Problem 1
Below is a sketch not to scale of the minority carrier distribution across the quasi-neutral regions of a forward biased p-n diode. For this diode, \( W_p - x_p = 4 \mu m \), \( W_n - x_n = 3 \mu m \), \( D_n = 25 \text{ cm}^2/\text{s} \) and \( D_p = 10 \text{ cm}^2/\text{s} \). The area of the junction is 10 \( \mu \text{m}^2 \).

![Diode Sketch](image)

a) Calculate the hole current injected into the n-side of the diode.

\[
I_p = qD_p \frac{dp_n}{dx} A \equiv qD_p \frac{p(x_n)}{W_n - x_n} A = 53nA
\]

b) Calculate the electron current injected into the p-side of the diode.

\[
I_n = qD_n \frac{dn_p}{dx} A \equiv qD_n \frac{n(-x_p)}{W_p - x_p} A = 10nA
\]
c) Calculate the diffusion capacitance associated with the carrier storage on the n-side of the diode

\[ C_{d_n} = \frac{dq p_n}{dv_d} = \frac{qA}{2V_{th}} (w_n - x_n) p(x_n) = 9.3 \text{fF} \]

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d) Calculate the diffusion capacitance associated with the carrier storage on the p-side of the diode.

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e) How much should the voltage across the junction increase if we wish to double the total current through the diode?

\[ I_D = I_0 e^{V_D/V_{th}} \]

\[ 2I_D = I_0 e^{V_D/V_{th}} \]

The voltage across the diode should increase than 18\( \mu \text{A} \).

\[ V_D' = V_D + V_{th} \ln 2 \]

f) Compute the diffusion capacitance of the diode when we increase the voltage in the manner suggested in the previous question.

\[ C_d(V_D') = \frac{qA}{2V_{th}} \left[ (w_p - x_p) n_p + (w_n - x_n) p_n \right] e^{V_D'/V_{th}} = k e^{V_D'/V_{th}} = 2C_d(V_D) \]

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g) What is the ratio of the doping levels across the junction: \( Na/Nd \)?

\[ Na / Nd = p(x_n) / n(-x_p) = 10 \]

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h) In what direction should \( Na/Nd \) change if we wish to redesign the diode so as to get less diffusion capacitance at the same current level? (Assume that in redesigning the diode \( D_n, D_p, W_n - x_n, \) and \( W_p - x_p \) do not change). Should \( Na/Nd \) increase or decrease? Explain.

\[ C_d = \frac{I_p \tau_p + I_n \tau_n}{V_{th}} \]

Since \( \tau_p > \tau_n \) and since \( I_D = I_n + I_p \) is constant, to decrease the diffusion capacitance we should increase the electron current (proportional to \( 1/Na \)) and decrease the hole current (proportional to \( 1/Nd \)). That can be accomplished by decreasing \( Na / Nd \).
Problem 2
Consider an abrupt asymmetric $n^+p$ junction diode with a junction area of 100 $\mu m^2$. All the action in this device is dominated by the lightly-doped p-type region. Due to processing reasons, the diffusion coefficient of holes across the quasi neutral p-type region is not uniform and changes half way down the n-QNR at a location $x_1$. As a result at a current level of 400 $\mu A$, the excess minority carrier concentration in the quasi neutral p-type region has the distribution sketched below:

\[ n'(x) \text{ (cm}^{-3}) \]

\[ n'(x_0)=10^{16} \quad n'(x_1)=8 \times 10^{15} \]

\[ 0.1 \mu m \quad 0.2 \mu m \]

\[ x_p \quad x_1 \quad x (\mu m) \]

\[ Q_1 \quad Q_2 \quad Q_3 \]

\textbf{a)} Calculate the electron diffusion coefficient in both portions of the quasi neutral regions.

The current is constant in the QNR since there is no recombination.

\[ I = qD_p \frac{dn_p}{dx} A \quad \text{Thus} \quad D_p = \frac{I}{q} \frac{dn_p}{dx} A \]

\[ D_{p1} = 12.5 \text{ cm}^2/\text{s}; \quad D_{p2} = 3.125 \text{ cm}^2/\text{s} \]

\textbf{b)} Calculate the total amount of excess minority carrier charge in the diode.

\[ Q = -\int_{x_p}^{x_0} qAn'(x)dx = Q_1 + Q_2 + Q_3 = -2.08 \times 10^{-14} C \]

\textbf{c)} Calculate the diffusion capacitance of the diode.

\[ |Q| = \tau_r I_D \quad \text{Thus} \quad \tau_r = |Q| / I_D = 52 \text{ ps} \quad \text{and} \quad C_d = \frac{\tau_r I_D}{V_{th}} = 804 fF \]
d) Calculate the electron diffusion velocity at $x_1^-$ and at $x_1^+$. 

\[ I(x) = qAn'(x)v_{\text{diff}} \quad \text{Thus} \quad v_{\text{diff}}(x_1^-) = v_{\text{diff}}(x_1^+) = \frac{I}{qAn(x)} = 3.125 \cdot 10^5 \text{cm/s} \]

Problem 3

A pn junction diode can be used as a tunable capacitor. For a short-base diode with area \(A = 20 \times 20 \ \mu\text{m}^2\), \(N_a = 10^{17} \ \text{cm}^{-3}\), \(W_p = 1 \ \mu\text{m}\), \(N_d = 10^{19} \ \text{cm}^{-3}\), \(W_n = 0.25 \ \mu\text{m}\):

a) Compute the depletion capacitance for a \(V_D\) of -2V and for a \(V_D\) of \(\phi_B / 2\).

\[ \phi_B = 0.96V \]

\[ C_j = \frac{C_{j0}}{\sqrt{1 - \frac{V_D}{\phi_B}}} = \frac{0.37 \text{pF}}{\sqrt{1 - \frac{V_D}{\phi_B}}} \]

Thus for \(V_D = -2V\), \(C_j = 0.21\text{pF}\) and for \(V_D = 0.48V\) \(C_j = 0.52\text{pF}\)

b) Compute the diffusion capacitance and the total capacitance for a \(V_D\) of -2V and for a \(V_D\) of \(\phi_B / 2\).

\[ \mu_n = 750 \text{cm}^2/\text{Vs} \quad \text{Thus} \quad D_n = 19.5 \text{cm}^2/\text{s} \]

\[ \mu_p = 75 \text{cm}^2/\text{Vs} \quad D_p = 1.95 \text{cm}^2/\text{s} \quad \text{And} \]

For \(V_D = -2V\), \(C_d\) is negligible compared \(C_j\) since the minority carriers are being “extracted” from the junction. For \(V_D = 0.48V\) we are in forward bias.

\[ C_d = \frac{I_p \tau_{ip} + I_n \tau_{in}}{V_{th}} \]

In forward bias \(x_n = 7.8 \ \text{nm}\) and \(x_p = 78 \text{nm} < \text{than} \ w_n \text{ and} \ w_p \text{ thus} \ \tau_{ip} = 160 \text{ps} \quad \text{and} \quad \tau_{in} = 256 \text{ps} \]

\[ I_p = qD_p \frac{dp_n}{dx} A \approx qD_p \frac{p(x_n)}{w_n} A = 52 \text{pA} \]
\[ I_n = q D_n \frac{dn_p}{dx} A \approx q D_n \frac{n(-x_p)}{w_p} A = 13 nA \]

Thus \[ C_d = \frac{I_n \tau_m}{V_{th}} = 0.128 \text{fF} \]

The total capacitance is therefore dominated by the junction capacitance:

Thus for \( V_D = -2 \text{V} \), \( C_j = \) and for \( V_D = 0.48 \text{V} \) \( C_j = \)

For \( V_D = -2 \text{V} \), \( C_T = 0.21 \text{pF} \) and for \( V_D = 0.48 \text{V} \) \( C_T = 0.52 \text{pF} \)

c) Compute the conductance for a \( V_D \) of -2V and for a \( V_D \) of \( \phi_B / 2 \).

The diode conductance is negligible in reverse bias.

In forward bias

\[ g_d = \frac{I}{V_{th}} = \frac{I_n}{V_{th}} = 5 \mu S \]
Problem 4
You are given a pn junction diode with the device data shown below.

Device Data:
Na = \(10^{16}\) cm\(^{-3}\)
Nd = \(10^{15}\) cm\(^{-3}\)
\(\mu_n = 1400\) cm\(^2\)/Vs
\(\mu_p = 500\) cm\(^2\)/Vs
A = 50 \(\mu m\) x 50 \(\mu m\)
W\(_{n}\) = W\(_{p}\) = 2 \(\mu m\)

a) What is the maximum applied voltage we can place across the diode and still satisfy
the Low Level Injection constraint? We define this onset as when the minority carrier
concentration equals \(1/10\) of the majority carrier concentration in thermal
equilibrium.

\[
V_D = Vth \cdot \ln \left( \frac{n_{max} \left( -x_p \right)}{n_{p0}} \right) = 0.026 \cdot \ln \left( \frac{10^{15}}{10^{4}} \right) = 0.66V
\]

\[
V_D = Vth \cdot \ln \left( \frac{p_{max} \left( -x_p \right)}{p_{n0}} \right) = 0.026 \cdot \ln \left( \frac{10^{14}}{10^{3}} \right) = 0.54V
\]

Thus the max voltage that satisfies LLI is due to the doping in the N region \(V_D = 0.54V\).

b) Calculate \(x_d\) at \(V_D = 0.54V\)

Since \(V_D > \phi_B / 2 = 0.66\) V

\[
x_d = \frac{x_{d0}}{\sqrt{2}} = 0.685\mu m
\]

\[
x_d = \sqrt{\frac{\varepsilon \phi_B}{q} \left( \frac{1}{Na} + \frac{1}{Nd} \right)} = 0.685\mu m
\]

\(x_p = 62.2\)\(nm\)

\(x_n = 0.623\)\(\mu m\)

c) Calculate \(I_0\)

\[
I_0 = qn_i^2 A \left( \frac{D_p}{Na \cdot (w_n - w_p)} + \frac{D_n}{Na \cdot w_p} \right) = 4.69 \cdot 10^{-14} A
\]
Note that $w_p$ and $w_n$ are much larger than $x_p$, but that $x_n$ is not negligible.

d) Calculate the depletion capacitance (in pF) under the applied bias $V_D = 0.54$

Since $V_D > \phi_B / 2$

$$C_j = C_{j0} \sqrt{2} = 0.378 \text{pF}$$

e) Calculate the diffusion capacitance (in pF) at $V_D = 0.54 \text{V}$

$$C_d = \frac{qA}{2V_{th}} \left[ w_p n_{p0} + (w_n - x_n) p_{n0} \right] e^{V_D/V_{th}} = 1.22 \text{pF}$$

f) Calculate $g_d$

$$g_d = \frac{I}{V_{th}} = 1.89 \text{mS}$$