Lecture 11 - MOSFET (III)

MOSFET Equivalent Circuit Models

March 15, 2001

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Reading assignment:

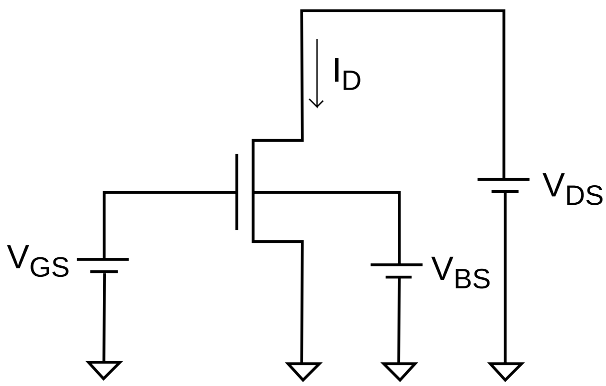
Howe and Sodini, Ch. 4, §4.5-4.6

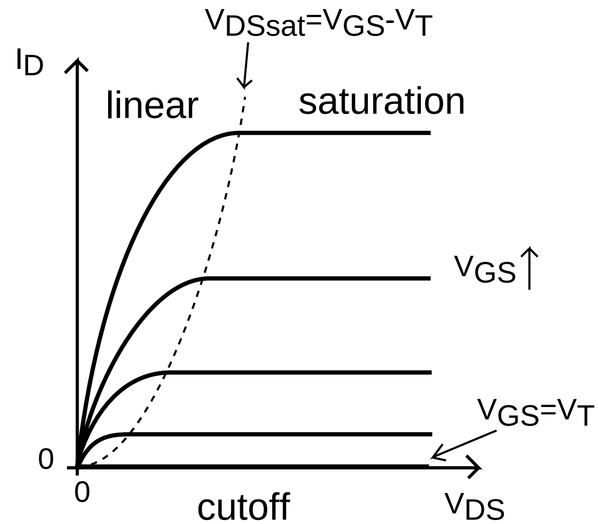
Key questions

- What is the topology of a small-signal equivalent circuit model of the MOSFET?
- What are the key dependencies of the leading model elements in saturation?

1. Low-frequency small-signal equivalent circuit model

Regimes of operation of MOSFET:





- *Cut-off*:

$$I_D = 0$$

- *Linear*:

$$I_D = \frac{W}{L}\mu_n C_{ox}(V_{GS} - \frac{V_{DS}}{2} - V_T)V_{DS}$$

- *Saturation*:

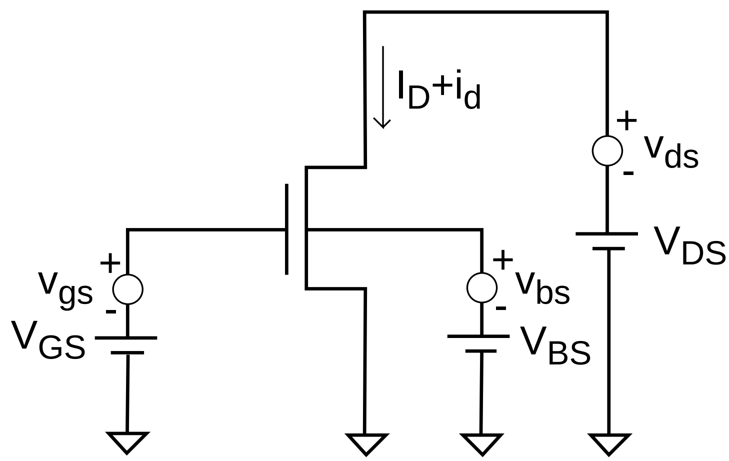
$$I_D = I_{Dsat} = \frac{W}{2L}\mu_n C_{ox}(V_{GS}-V_T)^2[1+\lambda(V_{DS}-V_{DSsat})]$$

Effect of back bias:

$$V_T(V_{BS}) = V_{To} + \gamma(\sqrt{-2\phi_p - V_{BS}} - \sqrt{-2\phi_p})$$

Small-signal device modeling

In many applications, interested in response of device to a *small-signal* applied on top of bias:



Key points:

- small-signal is *small*
  ⇒ response of non-linear components becomes linear
- since response is linear, *superposition* can be used
  ⇒ effects of different small signals are independent from each other

Mathematically:

$$i_D(V_{GS}, V_{DS}, V_{BS}; v_{gs}, v_{ds}, v_{bs}) \simeq$$

$$I_D(V_{GS}, V_{DS}, V_{BS}) + i_d(v_{gs}, v_{ds}, v_{bs})$$

with i_d linear on small-signal drives:

$$i_d \simeq g_m v_{gs} + g_o v_{ds} + g_{mb} v_{bs}$$

Define:

$g_m \equiv$ *transconductance* $[S]$
$g_o \equiv$ *output* or *drain conductance* $[S]$
$g_{mb} \equiv$ *backgate transconductance* $[S]$

Approach to computing g_m, g_o, and g_{mb}:

$$g_m \simeq \frac{\partial I_D}{\partial V_{GS}}\Big|_Q$$

$$g_o \simeq \frac{\partial I_D}{\partial V_{DS}}\Big|_Q$$

$$g_{mb} \simeq \frac{\partial I_D}{\partial V_{BS}}\Big|_Q$$

$Q \equiv$ *bias point* (V_{GS}, V_{DS}, V_{BS})

□ Transconductance

In saturation regime:

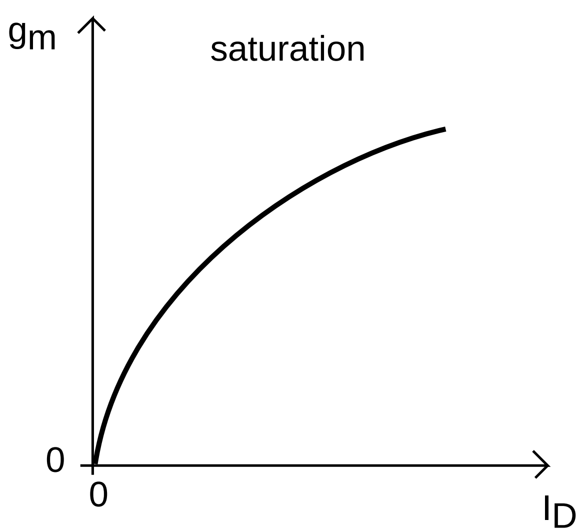
$$I_D = \frac{W}{2L}\mu_n C_{ox}(V_{GS} - V_T)^2[1 + \lambda(V_{DS} - V_{DSsat})]$$

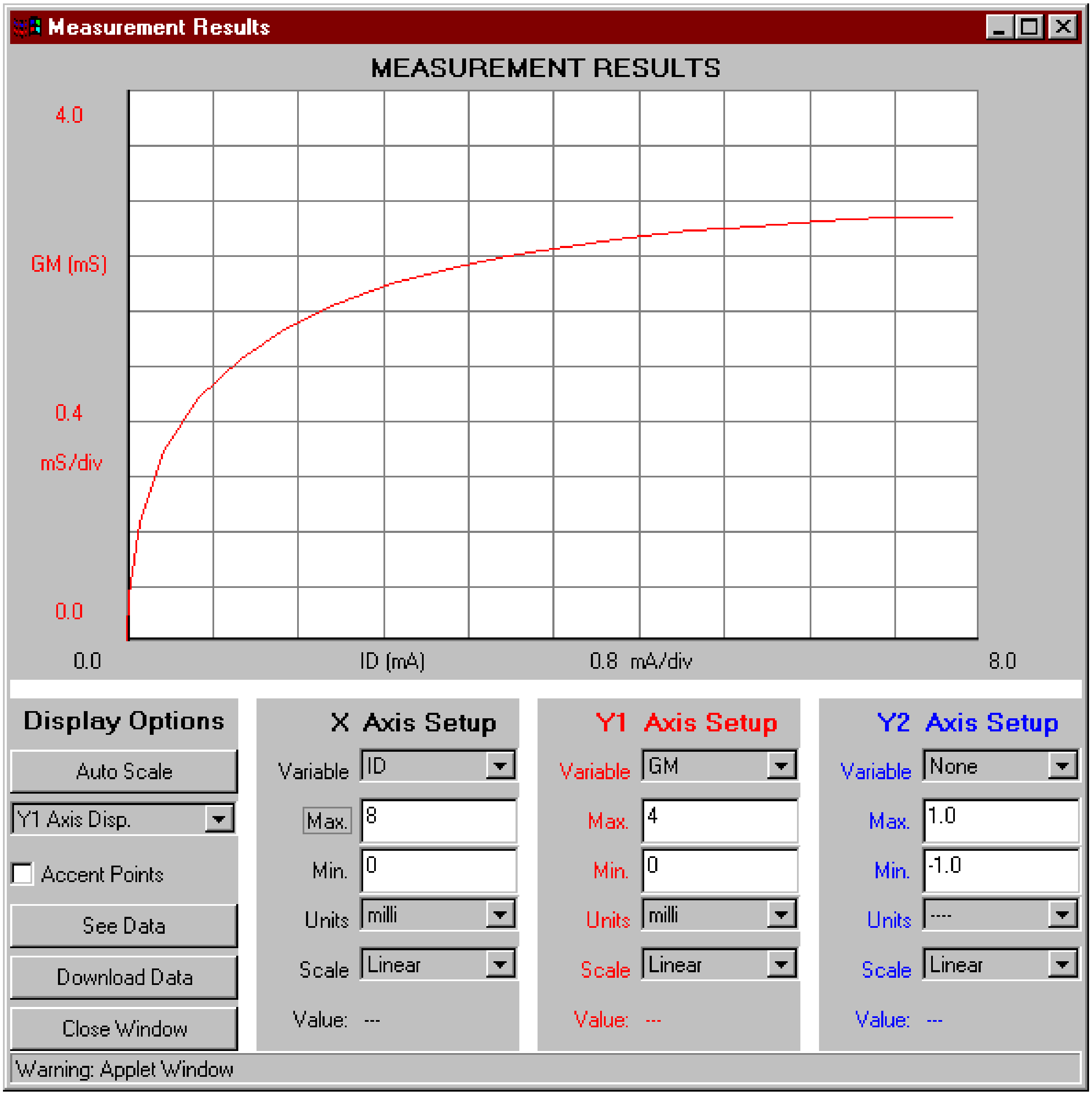
Then (neglecting channel length modulation):

$$g_m = \frac{\partial I_D}{\partial V_{GS}}\Big|_Q \simeq \frac{W}{L}\mu_n C_{ox}(V_{GS} - V_T)$$

Rewrite in terms of I_D:

$$g_m = \sqrt{2\frac{W}{L}\mu_n C_{ox} I_D}$$




Measurement Results
MEASUREMENT RESULTS
4.0
GM (mS)
0.4
mS/div
0.0
0.0
ID (mA)
0.8 mA/div
8.0
Display Options
Auto Scale
Y1 Axis Disp.
Accent Points
See Data
Download Data
Close Window
X Axis Setup
Variable ID
Max. 8
Min. 0
Units milli
Scale Linear
Value: ---
Y1 Axis Setup
Variable GM
Max. 4
Min. 0
Units milli
Scale Linear
Value: ---
Y2 Axis Setup
Variable None
Max. 1.0
Min. -1.0
Units ----
Scale Linear
Value: ---
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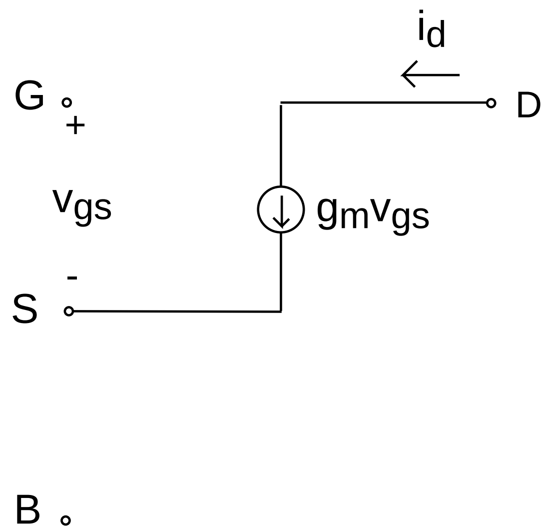
Equivalent circuit model representation of g_m:



□ Output conductance

In saturation regime:

$$I_D = \frac{W}{2L}\mu_n C_{ox}(V_{GS} - V_T)^2[1 + \lambda(V_{DS} - V_{DSsat})]$$

Then:

$$g_o = \frac{\partial I_D}{\partial V_{DS}}\Big|_Q = \frac{W}{2L}\mu_n C_{ox}(V_{GS} - V_T)^2\lambda \simeq \lambda I_D$$

Output resistance is inverse of output conductance:

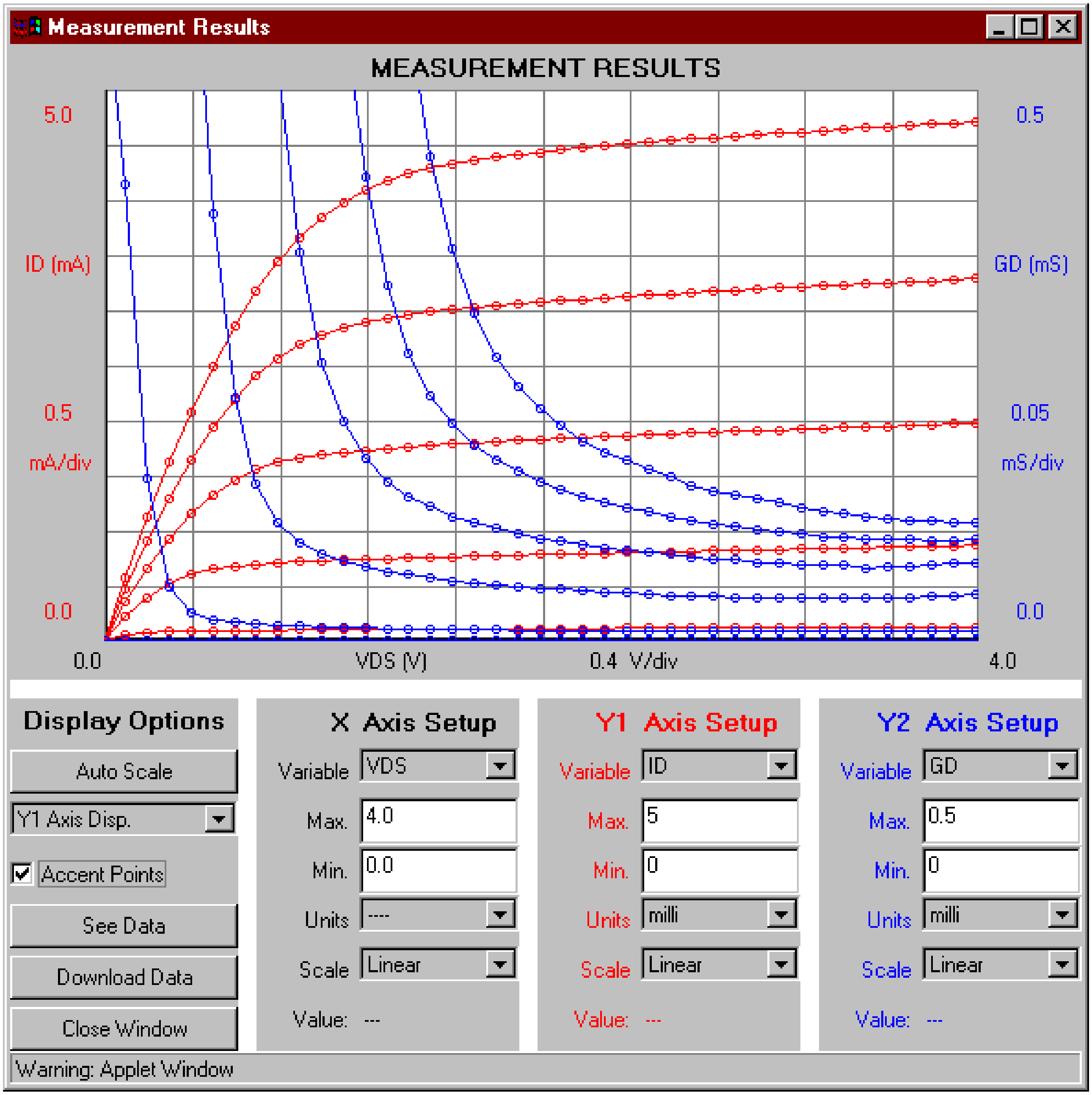
$$r_o = \frac{1}{g_o} = \frac{1}{\lambda I_D}$$

Remember also:

$$\lambda \propto \frac{1}{L}$$

Hence:

$$r_o \propto L$$



Measurement Results
MEASUREMENT RESULTS
5.0
ID (mA)
0.5
mA/div
0.0
0.5
GD (mS)
0.05
mS/div
0.0
0.0
VDS (V)
0.4 V/div
4.0
Display Options
Auto Scale
Y1 Axis Disp.
Accent Points
See Data
Download Data
Close Window
X Axis Setup
Variable VDS
Max. 4.0
Min. 0.0
Units ----
Scale Linear
Value: ---
Y1 Axis Setup
Variable ID
Max. 5
Min. 0
Units milli
Scale Linear
Value: ---
Y2 Axis Setup
Variable GD
Max. 0.5
Min. 0
Units milli
Scale Linear
Value: ---
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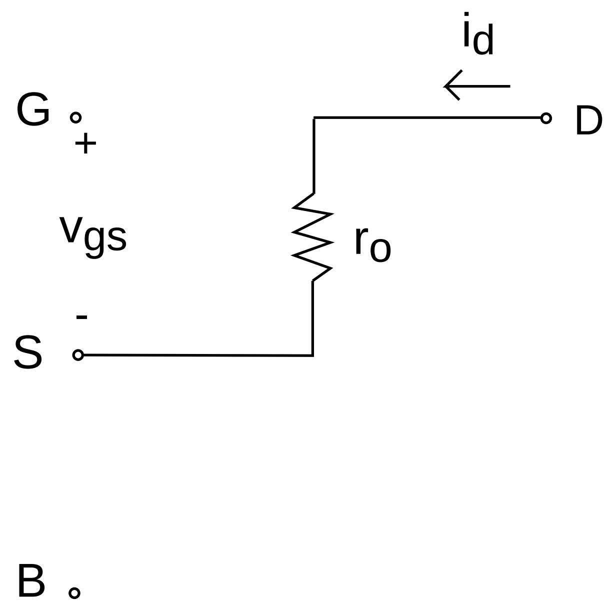
Equivalent circuit model representation of g_o:



□ Backgate transconductance

In saturation regime (neglect channel length modulation):

$$I_D \simeq \frac{W}{2L}\mu_n C_{ox}(V_{GS} - V_T)^2$$

Then:

$$g_{mb} = \frac{\partial I_D}{\partial V_{BS}}\Big|_Q = \frac{W}{L}\mu_n C_{ox}(V_{GS} - V_T)(-\frac{\partial V_T}{\partial V_{BS}}\Big|_Q)$$

Since:

$$V_T(V_{BS}) = V_{To} + \gamma(\sqrt{-2\phi_p - V_{BS}} - \sqrt{-2\phi_p})$$

Then:

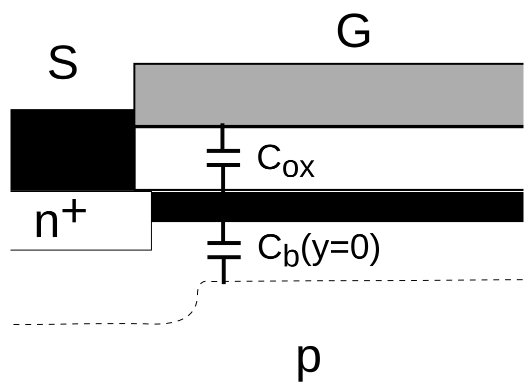
$$\frac{\partial V_T}{\partial V_{BS}}\Big|_Q = \frac{-\gamma}{2\sqrt{-2\phi_p - V_{BS}}}$$

All together:

$$g_{mb} = \frac{\gamma g_m}{2\sqrt{-2\phi_p - V_{BS}}}$$

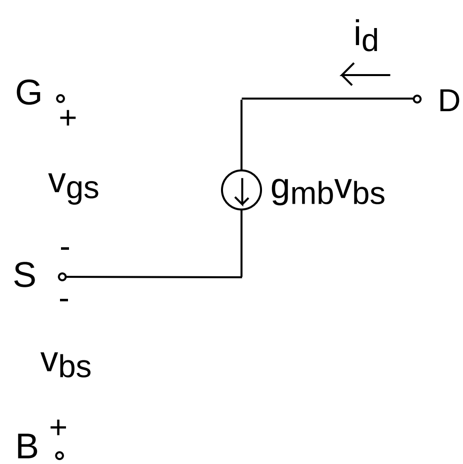
Another way to write this (see book):

$$\frac{g_{mb}}{g_m} = \frac{C_b(y=0)}{C_{ox}}$$

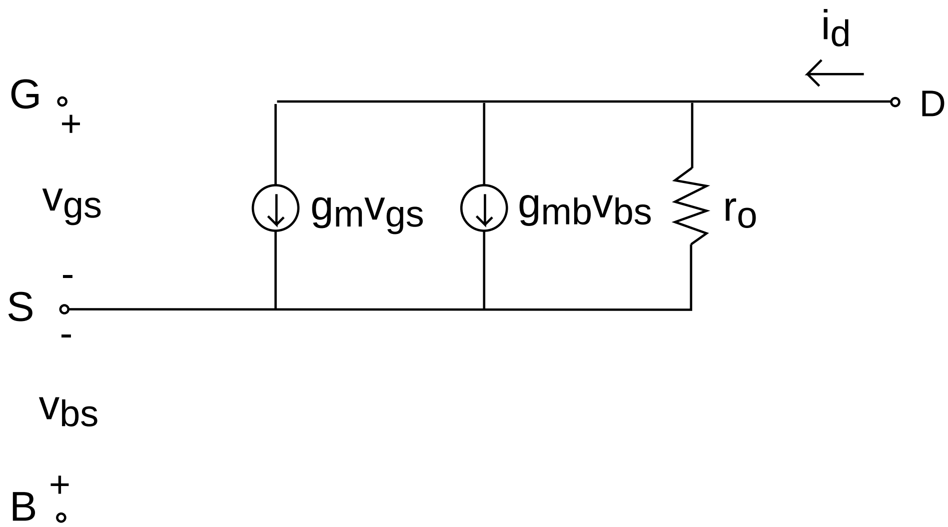


Handy rule: ratio of back transconductance to transconductance is equal to ratio of depletion capacitance at source to oxide capacitance.

Equivalent circuit model representation of g_{mb}:



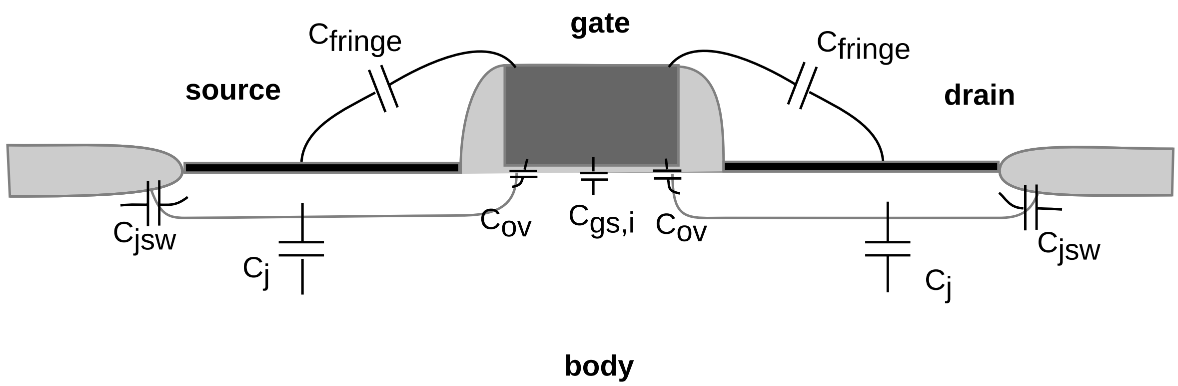
Complete MOSFET small-signal equivalent circuit model for low frequency:



2. High-frequency small-signal equivalent circuit model

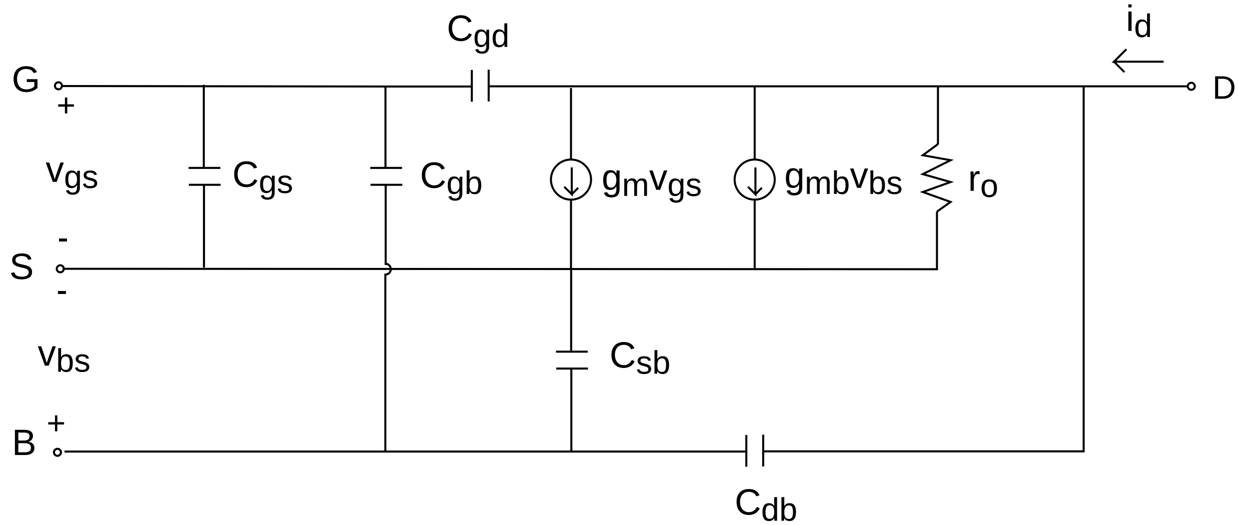
Need to add capacitances.

In saturation:



$C_{gs} \equiv$ channel charge + overlap capacitance, C_{ov}
$C_{gd} \equiv$ overlap capacitance, C_{ov}
$C_{gb} \equiv$ only parasitic capacitance
$C_{sb} \equiv$ source junction depletion capacitance (+sidewall)
$C_{db} \equiv$ drain junction depletion capacitance (+sidewall)

Complete MOSFET high-frequency small-signal equivalent circuit model:



Plan for development of capacitance model:

- Start with $C_{gs,i}$
  - compute gate charge $Q_G = Q_N + Q_B$
  - compute how Q_G changes with V_{GS}
- Add pn junction capacitances

Inversion layer charge in saturation

$$Q_N(V_{GS}) = W \int_0^L Q_n(y)dy = W \int_0^{V_{GS}-V_T} Q_n(V_c)\frac{dy}{dV_c}dV_c$$

But:

$$\frac{dV_c}{dy} = -\frac{I_D}{W\mu_n Q_n(V_c)}$$

Then:

$$Q_N(V_{GS}) = -\frac{W^2 L\mu_n}{I_D}\int_0^{V_{GS}-V_T} Q_n^2(V_c)dV_c$$

Remember:

$$Q_n(V) = -C_{ox}(V_{GS} - V_c - V_T)$$

Then:

$$Q_N(V_{GS}) = -\frac{W^2 L\mu_n C_{ox}^2}{I_D}\int_0^{V_{GS}-V_T}(V_{GS} - V_c - V_T)^2 dV_c$$

Do integral, substitute I_D in saturation and get:

$$Q_N(V_{GS}) = -\frac{2}{3}WLC_{ox}(V_{GS} - V_T)$$

Gate charge:

$$Q_G(V_{GS}) = -Q_N(V_{GS}) - Q_{B,max}$$

Intrinsic gate-to-source capacitance:

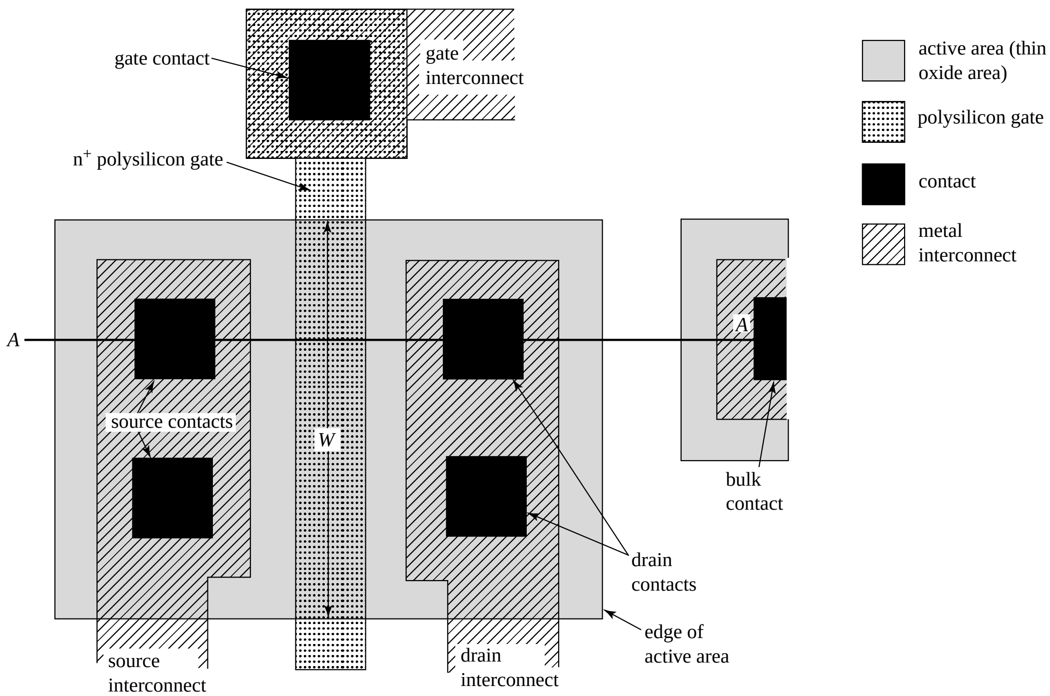
$$C_{gs,i} = \frac{dQ_G}{dV_{GS}} = \frac{2}{3}WLC_{ox}$$

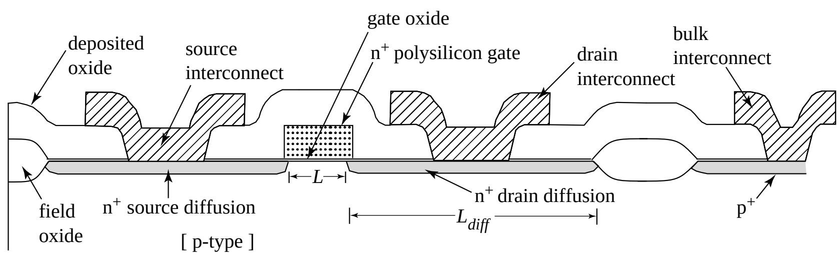
Must add overlap capacitance:

$$C_{gs} = \frac{2}{3}WLC_{ox} + WC_{ov}$$

Gate-to-drain capacitance - only overlap capacitance:

$$C_{gd} = WC_{ov}$$





Body-to-source capacitance = source junction capacitance:

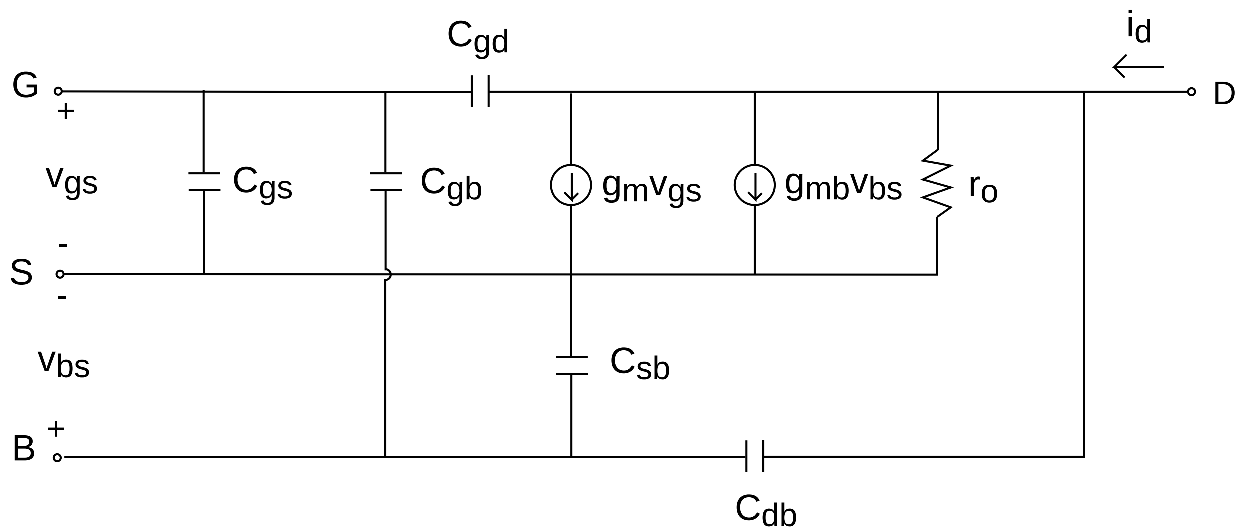
$$C_{bs} = C_j + C_{jsw} = W L_{diff} \sqrt{\frac{q\epsilon_s N_a}{2(\phi_B - V_{BS})}} + (2L_{diff} + W) C_{JSW}$$

Body-to-drain capacitance = drain junction capacitance:

$$C_{bd} = C_j + C_{jsw} = W L_{diff} \sqrt{\frac{q\epsilon_s N_a}{2(\phi_B - V_{BD})}} + (2L_{diff} + W) C_{JSW}$$

Key conclusions

High-frequency small-signal equivalent circuit model of MOSFET:



In saturation:

$$g_m \propto \sqrt{\frac{W}{L} I_D}$$

$$r_o \propto \frac{L}{I_D}$$

$$C_{gs} \propto W L C_{ox}$$