Problem Set #5 Solutions

Problem #1: E4.20

$V_{SG} = 1.5\text{ V} \qquad V_{SD} = 4\text{ V} \qquad V_{SB} = -2\text{ V} \qquad L_D = 0.1\text{ V}$

$$V_{Tp} = V_{Top} - \gamma_p\left(\sqrt{2\phi_n - V_{SB}} - \sqrt{2\phi_n}\right)$$

$$V_{Tp} = -1.0 - 0.6\left(\sqrt{0.84 + 2} - \sqrt{0.84}\right)$$

$$V_{Tp} = -1.461\text{ V}$$

$V_{SG} \geq -V_{Tp}$ & $V_{SD} \geq V_{SG} + V_{Tp} \Rightarrow$ inversion.

$L_{eff} = L - 2L_D = 1.8\,\mu m$

(a) $g_m = \left(\frac{W}{L}\right)\mu_p C_{ox}(V_{SG} + V_{Tp}) = \left(\frac{26}{1.8}\right)(25 \times 10^{-6})(0.039)$

$$\boxed{g_m = 1.41 \times 10^{-5}\ \mho}$$

$$g_{mb} = \frac{\gamma_p g_m}{2\sqrt{2\phi_n - V_{SB}}} = \frac{(0.6)(1.41 \times 10^{-5})}{2\sqrt{0.84 + 2}}$$

$$\boxed{g_{mb} = 2.51 \times 10^{-6}\ \mho}$$

$$r_o = \left(\frac{2L}{W}\right)\frac{1}{\mu_p C_{ox}(V_{SG} + V_{Tp})^2 \lambda_p} = \left(\frac{3.6}{26}\right)\frac{1}{(25 \times 10^{-6})(0.039)^2\left(\frac{0.1}{1.8}\right)}$$

$$\boxed{r_o = 65.5\ M\Omega}$$

(b) $\mu_p(N_D = 10^{17}) = 350\ \frac{cm^2}{V \cdot s}$ from table

$C_{ox} = 7.14 \times 10^{-7}\ F/cm^2$

$C_{ov} = C_{ox} L_D = 7.14 \times 10^{-12}\ F/cm$

$C_{gs} = \frac{2}{3} W L C_{ox} + W C_{ov} = \boxed{242\ fF}$

$\boxed{C_{gd} = W C_{ov} = 19\ fF}$

$C_{db} = WL_D\sqrt{\frac{q\epsilon_{Si}N_a}{2(\phi_B - V_{DB})}}$

$\boxed{C_{db} = 53.8\ fF}$

$C_{sb} = WL_D\sqrt{\frac{q\epsilon_{Si}N_a}{2(\phi - V_{SB})}}$

$\boxed{C_{sb} = 82\ fF}$

Problem #2: P4-1

(a) $V_{Tn} = V_{Ton} + \gamma_n \left(\sqrt{-2\phi_p - V_{BS}} - \sqrt{-2\phi_p}\right)$

$V_{Ton} = 1\ V$

$V_{Tn1}(V_{BS} = -3.2\ V) = 3\ V$

$V_{Tn2}(V_{BS} = -8.2\ V) = 4.8\ V$ All in Linear region of operation.

$$V_{Tn1} = 3 = 1 + \gamma_n \left(\sqrt{-2\phi_p + 3.2} - \sqrt{-2\phi_p}\right)$$

$$V_{Tn2} = 4.8 = 1 + \gamma_n \left(\sqrt{-2\phi_p + 8.2} - \sqrt{-2\phi_p}\right)$$

$$\frac{\cancel{\gamma_n}\left(\sqrt{-2\phi_p + 3.2} - \sqrt{-2\phi_p}\right)}{\cancel{\gamma_n}\left(\sqrt{-2\phi_p + 8.2} - \sqrt{-2\phi_p}\right)} = \frac{2}{3.8}$$

$$\sqrt{-2\phi_p + 3.2} - \sqrt{-2\phi_p} = 0.526\sqrt{-2\phi_p + 8.2} - 0.526\sqrt{-2\phi_p}$$

$\boxed{\phi_p = -0.387\ V}$

$\boxed{N_a = 2.91 \times 10^{16}\ cm^{-3}}$

$\boxed{\mu_n = 950\ \frac{cm^2}{V \cdot s}}$ from figure 2.8

$$V_{Ton} = V_{FB} - 2\phi_p + \frac{\sqrt{2 q \epsilon_{Si} N_a (-2\phi_p)}}{C_{ox}}$$

$$1 = -(0.55 + 0.387) + 2(0.387) + \frac{\sqrt{2(1.6\times10^{-19})(11.8)(8.854\times10^{-14})(2.91\times10^{16})(2)(0.387)}}{C_{ox}}$$

$$C_{ox} = 7.46 \times 10^{-8} = \frac{\epsilon_{ox}}{t_{ox}}$$

$$t_{ox} = 4.62 \times 10^{-6}\ cm = \boxed{46.2\,nm = t_{ox}}$$

Problem # 2 ; P4.1 (cont.)

(b) Uncertainty in I_D measurement.

$$I_D = \frac{W}{L}\mu_n C_{ox}\left(v_{GS} - V_{Tn} - \frac{v_{DS}}{2}\right)v_{DS}$$

Assume no variance in threshold voltage.

$I_D \propto \frac{1}{t_{ox}}$

N_a effects V_{Tn}

μ_n decreases w/ increasing N_a

$\mu_n \propto I_D$

Problem # 3 : P4.11

a) $$q_N(v_{GS}, v_{DS}) = -W\int_0^L C_{ox}\left(v_{GS} - V_{Tn} - v_C(y)\right)^2 dy$$

$$I_D = W\mu_n C_{ox}\left(v_{GS} - V_{Tn} - v_C(y)\right)\frac{dv_C}{dy}$$

$$dy = \left(\frac{W\mu_n C_{ox}}{I_D}\right)(v_{GS} - V_{Tn} - v_C)\,dv_C$$

$$q_N(v_{GS}, v_{DS}) = -\left(\frac{W^2\mu_n C_{ox}^2}{i_D}\right)\int_0^{v_{DS}}(v_{GS} - V_{Tn} - v_C)^2\,dv_C$$

$$i_D = \frac{W}{L}C_{ox}\mu_n\left(v_{GS} - V_{Tn} - \frac{v_{DS}}{2}\right)v_{DS} \leftarrow \text{in linear region.}$$

* Limit of integration is v_{DS} not $v_{DS,sat}$ as it is in saturation.

Everything else is same as derivation for saturation.

$$q_N(v_{GS}, v_{DS}) = \frac{-W^2\mu_n C_{ox}^2}{i_D}\,\frac{1}{3}\left(v_{GS} - V_{Tn} - v_C\right)^3\Big|_0^{v_{DS}}$$

$$\boxed{q_N(v_{GS}, v_{DS}) = -\frac{WLC_{ox}}{3}\,\frac{\left[(v_{GS} - V_{Tn} - v_{DS})^3 - (v_{GS} - V_{Tn})^2\right]}{(v_{GS} - V_{Tn} - \frac{v_{DS}}{2})\,v_{DS}}}$$

b) P.4.11 (cont.)

$$q_N(v_{GS}, v_{DS}) = \frac{-WLC_{ox}}{3} \; \frac{\left[(v_{GS} - V_{Tn} - v_{DS})^3 - (v_{GS} - V_{Tn})^3\right]}{\left(v_{GS} - V_{Tn} - \frac{v_{DS}}{2}\right) v_{DS}}$$

$$\frac{dq_N}{dv_{GS}} = C_{gs} = \frac{-WLC_{ox}}{3} \cdot \left[\frac{\left[3(v_{GS} - V_{Tn} - v_{DS})^2 - 3(v_{GS} - V_{Tn})^2\right]\left[\left(v_{GS} - V_{Tn} - \frac{v_{DS}}{2}\right) v_{DS}\right]}{\left[\left(v_{GS} - V_{Tn} - \frac{v_{DS}}{2}\right) v_{DS}\right]^2} - \frac{\left[v_{DS}\right]\left[(v_{GS} - V_{Tn} - v_{DS})^3 - (v_{GS} - V_{Tn})^3\right]}{\left[\left(v_{GS} - V_{Tn} - \frac{v_{DS}}{2}\right) v_{DS}\right]^2} \right]$$

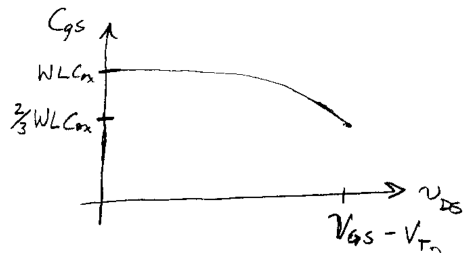
$$= \frac{-WLC_{ox}}{3} \cdot \left[\frac{3(v_{GS} - V_{Tn} - v_{DS})^2 \left(v_{GS} - V_{Tn} - \frac{v_{DS}}{2}\right) v_{DS}}{\left[v_{GS} - V_{Tn} - \frac{v_{DS}}{2}\right]^2 v_{DS}^2} - \frac{3(v_{GS} - V_{Tn})^2 \left(v_{GS} - V_{Tn} - \frac{v_{DS}}{2}\right) v_{DS}}{\left(v_{GS} - V_{Tn} - \frac{v_{DS}}{2}\right)^2 v_{DS}^2} + \frac{(v_{GS} - V_{Tn})^3 \, v_{DS}}{\left(v_{GS} - V_{Tn} - \frac{v_{DS}}{2}\right)^2 v_{DS}^2} - \frac{(v_{GS} - V_{Tn} - v_{DS})^3 \, v_{DS}}{\left(v_{GS} - V_{Tn} - \frac{v_{DS}}{2}\right)^2 v_{DS}^2} \right]$$

$$\boxed{C_{gs} = \frac{-WLC_{ox}}{3} \left[\frac{3(v_{GS} - V_{Tn} - v_{DS})^2}{\left(v_{GS} - V_{Tn} - \frac{v_{GS}}{2}\right) v_{DS}} - \frac{3(v_{GS} - V_{Tn})^2}{\left(v_{GS} - V_{Tn} - \frac{v_{DS}}{2}\right) v_{DS}} + \frac{(v_{GS} - V_{Tn})^3}{\left(v_{GS} - V_{Tn} - \frac{v_{DS}}{2}\right)^2 v_{DS}} - \frac{(v_{GS} - V_{Tn} - v_{DS})^3}{\left(v_{GS} - V_{Tn} - \frac{v_{DS}}{2}\right)^2 v_{DS}} \right]}$$

$$v_{GD} = v_{GS} - v_{DS} \qquad \frac{dv_{GS}}{dv_{GD}} = 1$$

$$C_{gd} = \frac{dq_N}{v_{GD}} = \frac{dq_N}{dv_{GS}} \cdot \frac{dv_{GS}}{dv_{GD}} = \boxed{C_{gs} = C_{gd}}$$

c) P4.11 (cont.)

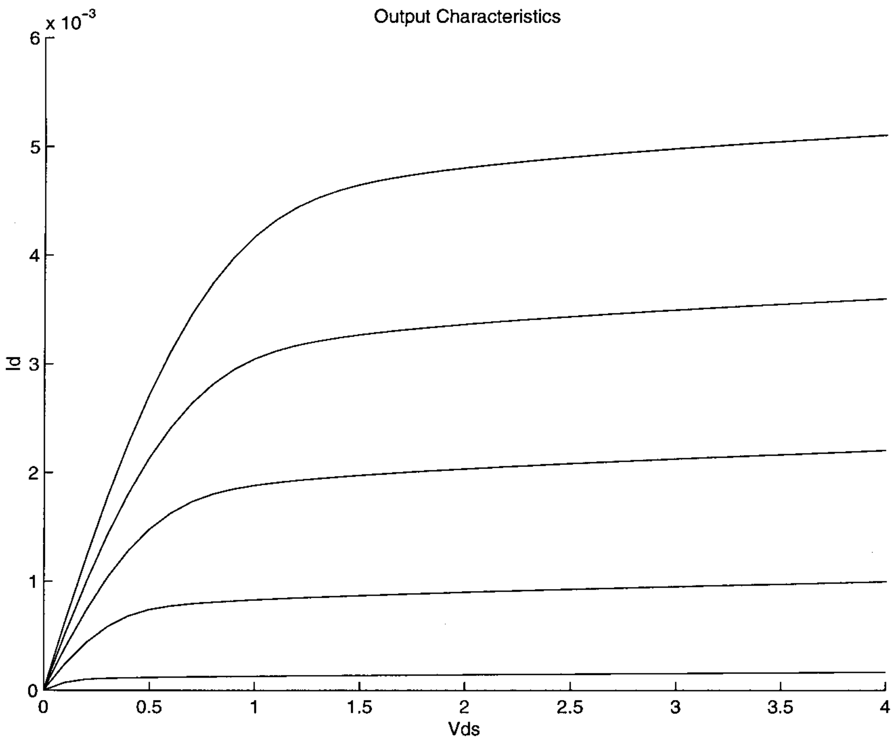


It can be seen here that as

$v_{DS} \rightarrow v_{DS_{sat}} = v_{GS} - V_{Tn}$, $C_{gs} \rightarrow \frac{2}{3} WLC_{ox}$.

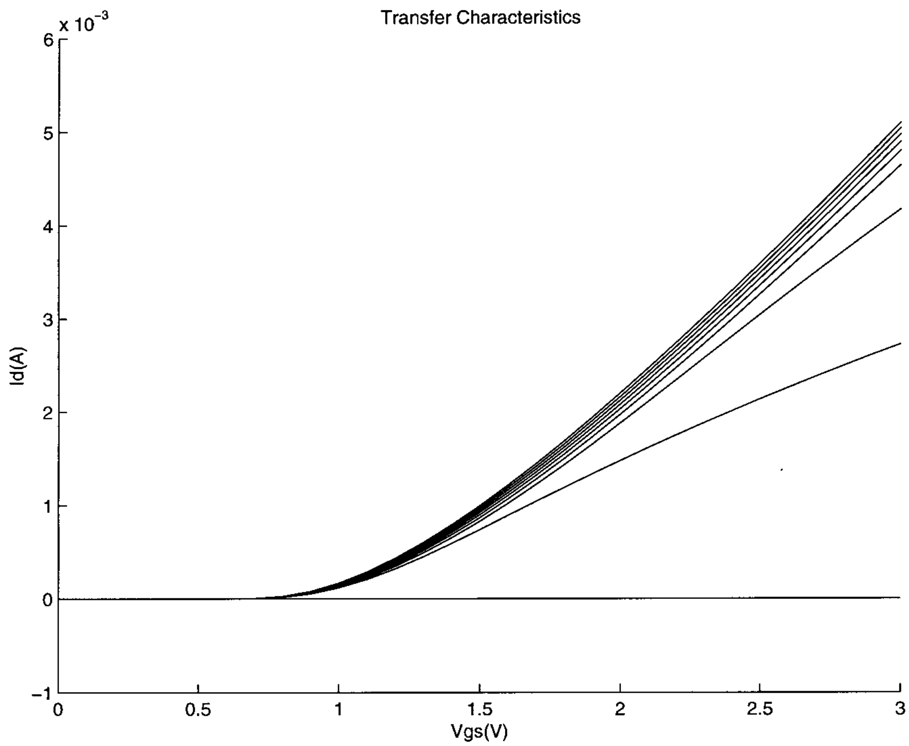
4)

a)



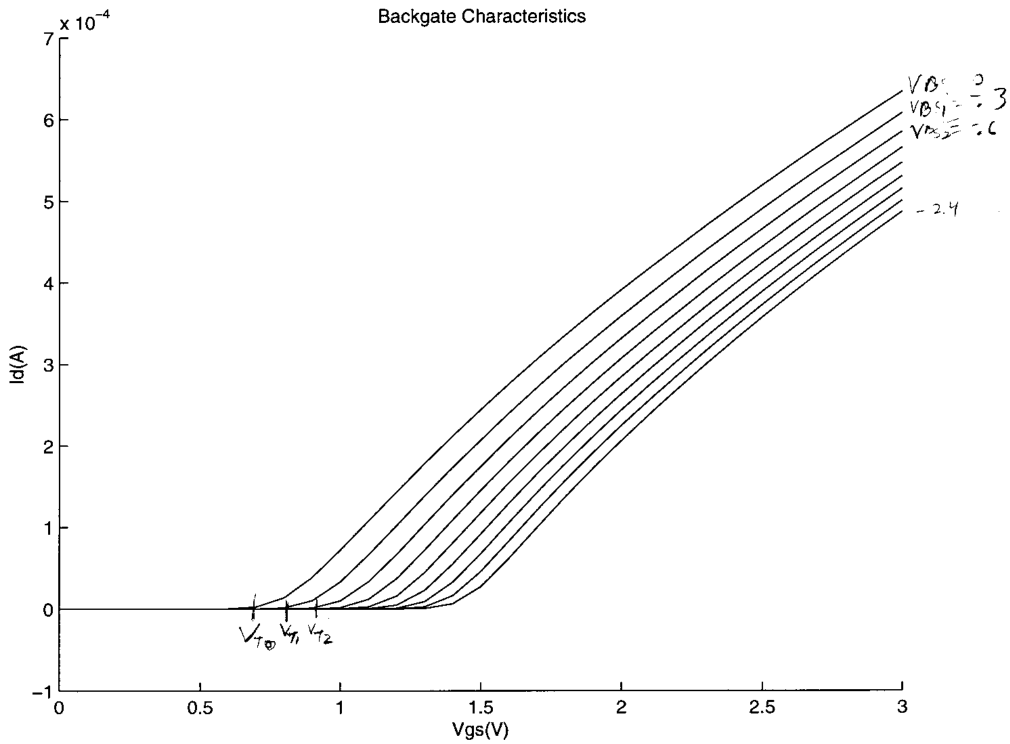
Output Characteristics
x 10^-3
6
5
4
3
2
1
0
Id
0
0.5
1
1.5
2
2.5
3
3.5
4
Vds

b)



Transfer Characteristics
x 10^-3
6
5
4
3
2
1
0
−1
Id(A)
0
0.5
1
1.5
2
2.5
3
Vgs(V)

c)


Backgate Characteristics
x 10^-4
7
6
5
4
3
2
1
0
-1
Id(A)
0
0.5
1
1.5
2
2.5
3
Vgs(V)
VBS 0
VBS1 -.3
VBS2 -.6
-2.4
VT0 VT1 VT2

d) $V_T = V_{TO} + GAMMA(\sqrt{2 \cdot PHI - V_{BS}} - \sqrt{2 \cdot PHI})$

get V_T from backgate characteristic plot
$V_T \approx .7V$

choose phi

solve for gamma using the different values for V_T ($V_{T1}, V_{T2}, \ldots$) and their corresponding values of V_{bs} ($V_{bs1}, V_{bs2}, \ldots$)

$GAMMA \approx .75$

e) $I_{DS} = \frac{KP}{2}(W/L) V_{DS}[2(V_{GS} - V_{TH}) - V_{DS}][1 + LAMBDA \cdot V_{DS}]$
$(0 \leq V_{DS} \leq V_{GS} - V_{TH})$ triode

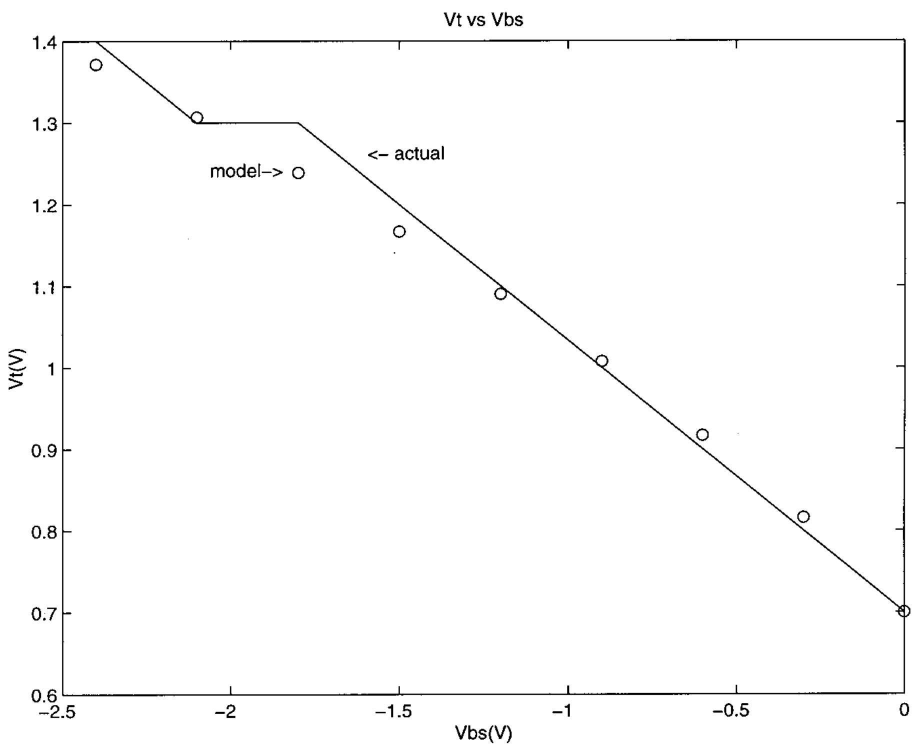
$I_{DS} = \frac{KP}{2}(W/L)(V_{GS} - V_{TH})^2[1 + LAMBDA \cdot V_{DS}]$
$(0 \leq V_{GS} - V_{TH} \leq V_{DS})$ saturation

Solve for Kp and LAMBDA using measured data.

$K_p \approx 50 \mu A/V^2 - 100 \mu A/V^2$

Lambda $\approx .05 V^{\frac{1}{2}} - .1 V^{\frac{1}{2}}$

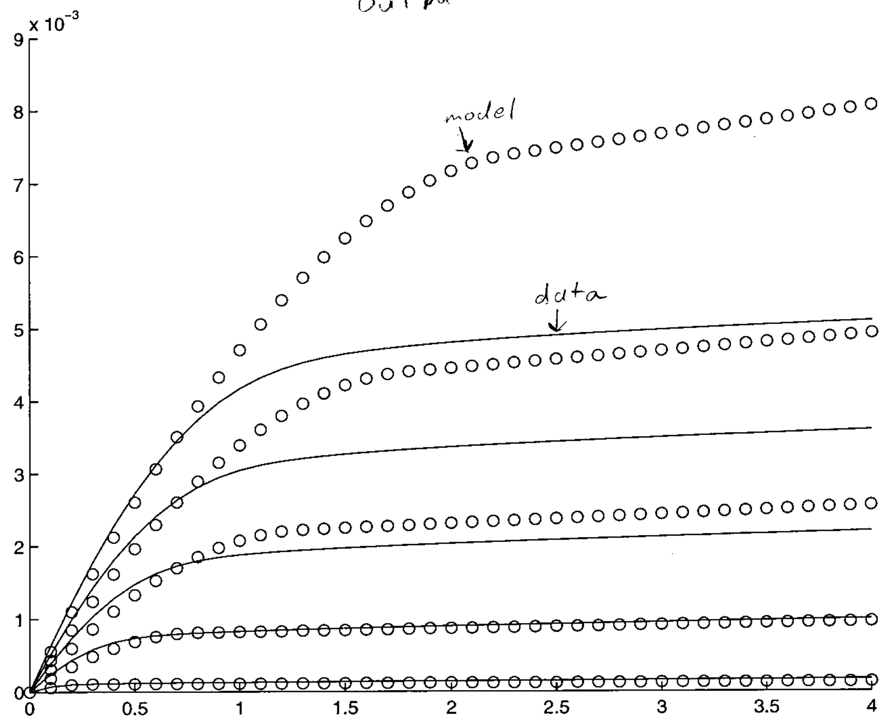
e)



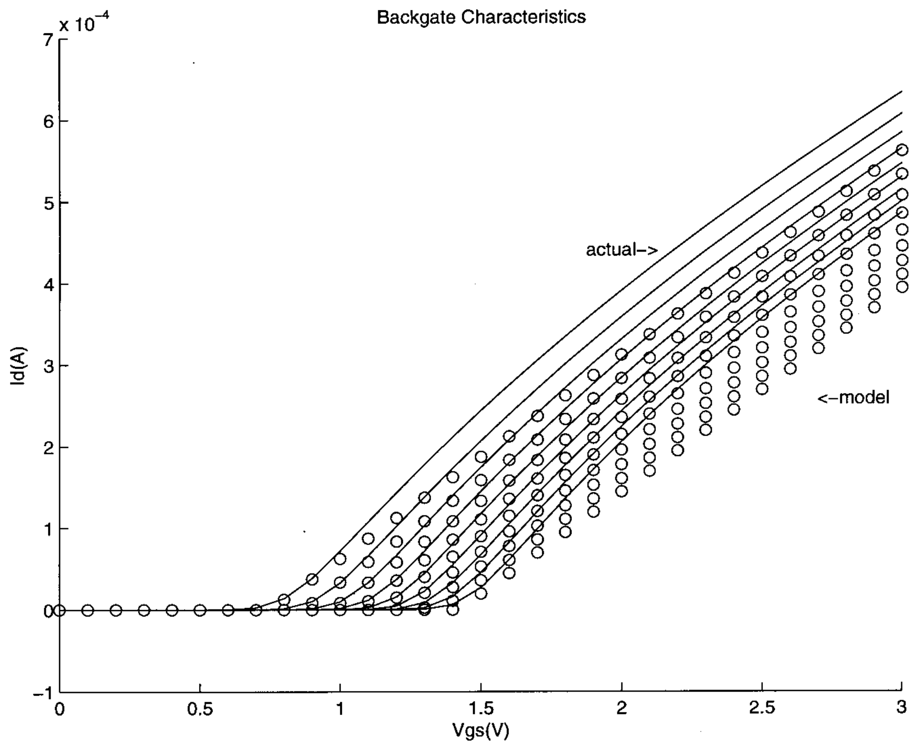
Vt vs Vbs
1.4
1.3
1.2
1.1
1
0.9
0.8
0.7
0.6
-2.5
-2
-1.5
-1
-0.5
0
Vt(V)
Vbs(V)
model->
<- actual

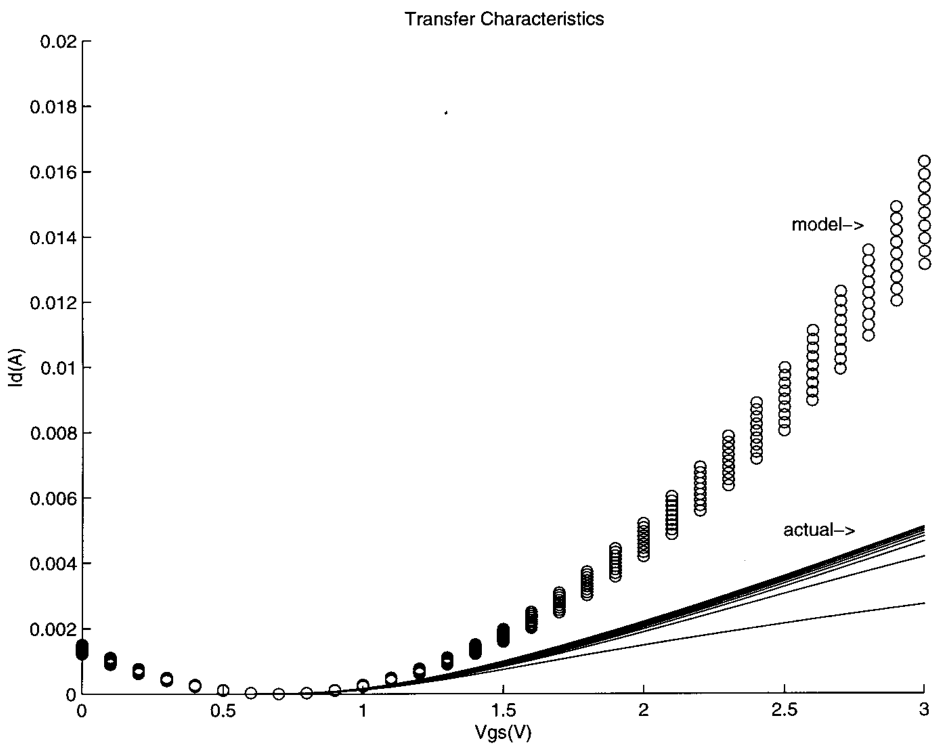
f)

Output Characteristics



f)


Backgate Characteristics
x 10^-4
7
6
5
4
3
2
1
0
-1
Id(A)
actual->
<-model
0
0.5
1
1.5
2
2.5
3
Vgs(V)


Transfer Characteristics
model->
actual->
Id(A)
Vgs(V)
0.02
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
0
0.5
1
1.5
2
2.5
3