6.012 - Problem Set #6 Solutions

Problem #1: E6.2

$A = 25 \times 25\ \mu m^2 \qquad N_a = 10^{17}\ cm^{-3} \qquad N_d = 10^{17}\ cm^{-3}$

$W_p = 2\mu m \qquad W_n = 2\mu m$

(a) $V_D = 700\ mV$

$$n_p(-x_p) = n_{po}\, e^{\frac{qV_D}{kT}} = 10^3\, e^{\frac{0.7}{0.026}} = \underline{4.93 \times 10^{14} \ll 10^{17}} = p_{po}$$

$$p_n(x_n) = p_{no}\, e^{\frac{qV_D}{kT}} = 10^3\, e^{\frac{0.7}{0.026}} = \underline{4.93 \times 10^{14} \ll 10^{17}} = n_{no}$$

(b)
$$\frac{I_D^{p}}{I_D^{n+p}} = \frac{\frac{D_p}{N_d W_n}}{\frac{D_p}{N_d W_n} + \frac{D_n}{N_a W_p}} = \boxed{0.304 = \frac{I_D^{p}}{I_D^{n+p}}}$$

$\mu_n(10^{17}) = 800\ \frac{cm^2}{V \cdot s} \qquad D_n = \frac{kT}{q}\mu_n = 20.8\ \frac{cm^2}{s}$

$\mu_p(10^{17}) = 350\ \frac{cm^2}{V \cdot s} \qquad D_p = \frac{kT}{q}\mu_p = 9.1\ \frac{cm^2}{s}$

(c)
$$I_o = q n_i^2 A \left[\frac{D_p}{N_d W_n} + \frac{D_n}{N_a W_p}\right] = (1.6 \times 10^{-19})(10^{10})^2 (25 \times 10^{-4})^2 \left[\frac{20.8}{(10^{17})(2 \times 10^{-4})} + \frac{9.1}{(10^{17})(2 \times 10^{-4})}\right]$$

$$I_o = 1.495 \times 10^{-16}\ A$$

$$I_D = I_o\left(e^{\frac{qV_D}{kT}} - 1\right)$$

$$I_D = (1.495 \times 10^{-16})\left(e^{\frac{0.7}{0.026}} - 1\right)$$

$$\boxed{I_D = 73.65\ \mu A}$$

Problem #2: E6-6

$V_D = 700\ mV$

(a) $r_d = \frac{1}{g_d} = \frac{kT}{q I_D} = \frac{0.026\ V}{73.65\ \mu A} = \boxed{353\ \Omega}$

(b) $C_{jo} = A\sqrt{\frac{q \epsilon_{si} N_a N_d}{2(N_a + N_d)\phi_B}} = (25 \times 10^{-4})^2 \sqrt{\frac{(1.6\times 10^{-19})(11.8)(8.854\times 10^{-14})(10^{17})(10^{17})}{2(10^{17}+10^{17})(0.84)}}$

$\phi_B = (60\ mV)\left(\log_{10}\left(\frac{N_d}{n_i}\right)\right) + 60\ mV\left(\log_{10}\left(\frac{N_a}{n_i}\right)\right) = 0.84\ V$

$C_{jo} = 4.41 \times 10^{-13}\ F$

$C_j = \frac{C_{jo}}{\sqrt{1 - V_D/\phi_B}} = \boxed{1.08\ pF}$

(c) $C_d = \frac{qA}{2\frac{kT}{q}}\left((W_p - x_p)\, n_{po} + (W_n - x_n) p_{no}\right) e^{\frac{qV_D}{kT}}$

$= \frac{(1.6\times 10^{-19})(25\times 10^{-4})^2}{2\,(0.026)}\left((2\times 10^{-4})(10^3) + (2\times 10^{-4})(10^3)\right) e^{\frac{0.7}{0.026}}$

$x_{no} = x_{po} \ll W_n,\ W_p \qquad x_{po} = x_{no} = 0.074\ \mu m$

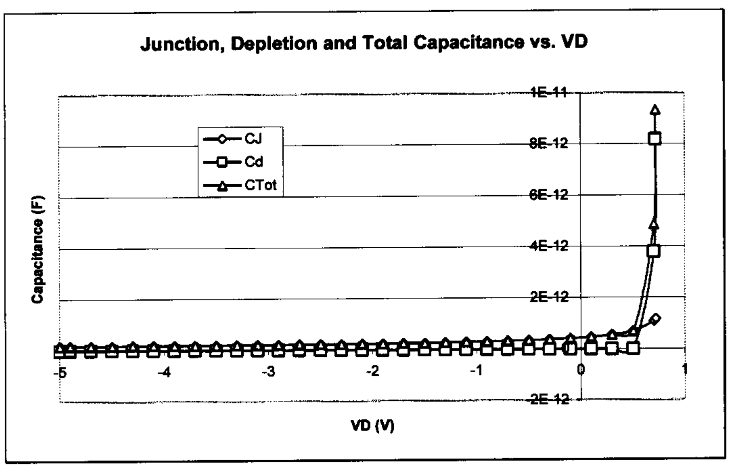
$\boxed{C_d = 3.79\ pF}$

Problem #3: E6.7

$x_n(V_D = -5 \text{ volts}) = 0.195\ \mu m$

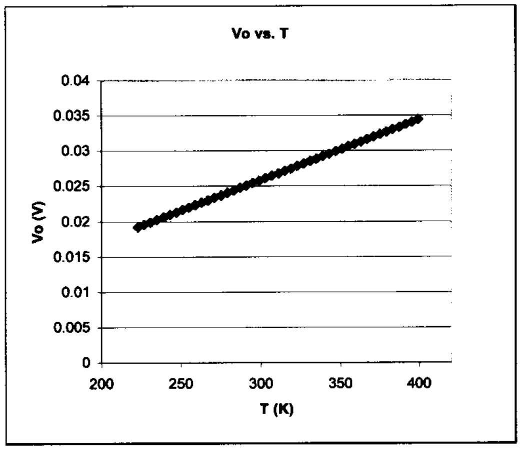
still small relative to $W_n = 2\ \mu m$

VD	CJ	Cd	CTot
-5	1.67252E-13	2.3322E-107	1.67252E-13
-4.9	1.68703E-13	1.0917E-105	1.68703E-13
-4.7	1.71721E-13	2.3925E-102	1.71721E-13
-4.5	1.74907E-13	5.2429E-99	1.74907E-13
-4.3	1.78278E-13	1.14895E-95	1.78278E-13
-4.1	1.81851E-13	2.51784E-92	1.81851E-13
-3.9	1.85647E-13	5.51765E-89	1.85647E-13
-3.7	1.89693E-13	1.20915E-85	1.89693E-13
-3.5	1.94014E-13	2.64977E-82	1.94014E-13
-3.3	1.98645E-13	5.80677E-79	1.98645E-13
-3.1	2.03625E-13	1.27251E-75	2.03625E-13
-2.9	2.08998E-13	2.78861E-72	2.08998E-13
-2.7	2.14821E-13	6.11103E-69	2.14821E-13
-2.5	2.21159E-13	1.33919E-65	2.21159E-13
-2.3	2.28094E-13	2.93473E-62	2.28094E-13
-2.1	2.35724E-13	6.43125E-59	2.35724E-13
-1.9	2.44176E-13	1.40936E-55	2.44176E-13
-1.7	2.53607E-13	3.08851E-52	2.53607E-13
-1.5	2.64223E-13	6.76824E-49	2.64223E-13
-1.3	2.76294E-13	1.48321E-45	2.76294E-13
-1.1	2.90187E-13	3.25034E-42	2.90187E-13
-0.9	3.0641E-13	7.12288E-39	3.0641E-13
-0.7	3.257E-13	1.56093E-35	3.257E-13
-0.5	3.49161E-13	3.42066E-32	3.49161E-13
-0.3	3.78552E-13	7.49611E-29	3.78552E-13
-0.1	4.16883E-13	1.64272E-25	4.16883E-13
0.1	4.69853E-13	3.59989E-22	4.69853E-13
0.3	5.50024E-13	7.8889E-19	5.50024E-13
0.5	6.93168E-13	1.72879E-15	6.94897E-13
0.7	1.08022E-12	3.78852E-12	4.86875E-12
0.72	1.16678E-12	8.17604E-12	9.34281E-12



Problem #4: E 6-16

T (K)	kT/q	VD1	VD2	Vo
223	0.019234	0.211571	0.192338	0.019234
227	0.019579	0.215366	0.195788	0.019579
231	0.019924	0.219161	0.199238	0.019924
235	0.020269	0.222956	0.202688	0.020269
239	0.020614	0.226751	0.206138	0.020614
243	0.020959	0.230546	0.209588	0.020959
247	0.021304	0.234341	0.213038	0.021304
251	0.021649	0.238136	0.216488	0.021649
255	0.021994	0.241931	0.219938	0.021994
259	0.022339	0.245726	0.223388	0.022339
263	0.022684	0.249521	0.226838	0.022684
267	0.023029	0.253316	0.230288	0.023029
271	0.023374	0.257111	0.233738	0.023374
275	0.023719	0.260906	0.237188	0.023719
279	0.024064	0.264701	0.240638	0.024064
283	0.024409	0.268496	0.244088	0.024409
287	0.024754	0.272291	0.247538	0.024754
291	0.025099	0.276086	0.250988	0.025099
295	0.025444	0.279881	0.254438	0.025444
299	0.025789	0.283676	0.257888	0.025789
303	0.026134	0.287471	0.261338	0.026134
307	0.026479	0.291266	0.264788	0.026479
311	0.026824	0.295061	0.268238	0.026824
315	0.027169	0.298856	0.271688	0.027169
319	0.027514	0.302651	0.275138	0.027514
323	0.027859	0.306446	0.278588	0.027859
327	0.028204	0.310241	0.282038	0.028204
331	0.028549	0.314036	0.285488	0.028549
335	0.028894	0.317831	0.288938	0.028894
339	0.029239	0.321626	0.292388	0.029239
343	0.029584	0.325421	0.295838	0.029584
347	0.029929	0.329216	0.299288	0.029929
351	0.030274	0.333011	0.302738	0.030274
355	0.030619	0.336806	0.306188	0.030619
359	0.030964	0.340601	0.309638	0.030964
363	0.031309	0.344396	0.313088	0.031309
367	0.031654	0.348191	0.316538	0.031654
371	0.031999	0.351986	0.319988	0.031999
375	0.032344	0.355781	0.323438	0.032344
379	0.032689	0.359576	0.326888	0.032689
383	0.033034	0.363371	0.330338	0.033034
387	0.033379	0.367166	0.333788	0.033379
391	0.033724	0.370961	0.337238	0.033724
395	0.034069	0.374756	0.340688	0.034069
399	0.034414	0.378551	0.344138	0.034414



(a)

$$V_D = \frac{kT}{q} \ln\left(\frac{I_D}{I_o}\right)$$

(b)

$$I_o = q n_i^2 A \left[\frac{D_p}{N_d W_n} + \frac{D_n}{N_a W_p}\right]$$

The most controllable way, from a process standpoint, to get $\frac{I_{o1}}{I_{o2}} = 0.1$ is to have $A_1 = \frac{1}{10} A_2$.