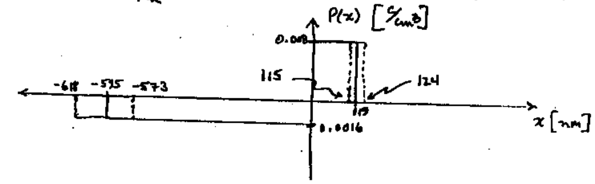
Problem #2: E3.14

(a) $\phi_B = \phi_n - \phi_p = (60\,\text{mV})\left(\log\left(\frac{5\times10^{16}}{10^{10}}\right) + \log\left(\frac{10^{16}}{10^{10}}\right)\right) = 0.762\ \text{volts.}$

$x_{no} = \sqrt{\left(\frac{2\epsilon_{si}\phi_B}{q N_d}\right)\left(\frac{N_a}{N_a + N_d}\right)} = 5.76 \times 10^{-6}\,\text{cm}$

$x_n(V_D = -2.5\,\text{V}) = x_{no}\sqrt{1 - (V_D/\phi_B)} = 1.19 \times 10^{-5}\ \text{cm} = 119\ \text{nm}$

$x_p = \frac{N_d}{N_a} x_n = 5 x_n = 5.95 \times 10^{-5}\ \text{cm} = 595\ \text{nm}$



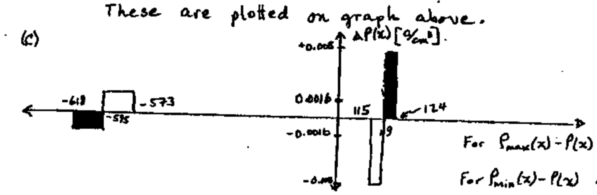
(b) Now Applying $v_D(t) = V_D + v_d(t)$ where $v_d(t) = (250\,\text{mV})\cos(\omega t)$

Extremes are when: $v_D(t) = -2.75$ volts, $x_n = 1.24 \times 10^{-5}$ cm & $x_p = 6.18 \times 10^{-5}$ cm

$v_D(t) = -2.25$ volts, $x_n = 1.15 \times 10^{-5}$ cm & $x_p = 5.73 \times 10^{-5}$ cm

These are plotted on graph above.

(c)



$\rho_{max}(x) - \rho(x) \Rightarrow$ ■

$\rho_{min}(x) - \rho(x) \Rightarrow$ □

For $\rho_{max}(x) - \rho(x)$: $q_j = (\Delta\rho(x))(\Delta x_n) = 4 \times 10^{-9}\ \text{C/cm}^2$

For $\rho_{min}(x) - \rho(x)$: $q_j = (\Delta\rho(x))(\Delta x_n) = -3.2 \times 10^{-9}\ \text{C/cm}^2$

These are not equal and opposite, but very close. It is not equal and opposite because x_n, x_p is not linear with V_D. At very small signals it seems linear because $x_n \propto \sqrt{1 - \left(\frac{V_D}{\phi_B}\right)} \approx 1 - \frac{1}{2}\left(\frac{V_D}{\phi_B}\right)$ for small V_D.

(d) $C_{jo} = \frac{q N_a x_{po}}{2\phi_B} = 3.02 \times 10^{-8}\ \text{C/cm}^2$ $\quad C_j(V_D) = \frac{C_{jo}}{\sqrt{1 - \frac{V_D}{\phi_B}}}$

$C_j(V_D = -2.5\,\text{volts}) = 1.46 \times 10^{-8}\ \text{C/cm}^2$

$q_j = (\Delta v_D)(C_j(2.5)) = \boxed{3.65 \times 10^{-9}\ \text{C/cm}^2}$ → This is similar to the figures found in part (b).

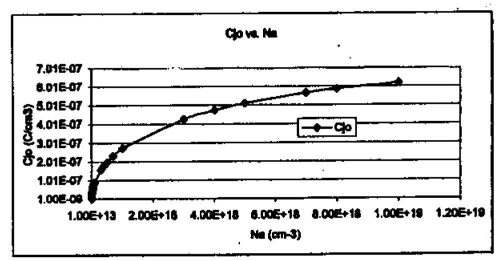
Problem #3: E3.15

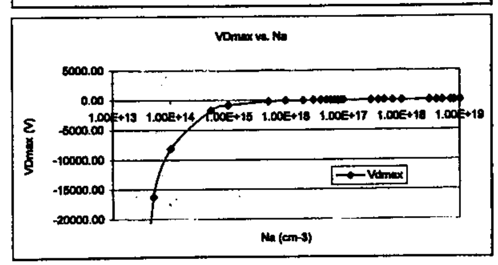
(a) Plot C_{jo} with N_a from $10^{13}\ cm^{-3}$ to $10^{19}\ cm^{-3}$ w/ $N_d = 10^{15}\ cm^{-3}$

$$C_{jo} = \sqrt{\frac{q\epsilon_{si}}{2\phi_B}\left(\frac{N_a N_d}{N_d + N_a}\right)} \quad \& \quad \phi_B = (60\ mV)\left[\log\left(\frac{N_d}{n_i}\right) + \log\left(\frac{N_a}{n_i}\right)\right]$$

Ploted on MS-Excel :

Na	phi_b	Cjo	Xno	Vdmax	C(Vdmax)	Cjo/C(Vdmax)
1.00E+13	7.20E-01	1.07743E-09	9.69668E-10	-81376.7	3.20E-12	3.36E+02
5.00E+13	7.62E-01	2.34196E-09	2.23064E-09	-16274.8	1.60E-11	1.46E+02
1.00E+14	7.80E-01	3.27345E-09	3.19162E-09	-8137.04	3.20E-11	1.02E+02
5.00E+14	8.22E-01	7.13034E-09	7.32567E-09	-1626.81	1.60E-10	4.45E+01
1.00E+15	8.40E-01	9.87457E-09	1.04733E-08	-813.015	3.20E-10	3.11E+01
5.00E+15	8.82E-01	2.17627E-08	2.39917E-08	-161.954	1.60E-09	1.36E+01
1.00E+16	9.00E-01	3.04591E-08	3.42665E-08	-80.5687	3.20E-09	9.51E+00
2.00E+16	9.18E-01	4.26286E-08	4.89198E-08	-39.852	6.40E-09	6.66E+00
3.00E+16	9.29E-01	5.16654E-08	6.02278E-08	-26.2765	9.59E-09	5.41E+00
4.00E+16	9.36E-01	5.9842E-08	6.97904E-08	-19.4696	1.28E-08	4.67E+00
5.00E+16	9.42E-01	6.64428E-08	7.8231E-08	-15.4146	1.59E-08	4.17E+00
6.00E+16	9.47E-01	7.25653E-08	8.58709E-08	-12.6976	1.91E-08	3.80E+00
7.00E+16	9.51E-01	7.81748E-08	9.29016E-08	-10.758	2.23E-08	3.51E+00
8.00E+16	9.54E-01	8.33785E-08	9.94482E-08	-9.29936	2.54E-08	3.28E+00
9.00E+16	9.57E-01	8.82506E-08	1.05596E-07	-8.16606	2.86E-08	3.09E+00
1.00E+17	9.60E-01	9.28452E-08	1.11414E-07	-7.25911	3.17E-08	2.93E+00
3.00E+17	9.89E-01	1.56921E-07	1.93921E-07	-1.80533	9.33E-08	1.68E+00
4.00E+17	9.96E-01	1.79644E-07	2.23684E-07	-1.11969	1.23E-07	1.46E+00
5.00E+17	1.00E+00	1.99306E-07	2.49618E-07	-0.70699	1.53E-07	1.31E+00
7.00E+17	1.01E+00	2.32595E-07	2.93866E-07	-0.23321	2.10E-07	1.11E+00
1.00E+18	1.02E+00	2.72935E-07	3.47992E-07	0.124949	2.91E-07	9.37E-01
3.00E+18	1.05E+00	4.28878E-07	5.62167E-07	0.695992	7.40E-07	5.80E-01
4.00E+18	1.06E+00	4.75515E-07	6.27763E-07	0.771303	9.16E-07	5.19E-01
5.00E+18	1.06E+00	5.12207E-07	6.79915E-07	0.817606	1.07E-06	4.79E-01
7.00E+18	1.07E+00	5.66951E-07	7.56797E-07	0.873075	1.32E-06	4.30E-01
8.00E+18	1.07E+00	5.88064E-07	7.89613E-07	0.891066	1.42E-06	4.13E-01
1.00E+19	1.08E+00	6.22056E-07	8.39774E-07	0.917245	1.60E-06	3.88E-01





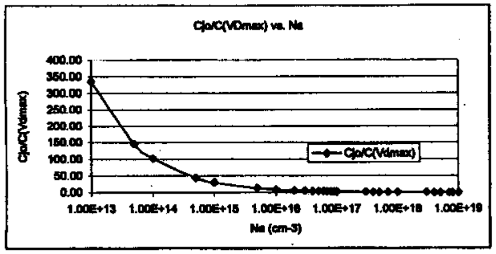
(b) $V_{Dmax} = -\phi_B\left(\left(\frac{x_{nmax}}{x_{no}}\right)^2 - 1\right)$ $\qquad x_{nmax} = \frac{\epsilon_{si} E_{max}}{q N_d} = 3.26 \times 10^{-7}$

$$x_{no} = \sqrt{\left(\frac{2\epsilon_{si}\phi_B}{qN_d}\right)\left(\frac{N_a}{N_d + N_a}\right)}$$

See Attached Plots

(c) $C(V_{Dmax}) = \frac{C_{jo}}{\sqrt{1 - \frac{V_{Dmax}}{\phi_B}}}$

See Attached Plots.



Problem #4: P3.6

(a) $x_{no} = \sqrt{\frac{2\epsilon_{si}\phi_B}{q N_D}\left(\frac{N_a}{N_a+N_d}\right)}$

$\phi_B = (60\,mV)\left(\log\left(\frac{10^{16}}{10^{10}}\right) + \log\left(\frac{5\cdot 10^{16}}{10^{10}}\right)\right)$

$\phi_B = 0.762$

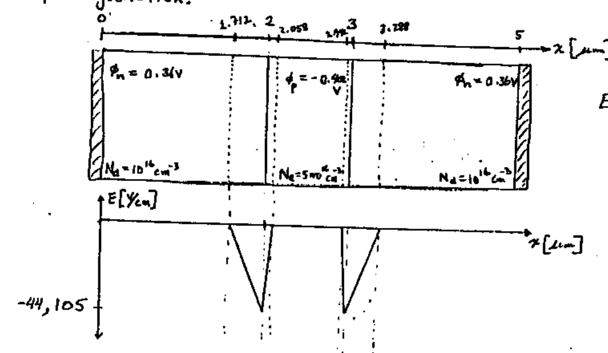
$x_{no} = 2.88 \times 10^{-5}\ cm = 0.288\ \mu m$

$x_{po} = \frac{N_d}{N_a} x_{no} = 5.76 \times 10^{-6}\ cm = 0.058\ \mu m$

$W = 0.288 + 0.058$

$\boxed{W = 0.346\ \mu m}$ for both junctions

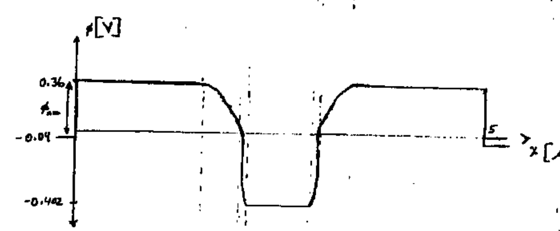
(b) There are two p-n junctions. Charge in depletion region is constant, so E-field is linear. Maximum $|E_o|$ is at the p-n junction.



$E_{max} = \frac{-q N_d}{\epsilon_{si}} x_n$

$= -44{,}105\ V/cm$

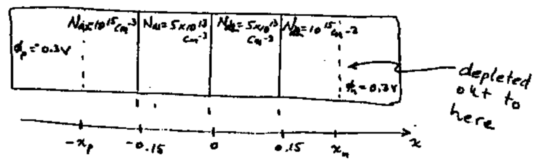
(c)

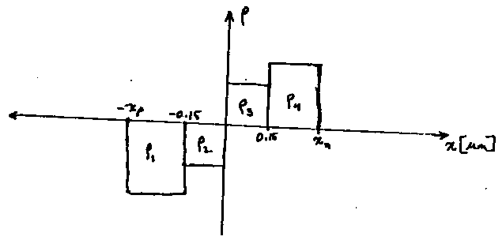


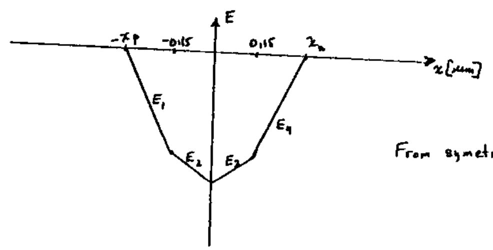
We know ϕ_n & ϕ_p in the quasi-neutral regions. In the depletion regions, ϕ will vary with the square of position.

Problem #5: P 3.8

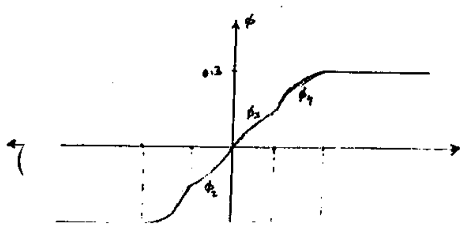
The depletion region extends to the higher doped regions. The low doped regions are fully depleted.







From symetry → $x_n = x_p$



Problem #5: P3.8 (cont.)

$\frac{dE}{dx} = \frac{\rho}{\epsilon}$

$E_1 = -\frac{qN_{A2}}{\epsilon_{si}}(x + x_p)$ $\qquad$ $E_2 = -\frac{qN_{A1}}{\epsilon_{si}}(x + 0.15) - \frac{qN_{A2}}{\epsilon_{si}}(x_p - 0.15)$

$E_1(0.15) = -\frac{qN_{A2}}{\epsilon_{si}}(x_p - 0.15)$ $\qquad$ $E_2(0) = -\frac{qN_{A1}}{\epsilon_{si}}(0.15) - \frac{qN_{A2}}{\epsilon_{si}}(x_p - 0.15)$

$\frac{d\phi}{dx} = -E(x)$;

$\phi_1 = \frac{qN_{A2}}{2\epsilon_{si}}(x + x_p)^2 + \phi_p$ $\qquad$ $\phi_2 = +\frac{qN_{A1}}{2\epsilon_{si}}(x + 0.15)^2 + \frac{qN_{A2}}{2\epsilon_{si}}(x_p - 0.15)^2 + \phi_p$

$\phi_1(-0.15) = \frac{qN_{A2}}{2\epsilon_{si}}(x_p - 0.15)^2 + \phi_p$ $\qquad$ $\phi_2(0) = \frac{qN_{A1}}{2\epsilon_{si}}(0.15)^2 + \frac{qN_{A2}}{2\epsilon_{si}}(x_p - 0.15)^2 + \phi_p$

By symetry, we know that $\phi_2(0) = 0$:

(convert to cm)

$$\frac{qN_{A1}}{2\epsilon_{si}}(0.15)^2 + \frac{qN_{A2}}{2\epsilon_{si}}(x_p - 0.15)^2 - 0.3 = 0$$

$x_p = 7.76\times10^{-5}$ cm. ← positive answer.

$x_n = 7.76\times10^{-5}$ cm $\qquad$ $W = 1.55\times10^{-4}$ cm.

(b) Previous pictures w/ $x_n = 1\ \mu m$ & $-x_p = -1\ \mu m$.

$E_{max} = E_2(0) = -13132$ V/cm.

$E_1(-0.15) = -13017$ V/cm

(c) $\phi_2(0) = \frac{qN_{A1}}{2\epsilon_{si}}(0.15\ \mu m)^2 + \frac{qN_{A2}}{2\epsilon_{si}}(0.85\times10^{-4})^2 + (\phi_p + V_D) = 0$

$V_D = -0.253$ volts.

Problem #6:

(a) Resonant frequency: $f_r = \frac{1}{2\pi\sqrt{LC}} = 2.4\ \text{GHz}$ w/ 1nH $\Rightarrow C_r = 4.40\ \text{pF}$

$$C_{jo} = \frac{q N_a x_{po}}{2\phi_B} = \sqrt{\frac{q\epsilon_{si}}{2\phi_B}\left(\frac{N_a N_d}{N_a + N_d}\right)} = 3.05 \times 10^{-8}\ \text{F/cm}^2$$

$$\phi_B = \phi_n - \phi_p = 0.54 + 0.36 = 0.9\ \text{volts}$$

$$C_r = A\, C_{jo} \Rightarrow \boxed{A = 1.44 \times 10^{-4}\ \text{cm}^2}$$

(b) $$x_{n,max} = \frac{\epsilon_{si} E_{max}}{q N_d} = 3.26 \times 10^{-7}\ \text{cm}$$ $$x_{no} = \sqrt{\frac{2\epsilon_{si}\phi_B}{q N_d}\left(\frac{N_a}{N_d + N_a}\right)}$$

$$x_n(V_D) = x_{no}\sqrt{1 - \left(\frac{V_D}{\phi_B}\right)}$$ $$x_{no} = 3.428 \times 10^{-8}$$

$$V_{D,max} = -\phi_B\left[\left(\frac{x_{n,max}}{x_{no}}\right)^2 - 1\right] = \boxed{-80.7\ \text{volts}}$$

(c) $$C_j(V_D) = \frac{C_{jo}}{\sqrt{1 - V_D/\phi_B}} \Rightarrow$$ $$C_j(0) = C_{jo} = 3.05 \times 10^{-8}\ \text{F/cm}^2$$ $$C_j(V_{D,max}) = 3.20 \times 10^{-9}\ \text{F/cm}^2$$

$C_r(0) = 4.40\ \text{pF}$ $\quad C_r(V_{D,max}) = 0.461\ \text{pF}$

$f_{r_{min}} = 2.4\ \text{GHz}$ $\quad f_{r_{max}} = 7.41\ \text{GHz}$

Plot increases as $\left(1 - \frac{V_D}{\phi_B}\right)^{1/4}$