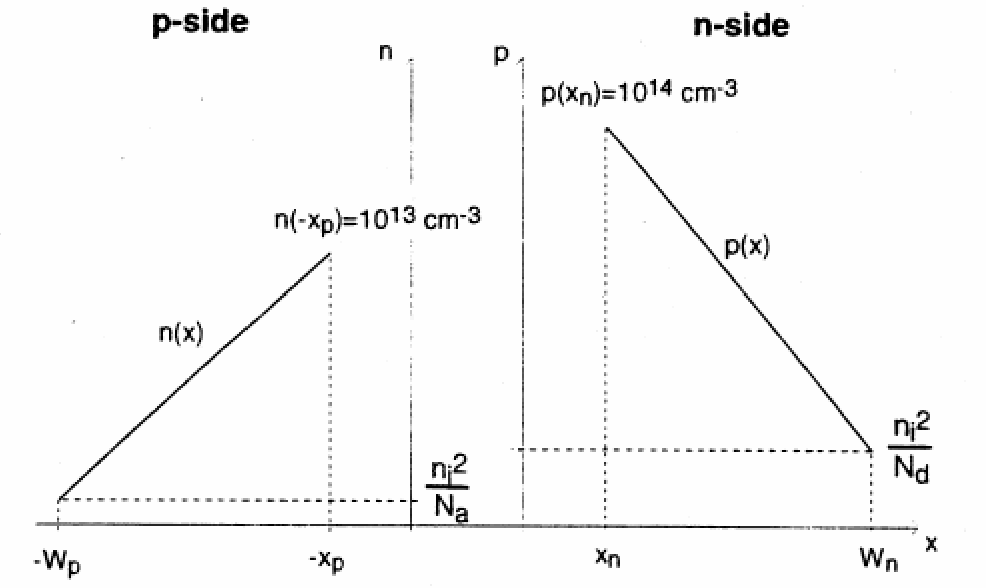
Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
6.012
Microelectronic Devices and Circuits
Spring 2007
Homework #5
Out: 04/04/2007 Due:04/13/2007

Problem 1

Below is a sketch not to scale of the minority carrier distribution across the quasi-neutral regions of a forward biased p-n diode. For this diode, $W_p - x_p = 4$ μm, $W_n - x_n = 3$ μm, $D_n = 25$ cm^2/s and $D_p = 10$ cm^2/s. The area of the junction is 10 μm^2.



a) Calculate the hole current injected into the n-side of the diode.

$$I_p = qD_p \frac{dp_n}{dx} A \cong qD_P \frac{p(x_n)}{w_n - x_n} A = 53nA$$

b) Calculate the electron current injected into the p-side of the diode.

$$I_n = qD_n \frac{dn_p}{dx} A \cong qD_N \frac{n(-x_p)}{w_p - x_p} A = 10nA$$

c) Calculate the diffusion capacitance associated with the carrier storage on the n-side of the diode

$$C_{d\,p} = \frac{dqp_n}{dv_d} = \frac{qA}{2Vth}(w_n - x_n)p(x_n) = 9.3\,fF$$

d) Calculate the diffusion capacitance associated with the carrier storage on the p-side of the diode.

$$C_{d\,n} = \frac{dqn_p}{dv_d} = \frac{qA}{2Vth}(w_p - x_p)n(-x_p) = 1.24\,fF$$

e) How much should the voltage across the junction increase if we wish to double the total current through the diode?

$I_D = I_0 e^{V_D/Vth}$

$2I_D = I_0 e^{V_D'/Vth}$ The voltage across the diode should increase than 18µA.

$V_D' = V_D + Vth \ln 2$

f) Compute the diffusion capacitance of the diode when we increase the voltage in the manner suggested in the previous question.

$$C_d(V_D') = \frac{qA}{2Vth}\left[(w_p - x_p)n_{p0} + (w_n - x_n)p_{n0}\right]e^{V_D'/Vth} = ke^{V_D'/Vth} = 2C_d(V_D)$$

g) What is the ratio of the doping levels across the junction: Na/Nd?

$Na/Nd = p(x_n)/n(-x_p) = 10$

h) In what direction should Na/Nd change if we wish to redesign the diode so as to get less diffusion capacitance at the same current level? (Assume that in redesigning the diode D_n, D_p, $W_n - x_n$, and $W_p - x_p$ do not change). Should Na/Nd increase or decrease? Explain.

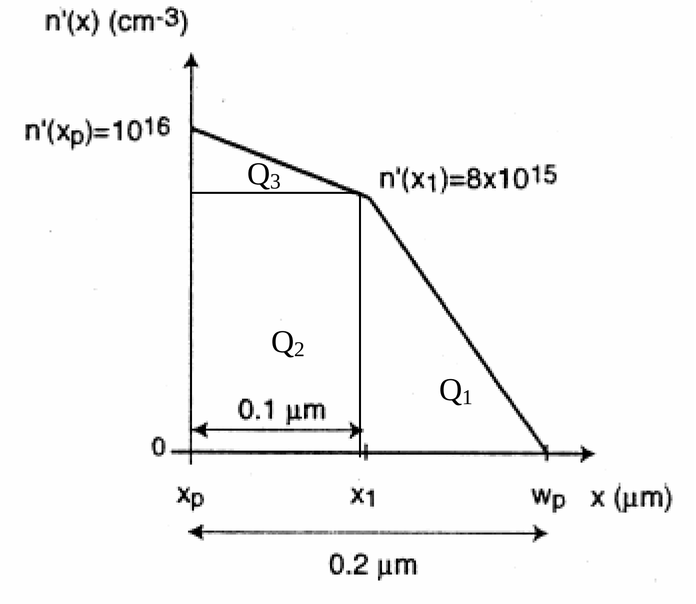
$$C_d = \frac{I_p\tau_{tp} + I_n\tau_{tn}}{Vth}$$

Since $\tau_{tp} > \tau_{tn}$ and since $I_D = I_n + I_p$ is constant, to decrease the diffusion capacitance we should increase the electron current (proportional to 1 / Na) and decrease the hole current (proportional to 1 / Nd). That can be accomplished by decreasing Na / Nd.

Problem 2

Consider an abrupt asymmetric n^+p junction diode with a junction area of 100 μm^2. All the action in this device is dominated by the lightly-doped p-type region.

Due to processing reasons, the diffusion coefficient of holes across the quasi neutral p-type region is not uniform and changes half way down the n-QNR at a location x_1. As a result at a current level of 400 µA, the excess minority carrier concentration in the quasi neutral p-type region has the distribution sketched below:



a) Calculate the electron diffusion coefficient in both portions of the quasi neutral regions.

The current is constant in the QNR since there is no recombination.

$I = qD_p \frac{dn_p}{dx} A$ Thus $D_p = \frac{I}{q\frac{dn_p}{dx}A}$

$D_{p1} = 12.5\ cm^2/s$; $D_{p2} = 3.125\ cm^2/s$

b) Calculate the total amount of excess minority carrier charge in the diode.

$$Q = -\int_{xp}^{wp} qAn'(x)dx = Q_1 + Q_2 + Q_3 = -2.08 \cdot 10^{-14} C$$

c) Calculate the diffusion capacitance of the diode.

$|Q| = \tau_t I_D$ Thus $\tau_t = |Q| / I_D = 52 ps$ and $C_d = \frac{\tau_t I_D}{Vth} = 804 fF$

d) Calculate the electron diffusion velocity at x_1^- and at x_1^+.

$I(x) = qAn'(x)v_{diff}$ Thus $v_{diff}(x_1^-) = v_{diff}(x_1^+) = \dfrac{I}{qAn'(x_1)} = 3.125 \cdot 10^5 cm/s$

Problem 3

A pn junction diode can be used as a tunable capacitor. For a short-base diode with area A = 20 x 20 µm^2, Na = 10^{17} cm^{-3}, W_p = 1 µm, Nd = 10^{19} cm^{-3}, W_n = 0.25 µm:

a) Compute the depletion capacitance for a V_D of -2V and for a V_D of $\phi_B / 2$.

ϕ_B = 0.96V

$$C_j = \frac{C_{j0}}{\sqrt{1-\frac{V_D}{\phi_B}}} = \frac{0.37pF}{\sqrt{1-\frac{V_D}{\phi_B}}}$$

Thus for V_D = -2V, C_j = 0.21pF and for V_D = 0.48V C_j = 0.52pF

b) Compute the diffusion capacitance and the total capacitance for a V_D of -2V and for a V_D of $\phi_B / 2$.

$$\begin{array}{l} \mu_n = 750 cm^2/Vs \\ \mu_p = 75 cm^2/Vs \end{array} \text{ Thus } \begin{array}{l} D_n = 19.5 cm^2/s \\ D_p = 1.95 cm^2/s \end{array} \text{ And}$$

For V_D = -2V, Cd is negligible compared C_j to since the minority carriers are being “extracted” from the junction.

For V_D = 0.48V we are in forward bias.

$$C_d = \frac{I_p \tau_{tp} + I_n \tau_{tn}}{Vth}$$

In forward bias x_n = 7.8 nm and x_p = 78nm<< than w_n and w_p thus $\tau_{tp} = 160 ps$ and $\tau_{tn} = 256 ps$

$$I_p = qD_p \frac{dp_n}{dx} A \cong qD_P \frac{p(x_n)}{w_n} A = 52 pA$$

$$I_n = qD_n \frac{dn_p}{dx} A \cong qD_n \frac{n(-x_p)}{w_p} A = 13nA$$

Thus $C_d = \frac{I_n \tau_{tn}}{Vth} = 0.128\,fF$

The total capacitance is therefore dominated by the junction capacitance:

Thus for V_D = -2V, C_j = and for V_D = 0.48V C_j =

For V_D = -2V, C_T = 0.21pF and for V_D = 0.48V C_T = 0.52pF

c) Compute the conductance for a V_D of -2V and for a V_D of ϕ_B / 2.

The diode conductance is negligible in reverse bias.

In forward bias

$$g_d = \frac{I}{Vth} = \frac{I_n}{Vth} = 5\mu S$$

Problem 4

You are given a pn junction diode with the device data shown below.

Device Data:
Na = 10^{16} cm^{-3}
Nd = 10^{15} cm^{-3}
μ_n = 1400 cm^2/Vs
μ_p = 500 cm^2/Vs
A = 50 µm x 50 µm
$W_n = W_p$ = 2 µm

a) What is the maximum applied voltage we can place across the diode and still satisfy the Low Level Injection constraint? We define this onset as when the minority carrier concentration equals 1 / 10 of the majority carrier concentration in thermal equilibrium.

$$V_D = Vth \cdot \ln\left(\frac{n_{\max}(-x_p)}{n_{p0}}\right) = 0.026 \cdot \ln\left(\frac{10^{15}}{10^4}\right) = 0.66V$$

$$V_D = Vth \cdot \ln\left(\frac{p_{\max}(-x_p)}{p_{n0}}\right) = 0.026 \cdot \ln\left(\frac{10^{14}}{10^5}\right) = 0.54V$$

Thus the max voltage that satisfies LLI is due to the doping in the N region V_D = 0.54V.

b) Calculate x_d at V_D = 0.54V

Since $V_D > \phi_B / 2 = 0.66$ V

$$x_d = \frac{x_{d0}}{\sqrt{2}} = 0.685\mu m$$

$$x_d = \sqrt{\frac{\varepsilon_s \phi_B}{q}\left(\frac{1}{Na} + \frac{1}{Nd}\right)} = 0.685\mu m$$

$$x_p = 62.2nm$$

$$x_n = 0.623\mu m$$

c) Calculate I_0

$$I_0 = qn_i^{\,2} A\left(\frac{D_p}{Nd \cdot (w_n - w_n)} + \frac{D_n}{Na \cdot w_p}\right) = 4.69 \cdot 10^{-14} A$$

Note that w_p and w_n are much larger than x_p, but that x_n is not negligible.

d) Calculate the depletion capacitance (in pF) under the applied bias $V_D = 0.54$

Since $V_D > \phi_B / 2$

$$C_j = C_{j0}\sqrt{2} = 0.378\,pF$$

e) Calculate the diffusion capacitance (in pF) at $V_D = 0.54V$

$$C_d = \frac{qA}{2Vth}\left[w_p n_{p0} + (w_n - x_n)\,p_{n0}\right]e^{V_D/Vth} = 1.22\,pF$$

f) Calculate g_d

$$g_d = \frac{I}{Vth} = 1.89mS$$