Lecture 16 - The pn Junction Diode (II)

Equivalent Circuit Model

April 10, 2001

Contents:

1. I-V characteristics (*cont.)*
2. Small-signal equivalent circuit model
3. Carrier charge storage: diffusion capacitance

Reading assignment:

Howe and Sodini, Ch. 6, §§6.4, 6.5, 6.9

Announcements:

Quiz 2: 4/18, 7:30-9:30 PM, Walker (lectures #10-17) open book, must bring calculator

Key questions

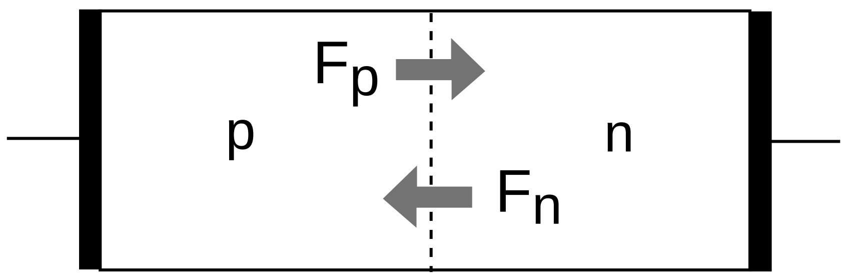
- How does a pn diode look like from a small-signal point of view?
- What are the leading dependences of the small-signal elements?
- In addition to the junction capacitance, are there any other capacitive effects in a pn diode?

1. I-V characteristics *(cont.)*

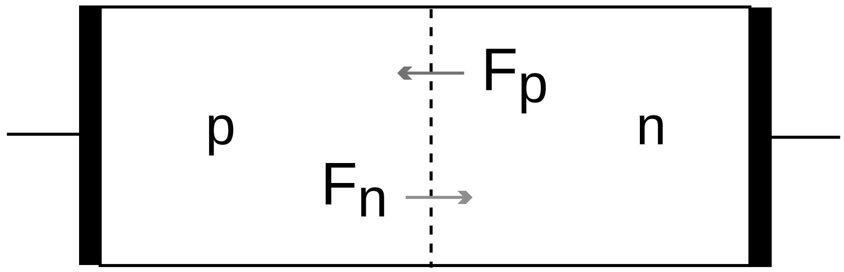
Diode current equation:

$$I = I_o(\exp\frac{qV}{kT} - 1)$$

Physics of forward bias:



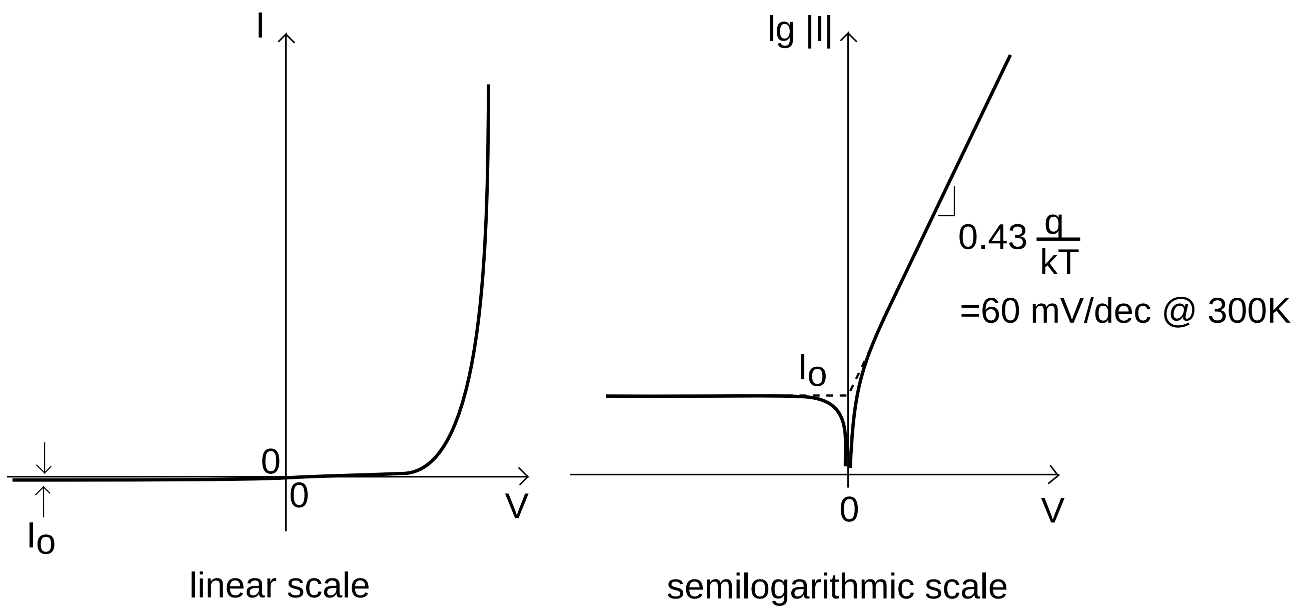
- potential difference across SCR reduced by $V \Rightarrow$ minority carrier injection in QNR's
- minority carrier diffusion through QNR's
- minority carrier recombination at surface of QNR's
- large supply of carriers available for injection
  $\Rightarrow I \propto e^{qV/kT}$



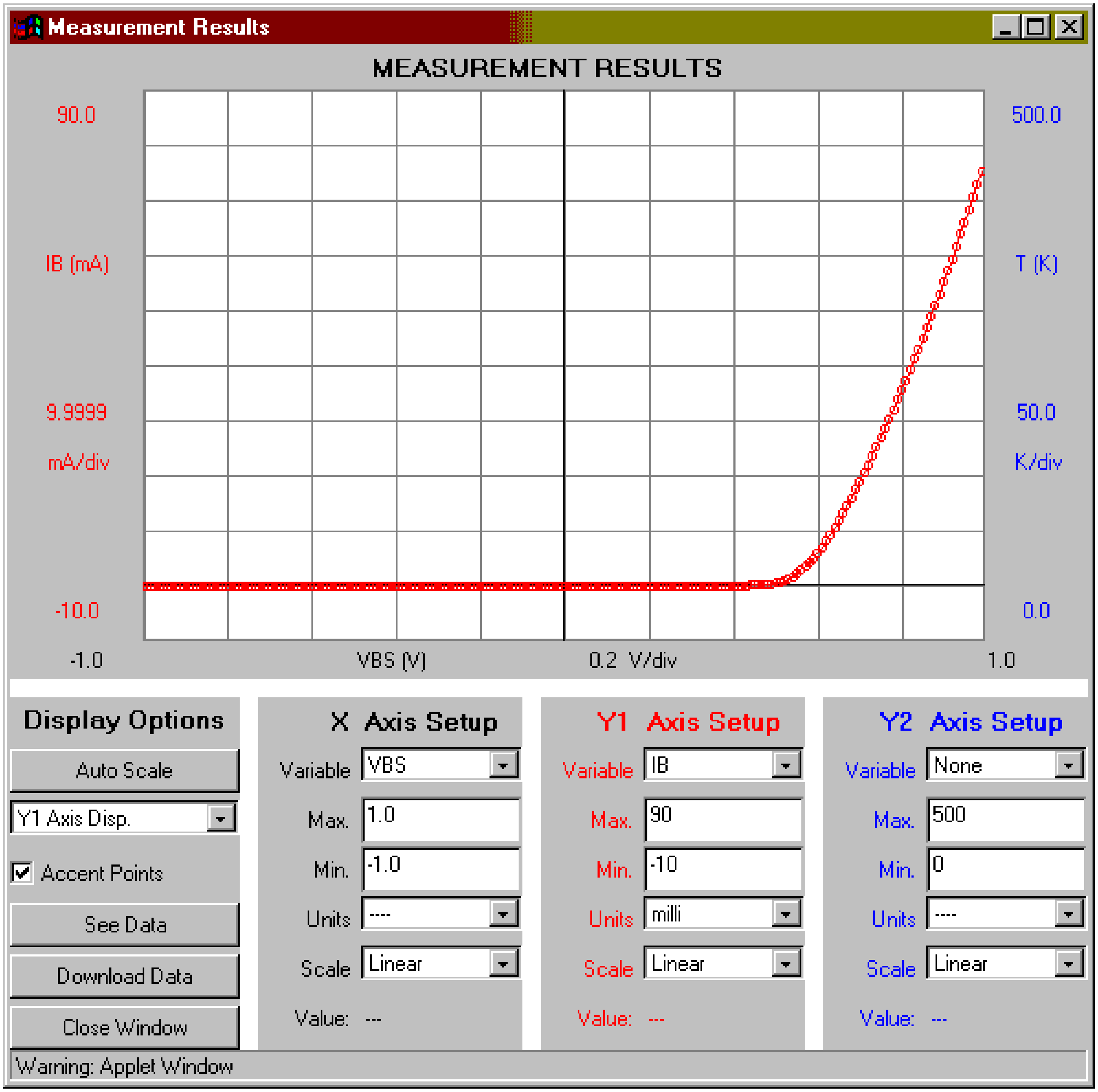
Physics of reverse bias:

- potential difference across SCR increased by V
  $\Rightarrow$ minority carrier extraction from QNR's
- minority carrier diffusion through QNR's
- minority carrier generation at surface of QNR's
- very small supply of carriers available for extraction
  $\Rightarrow$ I saturates to small value

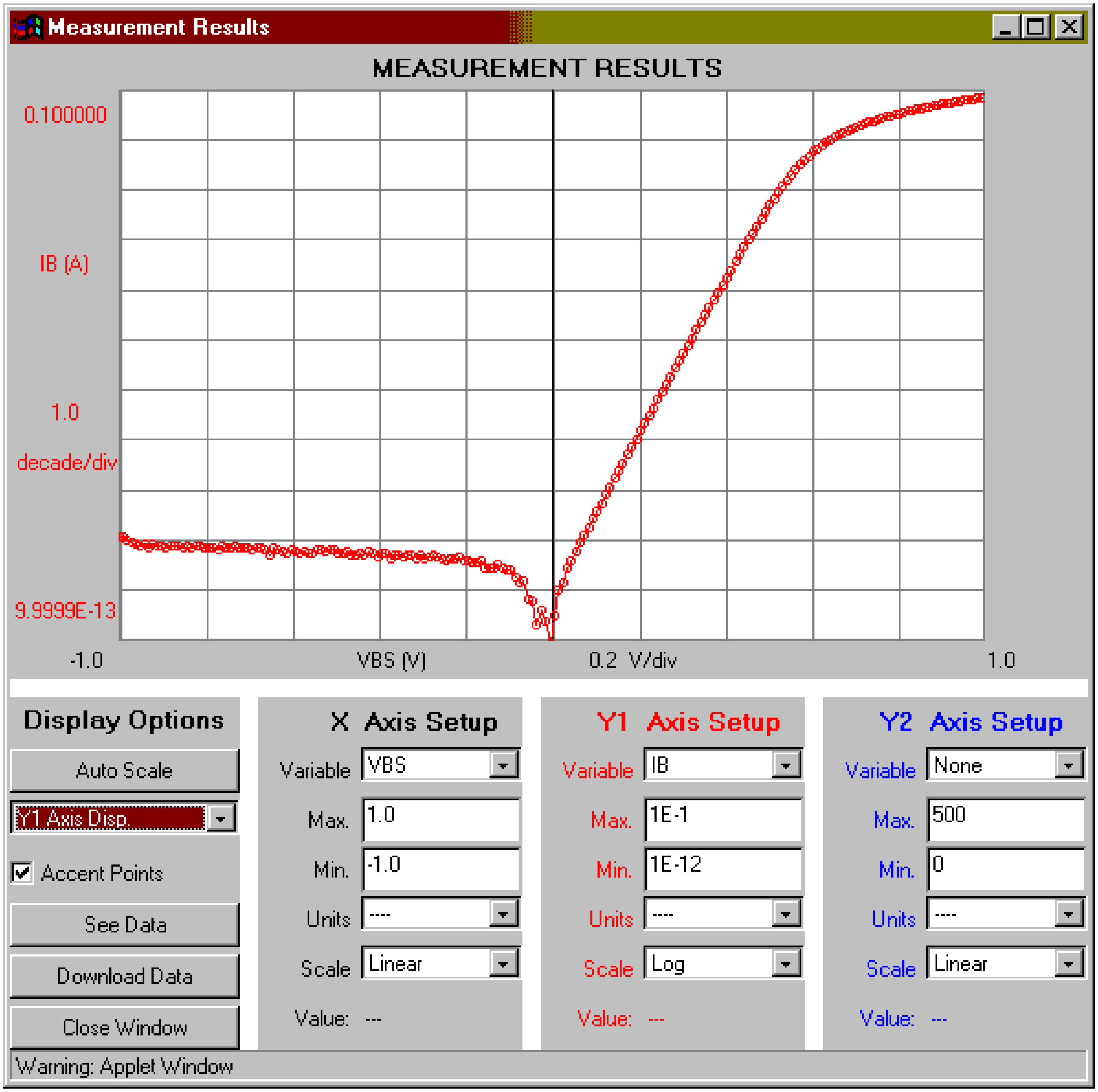
I-V characteristics: $I = I_o(\exp \frac{qV}{kT} - 1)$



Source-body pn diode of NMOSFET (linear scale):

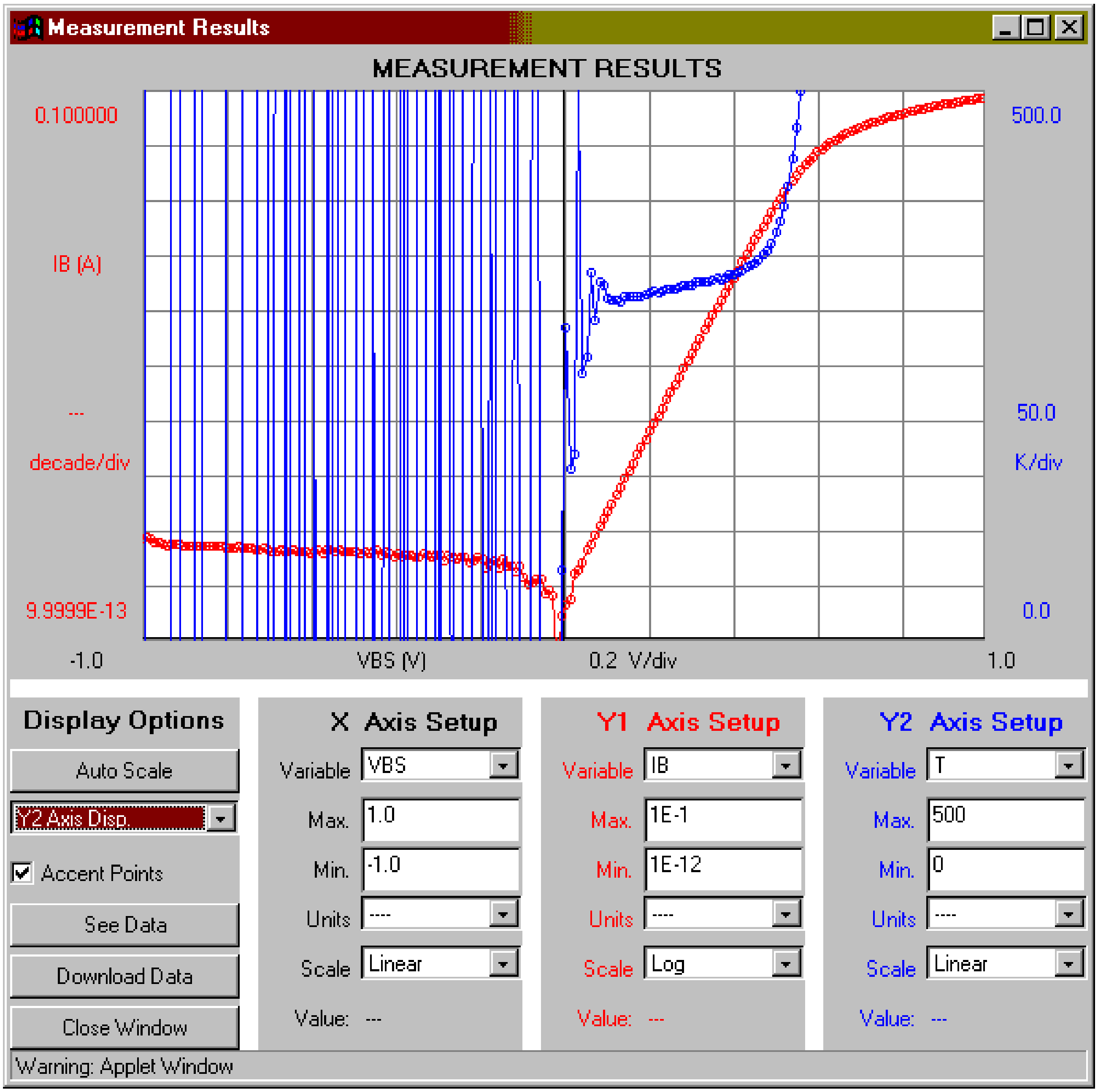


Source-body pn diode of NMOSFET (logarithmic scale):



Temperature extraction from forward bias characteristics:

$$T \simeq \frac{q}{k} \frac{1}{\frac{d(\ln I)}{dV}}$$



Key dependences of diode current:

$$I = qAn_i^2(\frac{1}{N_a}\frac{D_n}{w_p - x_p} + \frac{1}{N_d}\frac{D_p}{w_n - x_n})(\exp\frac{qV}{kT} - 1)$$

- $I \propto \frac{n_i^2}{N}(\exp\frac{qV}{kT} - 1) \equiv$ *excess* minority carrier concentration at edges of SCR
  - in forward bias: $I \propto \frac{n_i^2}{N}\exp\frac{qV}{kT}$: the more carrier are injected, the more current flows
  - in reverse bias: $I \propto -\frac{n_i^2}{N}$: when minority carrier concentration drops to zero, the current saturates
- $I \propto D$: faster diffusion $\Rightarrow$ more current
- $I \propto \frac{1}{w_{QNR}}$: shorter region to diffuse through $\Rightarrow$ more current
- $I \propto A$: bigger diode $\Rightarrow$ more current

2. Small-signal equivalent circuit model

Examine effect of small signal overlapping bias:

$$I + i = I_o[\exp \frac{q(V+v)}{kT} - 1]$$

If v small enough, linearize exponential characteristics:

$$I + i \simeq I_o(\exp \frac{qV}{kT} \exp \frac{qv}{kT} - 1) \simeq I_o[\exp \frac{qV}{kT}(1 + \frac{qv}{kT}) - 1]$$

$$= I_o(\exp \frac{qV}{kT} - 1) + I_o(\exp \frac{qV}{kT})\frac{qv}{kT}$$

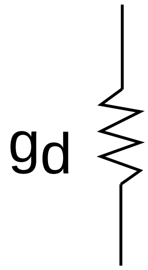
Then:

$$i = \frac{q(I + I_o)}{kT} v$$

From small signal point of view, diode behaves as conductance of value:

$$g_d = \frac{q(I + I_o)}{kT}$$

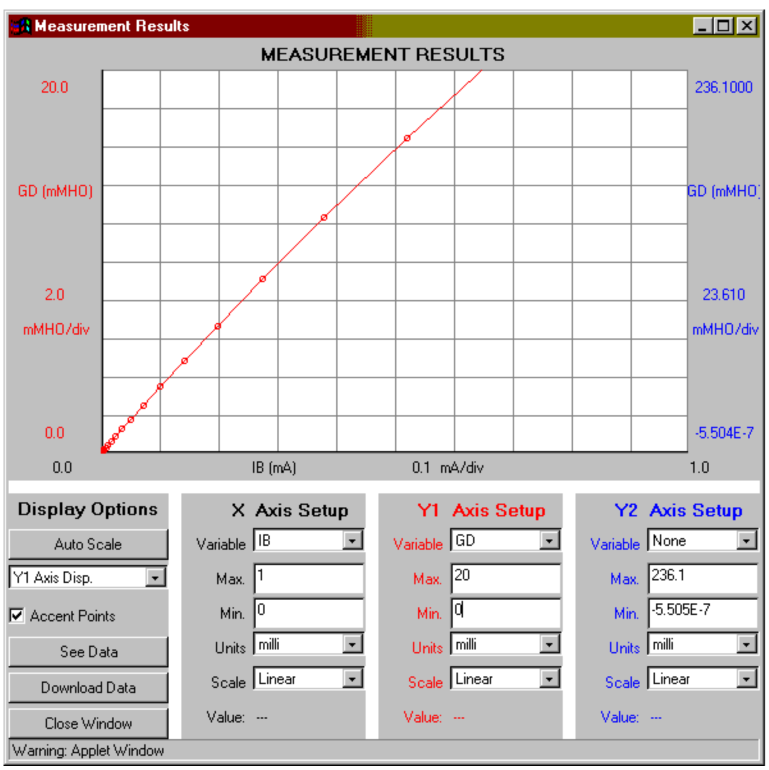
Small-signal equivalent circuit model, so far:



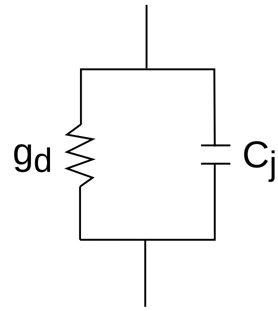
g_d depends on bias. In forward bias:

$$g_d \simeq \frac{qI}{kT}$$

g_d is linear in diode current.



Must add capacitance associated with depletion region:



Depletion or junction capacitance:

$$C_j = A\sqrt{\frac{q\epsilon_s N_a N_d}{2(N_a + N_d)(\phi_B - V)}}$$

Can rewrite as:

$$C_j = A\sqrt{\frac{q\epsilon_s N_a N_d}{2(N_a + N_d)\phi_B}}\sqrt{\frac{\phi_B}{\phi_B - V}}$$

Or,

$$C_j = \frac{C_{jo}}{\sqrt{1 - \frac{V}{\phi_B}}}$$

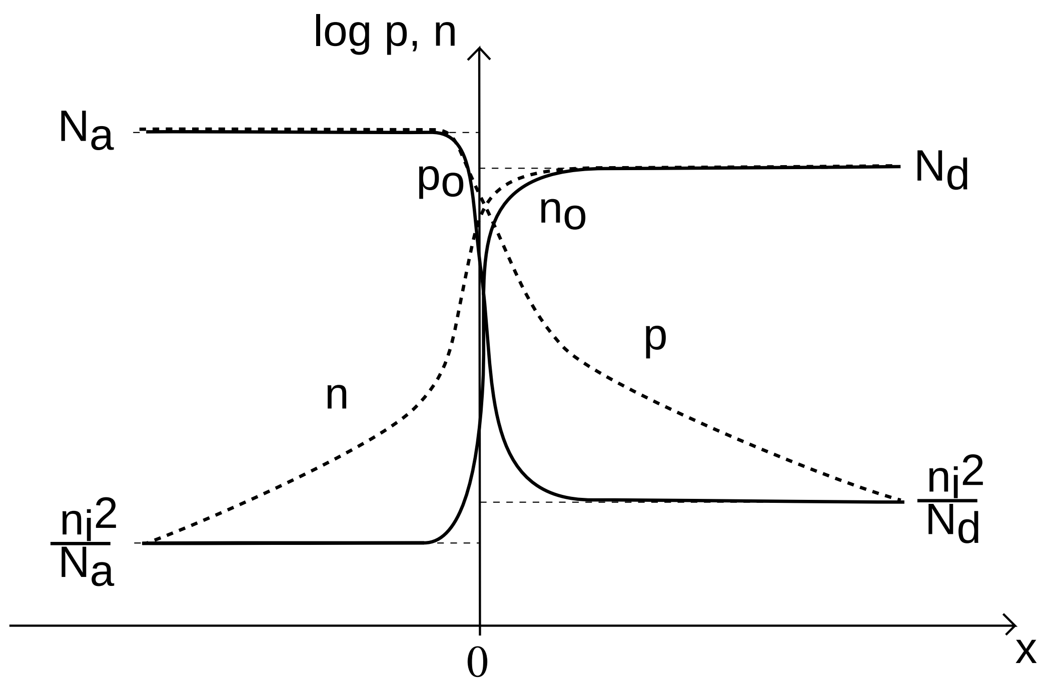
where

$$C_{jo} \equiv \textit{zero-voltage junction capacitance}$$

3. Carrier charge storage: diffusion capacitance

What happens to the majority carriers?

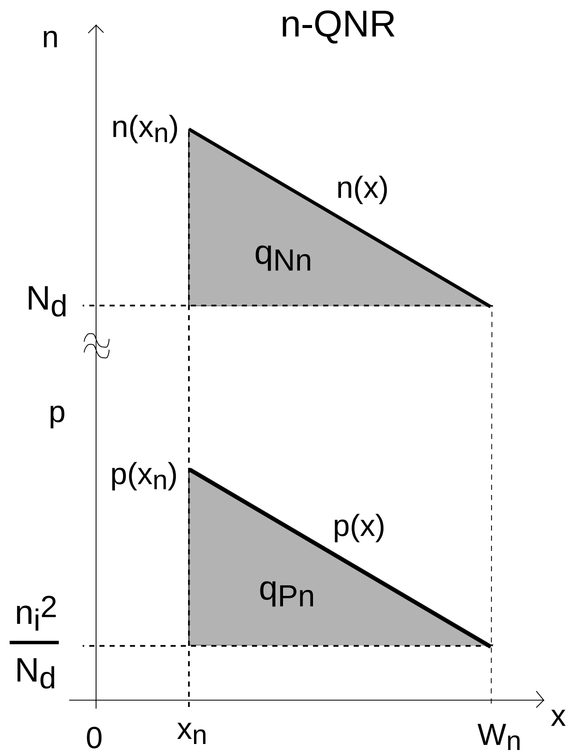
Carrier picture so far:



If in QNR minority carrier concentration ↑ but majority carrier concentration unchanged
$\Rightarrow$ quasi-neutrality is violated.

Quasi-neutrality demands that at every point in QNR:

excess minority carrier concentration
= excess majority carrier concentration



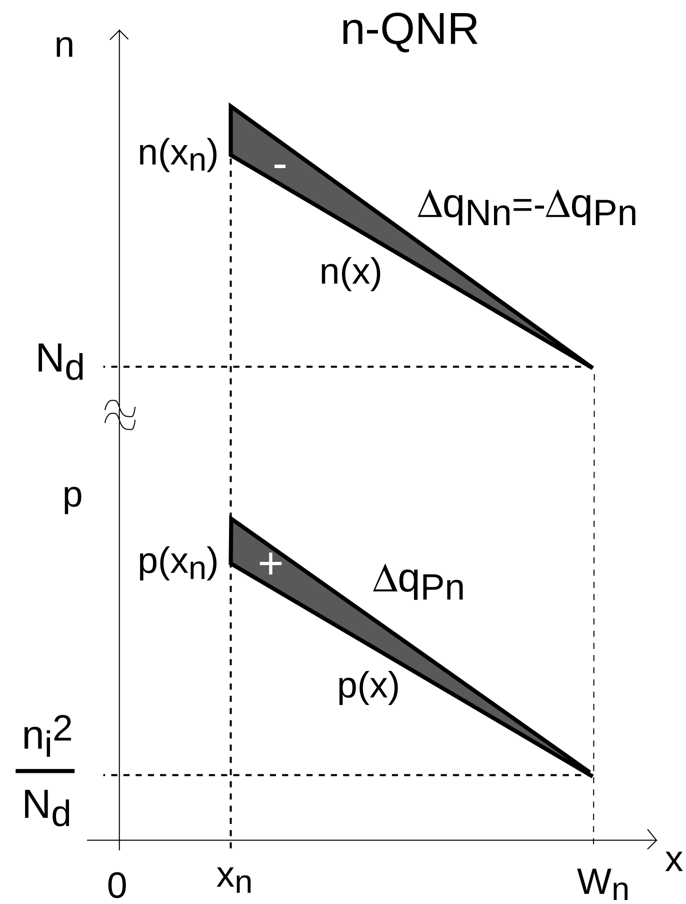
Mathematically:

$$p'(x) = p(x) - p_o \simeq n'(x) = n(x) - n_o$$

Define integrated carrier charge:

$$q_{Pn} = qA\tfrac{1}{2}p'(x_n)(w_n - x_n) =$$
$$= qA\tfrac{w_n - x_n}{2}\tfrac{n_i^2}{N_d}(\exp\tfrac{qV}{kT} - 1) = -q_{Nn}$$

Now examine small increase in V:



Small increase in $V \Rightarrow$ small increase in $q_{Pn} \Rightarrow$ small increase in $|q_{Nn}|$

Behaves as capacitor of capacitance:

$$C_{dn} = \frac{dq_{Pn}}{dV} = qA\frac{w_n - x_n}{2}\frac{n_i^2}{N_d}\frac{q}{kT}\exp(\frac{qV}{kT})$$

Can write in terms of I_p (portion of diode current due to holes in n-QNR):

$$C_{dn} = \frac{q}{kT}\frac{(w_n - x_n)^2}{2D_p} qA\frac{n_i^2}{N_d}\frac{D_p}{w_n - x_n}\exp(\frac{qV}{kT})$$

$$\simeq \frac{q}{kT}\frac{(w_n - x_n)^2}{2D_p} I_p$$

Define *transit time* of holes through n-QNR:

$$\tau_{Tp} = \frac{(w_n - x_n)^2}{2D_p}$$

Transit time is *average time for a hole to diffuse through n-QNR* [will discuss in more detail in BJT]

Then:

$$C_{dn} \simeq \frac{q}{kT}\tau_{Tp} I_p$$

Similarly for p-QNR:

$$C_{dp} \simeq \frac{q}{kT}\tau_{Tn} I_n$$

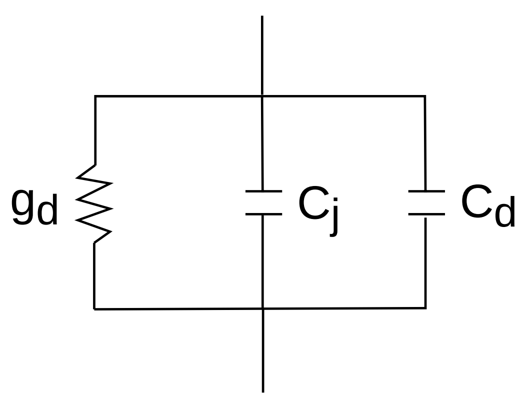
where τ_{Tn} is *transit time* of electrons through p-QNR:

$$\tau_{Tn} = \frac{(w_p - x_p)^2}{2D_n}$$

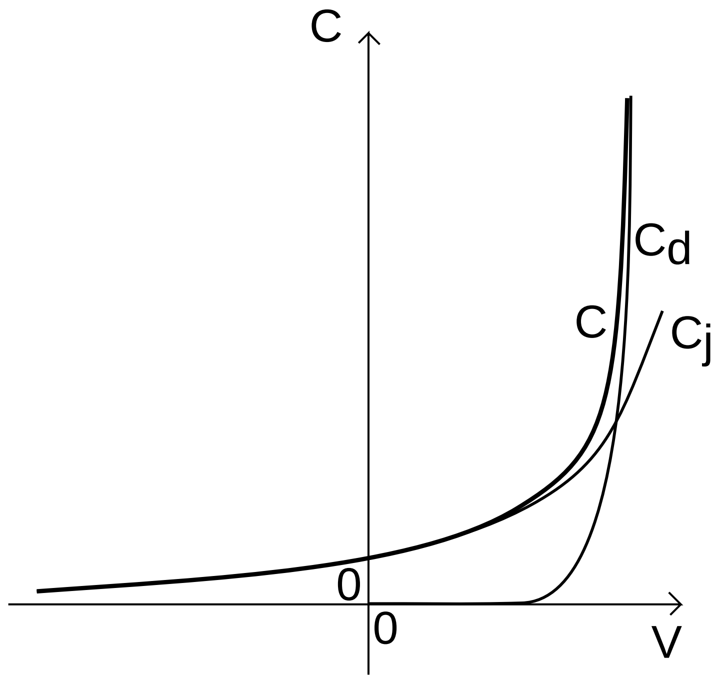
Both capacitors sit in *parallel* ⇒ total diffusion capacitance:

$$C_d = C_{dn} + C_{dp} = \frac{q}{kT}(\tau_{Tn} I_n + \tau_{Tp} I_p)$$

Complete small-signal equivalent circuit model for diode:



Bias dependence of C_j and C_d:



- C_j dominates in reverse bias and small forward bias ($\sim 1/\sqrt{\phi_B - V}$)

- C_d dominates in strong forward bias ($\sim e^{qV/kT}$)

Key conclusions

Small-signal behavior of diode:

- *conductance*: associated with current-voltage characteristics

$$g_d \sim I \text{ in forward bias, negligible in reverse bias}$$

- *junction capacitance*: associated with charge modulation in depletion region

$$C_j \sim 1/\sqrt{\phi_B - V}$$

- *diffusion capacitance*: associated with charge storage in QNR's to keep quasi-neutrality

$$C_d \sim e^{qV/kT}$$