Problem Set #9 - 6.012 - Solutions - December 5, 2000

Problem #1: P9.20

(a) $g_m = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_1 I_D} \Rightarrow \left(\frac{W}{L}\right)_1 = \frac{g_m^2}{2\mu_n C_{ox} I_D}$

$$\left(\frac{W}{L}\right)_1 = \frac{(1\times10^{-3})^2}{2(50\times10^{-6})(100\times10^{-6})}$$

$$\boxed{\left(\frac{W}{L}\right)_1 = 100}$$

(b) $I_{D1} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{GS} - V_{Tn})^2$ and $V_{GS} = V_{Bias}$

for $I_{OUT} = 0$, $I_{D1} = 100\mu A$

$$V_{Bias} = V_{Tn} + \sqrt{\frac{2 I_{D1}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1}} = 0.7 + \sqrt{\frac{2(100\times10^{-6})}{(50\times10^{-6})(100)}}$$

$$\boxed{V_{Bias} = 0.9 \text{ V}}$$

(c) This is a cascode amplifier stage with cascode current supply.

So $R_{OUT} = R_{OUT_{CS}} \,//\, R_{OUT_{Cascode}}$ $\qquad r_{o3} = r_{o4} = \frac{1}{\lambda_p I_D} = 200\text{ k}\Omega$

Current source: $R_{OUT_{CS}} = r_{o4} + r_{o3} + g_{m3} r_{o4} r_{o3}$ and $g_{m3} = \sqrt{2\mu_p C_{ox}\left(\frac{W}{L}\right)_3 I_{D_P}}$

$R_{OUT_{CS}} = 15\text{ M}\Omega$ $\qquad g_{m3} = 0.365\text{ mS}$

Stage: $R_{OUT_{Cascode}} = r_{o1} + r_{o2} + g_{m2} r_{o1} r_{o2}$ & $g_{m2} = \sqrt{2}\, g_{m3} = 0.516\text{ mS}$

$R_{OUT_{Cascode}} = 10.8\text{ M}\Omega$ $\qquad r_{o2} = r_{o1} = \frac{1}{\lambda_n I_D} = 142.9\text{ k}\Omega$

$$\boxed{R_{OUT} = 6.3\text{ M}\Omega}$$

(d) $i_{OUT} = i_{sc}\frac{R_{OUT}}{R_L + R_{OUT}}$ where i_{sc} is short circuit current.

$$\frac{R_{OUT}}{R_L^{MAX} + R_{OUT}} = 0.8 \Rightarrow \boxed{R_L^{MAX} = 1\text{ M}\Omega}$$

Problem # 2: pg. 22

(a) First leg (left most) $\Rightarrow I_8 = I_{Ref} = 100\,\mu A$

Second leg $\Rightarrow$ Current mirror with $\left(\frac{W}{L}\right)_8 = \left(\frac{W}{L}\right)_9 \Rightarrow I_9 = 100\,\mu A$

Third leg $\Rightarrow$ Current mirror with $\left(\frac{W}{L}\right)_{7B} = \left(\frac{W}{L}\right)_7 \Rightarrow I_7 = 100\,\mu A$

Fourth leg $\Rightarrow$ Current mirror with $\left(\frac{W}{L}\right)_7 = \left(\frac{W}{L}\right)_5 \Rightarrow I_5 = 100\,\mu A$

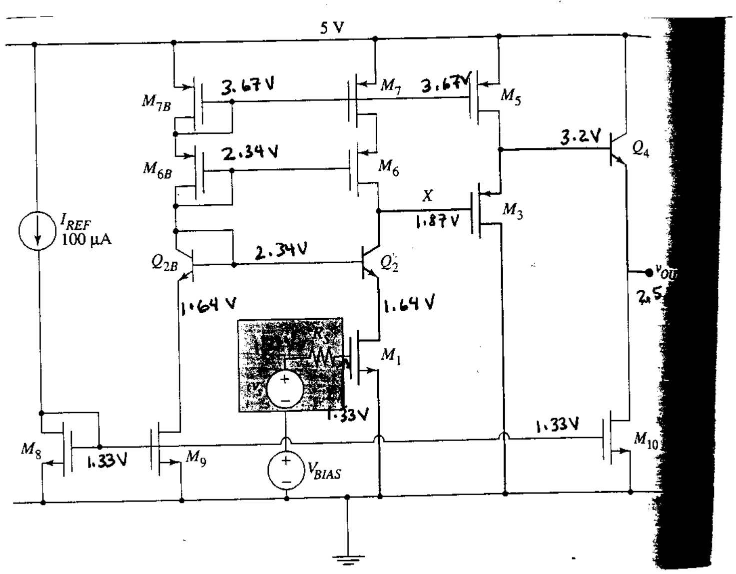
Fifth leg $\Rightarrow$ Current mirror with $\left(\frac{W}{L}\right)_9 = \left(\frac{W}{L}\right)_{10} \Rightarrow I_{10} = 100\,\mu A$

(b) $I_{D1} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{Bias} - V_{Tn})^2 = I_7$

$$V_{Bias} = V_{Tn} + \sqrt{\frac{I_7}{\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1}} = \boxed{1.33 \text{ volts.}}$$

(c) $$V_{GS8} = V_{Tn} + \sqrt{\frac{I_8}{\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_8}} = \text{~~1.09 volts~~}\ 1.33 \text{ volts}$$

$$V_{SG7B} = -V_{Tp} + \sqrt{\frac{I_{7B}}{\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_7}} = 1.33 \text{ volts}$$



Problem #2 : P9.22 (cont.)

(d) Voltage on high side limited by Q_4 going from FAR to cuttoff.

M5 must have V_{Tp} drop to stay in saturation. So, $V_{B4}^{Max} = 4.3\,V$

and $V_{OUT}^{Max} = 3.6\,V$.

Voltage on low side limited by M10 going from saturation to linear.

$V_{OUT}^{Min} > V_{GS} - V_{Tn} = 1.33 - 0.7 = 0.67\,V$

$$V_{Swing} = V_{OUT}^{Max} - V_{OUT}^{Min} = 3.6 - 0.67 = \boxed{2.93\,V}$$

(e) $R_{out_{cascode}} = \beta_{o2} r_{o2} \,||\, r_{oc} = 100\left(\frac{20\,V}{100\,\mu A}\right) // \left(r_{o6} + r_{o7} + g_{m6} r_{o6} r_{o7}\right)$

$r_{o6} = r_{o7} = \dfrac{1}{(0.05)(100\,\mu A)} = \dfrac{1}{\lambda_p I_{Dp}} = 200\,k\Omega$ & $g_{m6} = \sqrt{2 \mu_p C_{ox} \frac{W}{L} I_{Dp}} = 0.316 \times 10^{-3}$

$R_{OUT_{Cascode}} = (20\,M\Omega \,//\, 13\,M\Omega) = 7.9\,M\Omega$

$G_m = g_{m1} = \sqrt{2 \mu_n C_{ox} \frac{W}{L} I_{Dn}} = 0.316\,mS$

$$\boxed{A_v^{Cas} = G_m R_{out_{Cascode}} = 2490}$$

(f) Next stage is a common drain : $A_{v_2} \approx 1$

Next stage is a common collector : $A_{v_3} \approx 1$

$A_v = A_v^{Cas} A_{v_2} A_{v_3}$

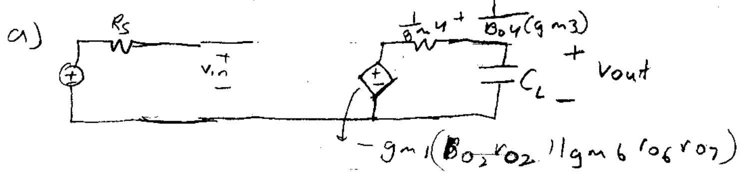
$$\boxed{A_v \approx 2490}$$

P10.24

cascode ^ voltage buffer ^

CS - CB - CD - CC

a)



$R_{in} = \infty$

$R_{out} = \frac{1}{g_{m4}} + \frac{1}{\beta_{o4}(g_{m3})}$ (9.12)

$A_{voc} = -g_{m1}(\beta_{o2} r_{o2} \| g_{m6} r_{o6} r_{o7})$

b) A_{voc} $-g_{m1}(\beta_{o2} r_{o2} \| g_{m6} r_{o6} r_{o7})$

$g_{m1} = \sqrt{2 \frac{W}{L} \mu_n C_{ox} I_{D1}}$ $k_n = k_p$ $I_{D1} = I_{REF}$

$= \sqrt{2 \; 10 \; 50 \times 10^{-6} \; 50 \times 10^{-6}}$

$= .223 \text{ mS}$

$\beta_{o2} = 100$

$r_{o2} = \frac{V_A}{I_C}$ $I_C = I_{D1}$ $r_{o2} = \frac{25}{50 \times 10^{-6}} = 500 \text{ k}\Omega$

$\beta_{o2} r_{o2} = 50 \text{ M}\Omega$

$g_{m6} = g_{m1}$ because $\left(\frac{W}{L}\right)_p = 2\left(\frac{W}{L}\right)_n$

$r_{o6} = r_{o7} = \frac{1}{\lambda I_D} = \frac{1}{.05 \; 50 \times 10^{-6}} = 400 \text{ k}\Omega$

$g_{m6} r_{o6} r_{o7} = 35.77 \text{ M}\Omega$

$A_{voc} = 4662.7$

c) $C_L = 0$

the dominate pole is at the gate of M_3 because of the cascode's high output resistance

$$\omega_{3dB} = \frac{1}{(\beta_o r_{o2} \| r_{oc})\,[C_{\mu 2} + C_{gd6} + C_{gd3} + (1 + A_{vcgs3})\,C_{gs3}]} \quad (10.101)$$

$\beta_o r_{o2} \| r_{oc} = \beta_o r_{o2} \| g_{m6}(r_{o6} r_{o7}) = 50\,M\Omega \| 35.77\,M\Omega = 20.85\,M\Omega$

$C_{\mu 2} = 10\,fF$

$C_{gd6} = C_{ovp} W_6 = .35\,fF/\mu m \times 40\,\mu m = 14\,fF$

$C_{gd3} = C_{gd6}$

$C_{gs3} = C_{ovn} W_3 + \frac{2}{3} W L C_{ox} = .5\,fF/\mu m \times 20\,\mu m + \frac{2}{3}(20)(2)(2.3)$

$= 10 + 61.3 = 71.3\,fF$

$A_{vcgs3} = \frac{g_{m3}}{g_{m3} + g_{mb3}}$ $\quad g_{mb3} = 0$ $\quad A_{vcgs} = 1$

$$\omega_{3dB} = \frac{1}{20.85\,M\Omega\,[10\,fF + 14 + 71.3\,fF + 0]} = 503270\,\frac{rad}{sec}$$

$f_{3dB} = 80.1\,kHz$

d) $\omega_{3dB_2} = \frac{1}{R_1 C_1 + R_{out} C_L} = .8\left(\omega_{3dB_1} = \frac{1}{R_1 C_1}\right)$

$\frac{1}{R_1 C_1 + R_{out} C_L} = \frac{.8}{R_1 C_1}$ $\qquad \frac{1}{\frac{1}{\omega_{3dB}} + R_{out} C_L} = .8\,\omega_{3dB_1}$

$C_L = \left(\frac{1}{.8} - 1\right)\frac{1}{R_{out}\,\omega_{3dB_1}} = \frac{1}{4}\left(\frac{1}{R_{out}\,\omega_{3dB_1}}\right)$

$R_{out} = \frac{1}{g_{m4}} * \frac{1}{\beta_{o4}\, g_{m3}}$

$g_{m3} = g_{m1}$

$\beta_{o4} = 100$

$$g_{m4} = \frac{I_{C4}}{V_{th}} \qquad I_{C4} = I_{REF}$$

$$g_{m4} = \frac{50 \times 10^{-6}}{.026} = 1.92 \text{ mS}$$

$$R_{out} = \frac{1}{1.92 \times 10^{-3}} + \frac{1}{100 \cdot .2236 \times 10^{-3}} = 565\ \Omega$$

$$C_L = \frac{1}{4}\left(\frac{1}{565(503270)}\right) = 878 \text{ pF}$$

P 11.12

a) $I_{BIAS} = I_{REF} = 100 \mu A$

$I_1 = \frac{2.5 - V_{O_1}}{R_D}$ set $V_{O_1} = 0$

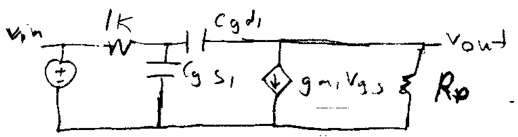
$I_1 = \frac{2.5}{R_D}$ $I_2 = \frac{2.5}{R_2}$

$I_{BIAS} = I_1 + I_2$

$I_1 = \frac{I_{BIAS}}{2}$

$R_D = \frac{(2.5)(2)}{I_{BIAS}} = 50 k\Omega$

b)



$A_{VOC} = -g_{m1} R_D = a_{dm}$ $I_{D_1} = \frac{I_{REF}}{2}$

$g_{m1} = \sqrt{\frac{W}{L} 2 \mu_n C_{ox} I_{D_1}} = \sqrt{50 (50 \times 10^{-6})(2)(50 \times 10^{-6})}$
$= .5 mS$

$a_{dm} = -25$

$\tau_1 = C_{gs1} R_S$ ← miller capacitor

$\tau_2 = C_{gd1}(1 + g_{m1} R_D) R_S$

$C_{gs1} = C_{ovn} W_1 + \frac{2}{3} W_1 L_1 C_{ox} = (.5)(100) + \frac{2}{3}(100)(2)(2.3)$
$= 357 fF$

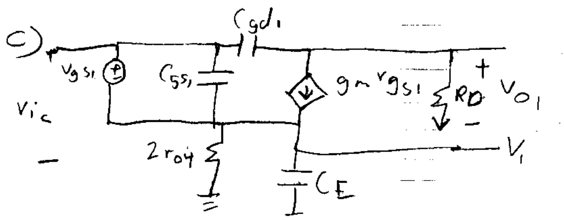
$\tau_1 = 3.57 \times 10^{-10}$

$C_{gd1} = C_{ovn} W_1 = 50 fF$

$\tau_2 = 50 fF \cdot (1 + 25)(1 \times 10^3)$
$= 1.3 \times 10^{-9}$

$\omega_{3dB} = 769.2M$ rad/sec   122.4 MHz

c)



$V_{o1} = \frac{-g_{m1} R_D}{1 + 2g_{m1} r_{o4}} \cdot V_{ic}$

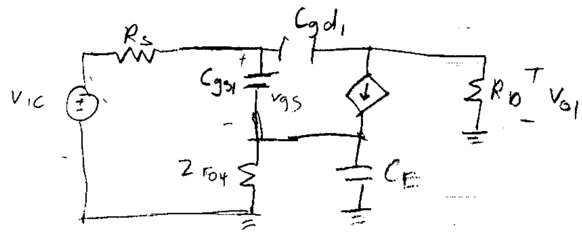
$a_{cm} = -\frac{g_{m1} R_D}{1 + 2g_{m1} r_{o4}}$

$r_{o4} = \frac{1}{\lambda I_{D4}} = \frac{1}{.025(100 \times 10^{-6})} = 400k\Omega$

$\lambda = \frac{.05}{4} \times 2$

$a_{cm} = \frac{.5 \times 10^{-3} \; 50 \times 10^3}{1 + 2(.5 \times 10^{-3})(400k)} = .062344$

(look at example 11.3)



$$C_E = C_{dg_3} + C_{db_3}$$

$$C_{gd_3} = w_3 C_{ov} = 50(.5) = 25\,fF$$

$$C_{db_3} = W_3 L_{diff} C_{jn} + (2L_{diff} + W) C_{jswn}$$
$$(50)(6)(.1) + (2(6) + 50)(.5)$$
$$61\,fF$$

$$C_E = 86\,fF$$

$$Z_e = 2r_{o4} // C_E = \frac{2r_{o4}}{1 + j\omega r_{o4} C_E}$$

$$a_{cm} = \frac{v_{oc}}{v_{ic}} = \frac{-g_m R_D}{1 + g_m Z_e}$$

$$|g_m Z_e| \gg 1 \qquad g_m z_e(\omega=0) = \frac{.5\times10^{-3}(2)(400\times10^3)}{1} = 400$$

$$a_{cm} = \frac{-R_D}{2r_{o4}}(1 + j\omega r_{o4} C_E)$$

$$\omega_{3dB} = \frac{1}{r_{o4} C_E} = 29\text{ Mrad} ; \quad 4.6\text{ MHz}$$

c) $CMRR = \frac{a_{dm}}{a_{cm}} = \frac{-25}{.062} = -403$