Problem 1
An npn transistor with area $A_E = 2.5 \, \mu m \times 2.5 \, \mu m$ is biased in the forward active region, with the collector current $I_C = 50 \, \mu A$. The emitter, base and collector dimensions and doping are:

$N_{dE} = 10^{19} \, \text{cm}^{-3}$, $W_E = 0.3 \, \mu m$, $N_{aB} = 10^{17} \, \text{cm}^{-3}$, $W_B = 0.25 \, \mu m$, and $N_{dC} = 10^{16} \, \text{cm}^{-3}$, $W_C = 1.5 \, \mu m$.

A) Draw a picture of the minority carrier concentration in the emitter and base (identify the minority carrier concentration at the base and emitter edges).

From page 36 of Professor Sodini’s book:

$$\mu_{nB} = 775 \, \text{cm}^2 / \text{Vs} \quad \text{Thus} \quad D_{nB} = 20.03 \, \text{cm}^2 / \text{s}$$

$$\mu_{pE} = 75 \, \text{cm}^2 / \text{Vs} \quad D_{pE} = 1.939 \, \text{cm}^2 / \text{s}$$

$$\beta_F = \frac{N_{dE}D_{nB}W_E}{N_{aB}D_{pE}W_B} = 1240 \quad \text{And} \quad I_B = \frac{I_C}{\beta_F} = 40.323 \, nA$$

Also

$$I_B = qA_E \frac{D_{pE}}{W_E} \frac{n_i^2}{N_{dE}} e^{\frac{V_{as}/\gamma_h}{V_{th}}}$$

Therefore $V_{BE} = 0.7616V$

Thus

$p_{aE}(0^-) = 6.23 \cdot 10^{13} \, \text{cm}^{-3}$

$n_{pB}(0^-) = 6.23 \cdot 10^{15} \, \text{cm}^{-3}$

$p_{aE}(-W_E) = 10 \, \text{cm}^{-3}$

$n_{pB}(W_B) = 0 \, \text{cm}^{-3}$

B) Find the base-emitter bias $V_{BE}$.

$V_{BE} = 0.7616V$

C) Find the base current $I_B$.

$$I_B = \frac{I_C}{\beta_F} = 40.323 \, nA$$
D) For the npn BJT biased as above, given that $V_{An}=25\ V$, find the transconductance $g_m$, the input resistance $r_\pi$, and the output resistance $r_0$.

$$g_m = \frac{I_C}{V_{th}} = 1.93mS;\quad r_\pi = \frac{\beta_F}{g_m} = 641k\Omega;\quad r_0 = \frac{V_A}{I_C} = 500k\Omega$$

E) For the npn BJT biased as above, given that the emitter-base depletion region width is $x_{BE}=0.05\ \mu m$, what is the minority electron charge storage in the base $Q_{NB}(V_{BE})$ at this operating point?

$$|Q_{nb}| = qA\int_{\text{Base}} n(x)dx = \frac{1}{2} q \cdot A_{np} \cdot (W_B - x_{BE}) = 6.24 \cdot 10^{-16} C$$

F) What is $C_\pi$ at this operating point?

$$C_\pi = A \frac{E_{Si}}{x_d} + g_m \frac{W_B^2}{2D_{nb}} = 12.9\ fF + 19.3\ fF = 32.2\ fF$$

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**Problem 2**

In this problem we will consider an important development of the late 1980s, the SiGe alloy base BJT. This Hetero Bipolar Transistor (HBT) is usually fabricated as an npn BJT with a base made of SiGe to increase the intrinsic carrier concentration in the base and with Si collector and emitter. The emitter, base, and collector dimensions are:

- $N_{dE} = 5x10^{19}\ \text{cm}^{-3}$, $W_E = 0.25\ \mu m$, $N_{aB} = 10^{18}\ \text{cm}^{-3}$, $W_B = 0.25\ \mu m$, and $N_{dC} = 10^{17}\ \text{cm}^{-3}$, $W_C = 1.5\ \mu m$. Note that at room temperature the intrinsic carrier concentration of SiGe is $n_{iSiGe} = 5x10^{10}\ \text{cm}^{-3}$.

For this problem assume that the concentration of Ge is low thus the mobility, dielectric constant of the SiGe base film remain unchanged from that of Si.

A) Find $\alpha_F$ and the forward active current gain $\beta_F$ for the npn SiGe HBT (SiGe Base Transistor) and npn BJT (Si Base) at room temperature.

$$\mu_{nb} = 325 cm^2/Vs\quad D_{nb} = 8.4 cm^2/s\quad D_{pE} = 1.29 cm^2/s$$

$$\mu_{pE} = 50 cm^2/Vs\quad \text{Thus}$$

$$\beta_{FSiGe} = \frac{N_{dE} D_{ab} n_{SiGe}^2 W_E}{N_{aB} D_{pE} n_{Si}^2 W_B} = 8125\quad \text{And}\quad \alpha_{FSiGe} = \frac{\beta_{FSiGe}}{1 + \beta_{FSiGe}} = 0.99988$$

$$\beta_{FSi} = \frac{N_{dE} D_{ab} W_E}{N_{aB} D_{pE} W_B} = 325\quad \text{And}\quad \alpha_{FSi} = \frac{\beta_{FSi}}{1 + \beta_{FSi}} = 0.9969$$
B) What is the ratio between forward active current gains for the npn SiGe HBT and the corresponding npn BJT?

\[
\frac{\beta_{FSiGe}}{\beta_{FSi}} = \frac{n_{SiGe}^2}{n_{Si}^2} = 25
\]

C) Determine the base doping of the npn BJT that will yield the same value of $\beta_F$ as in the npn SiGe HBT. (Note that the mobility depend on the doping, thus changing the doping would change the mobility. You should converge to the solution through few iterations)

For $N_{aB} = 9.2 \cdot 10^{16} \text{ cm}^{-3}$ the mobility is $\mu_{nB} = 750 \text{ cm}^2 / \text{Vs}$

\[
\beta_{FSi} = \beta_{FSiGe} = \frac{N_{aE} D_{aB} W_E}{N_{aB} D_{pt} W_B} = 8150
\]
**Problem 3**

A p⁺np bipolar transistor has the geometry and doping profile described below. For all the following questions the BJT is operating in a common-emitter mode in the forward active region.

BJT Data:

\[ D_{pB} = 5 \text{ cm}^2/\text{s}; \ D_{nE} = 10 \text{ cm}^2/\text{s}; \ W_E = 500 \text{ nm}; \ A = 25 \mu\text{m}^2; \ N_{aE} = 10^{19} \text{ cm}^{-3}; \ N_{db} = 10^{17} \text{cm}^{-3}; \ N_{aC} = 10^{16} \text{cm}^{-3}. \]

A) We want the current gain \( \beta_F \) to be 100, what should be the value for the base thickness \( W_B \)? Neglect depletion region widths.

\[
\beta_F = \frac{N_{aE} D_{pb} W_E}{N_{db} D_{nE} W_B} \quad \text{Therefore} \quad W_B = \frac{N_{aE} D_{pb} W_E}{N_{db} D_{nE} \beta_F} = 250 \text{nm}
\]

B) What is the saturation current \( I_S \) for the emitter-base p-n diode?

\[
I_E = q A_E \frac{D_{pb}}{W_B} n_i^2 e^{\frac{V_{EB}}{V_{th}}} = I_s e^{\frac{V_{EB}}{V_{th}}} \quad \text{Therefore} \quad I_s = q A_E \frac{D_{pb}}{W_B} n_i^2 e^{\frac{V_{EB}}{V_{th}}} = 8 \cdot 10^{-18} \text{A}
\]

C) What should be the EB voltage to obtain a collector current of \( I_C = 100 \mu\text{A} \)?

\[
I_C = I_s e^{\frac{V_{EB}}{V_{th}}} \quad \text{Therefore} \quad V_{EB} = V_{th} \cdot \ln\left(\frac{I_C}{I_s}\right) = -0.78V
\]

D) What is the transconductance at \( I_C = 100 \mu\text{A} \)?

\[
g_m = \frac{I_C}{V_{th}} = 3.87mS
\]

E) What is the capacitance \( C_\pi \) at \( I_C = 100 \mu\text{A} \)?

\[
C_\pi = \sqrt{2} C_{j0} A_E + g_m \frac{W_B^2}{2 D_{pB}} = 32.7 \text{fF} + 242 \text{fF} = 275 \text{fF}
\]

\[ C_{j0} = 92.5n\text{F/ cm}^2 \]

F) What is the input resistance at \( I_C = 100 \mu\text{A} \)?

\[
R_{in} = R_E = \frac{\beta_F}{g_m} = 25.8k\Omega
\]

G) What is the output resistance at \( I_C = 100 \mu\text{A} \) given an Early Voltage \( V_A = 30V \)?
\[ Ro = \frac{V_A}{I_c} = \frac{30V}{100\mu A} = 300k\Omega \]

H) In forward active regime find the frequency limit set by the base diffusion transit time?

The BJT needs to be in the forward active region.

\[ V_{BE} < 0 \text{ and } V_{BC} > 0 \]

\[ f_T = \frac{1}{\tau_f} = \frac{1}{W_B^2} = 16\text{GHz} \]

**Problem 4**

The figure below shows six possible ways of connecting an npn bipolar transistor that may yield a diode-like behavior. Using the Ideal Non-Linear Hybrid-π Model, write the I-V characteristics of the two-terminal device in each configuration. Express your results as a function of \( I_S \), \( \beta_F \), and \( \beta_R \).

**Configuration A**

\[ V_{BE} = V \text{ and } V_{BC} = 0 \]

\[ I = \frac{I_S}{\beta} \left( e^{\frac{V_{BE}}{V_{th}}} - 1 \right) + I_S \left( e^{\frac{V_{BE}}{V_{th}}} - 1 \right) \]

Therefore \[ I = I_S \frac{1 + \beta_F}{\beta_F} \left( e^{\frac{V}{V_{th}}} - 1 \right) \]

For \( \beta_F >> 1 \) \[ I \approx I_S \left( e^{\frac{V}{V_{th}}} - 1 \right) \]

**Configuration B**

\[ V_{BE} = V \text{ and } V_{BC} = V \text{ (because there is a short circuit between E and C)} \]

...
\[ I = \frac{I_s}{\beta_F} \left( e^{\frac{V_{ae}}{V_{th}}} - 1 \right) + \frac{I_s}{\beta_R} \left( e^{\frac{V_{ae}}{V_{th}}} - 1 \right) \] Therefore \[ I = \frac{I_s}{\beta_F} \left( e^{\frac{V_{ae}}{V_{th}}} - 1 \right) + \frac{I_s}{\beta_R} \left( e^{\frac{V_{ae}}{V_{th}}} - 1 \right) \]

For \( \beta_F >> \beta_R \)

\[ I \approx \frac{I_s}{\beta_R} \left( e^{\frac{V_{ae}}{V_{th}}} - 1 \right) \]

Configuration C

\( V_{BE} = 0 \) and \( V_{BC} = -V \)

\[ I = -\frac{I_s}{\beta_R} \left( e^{-\frac{V}{V_{th}}} - 1 \right) + I_s \left( 1 - e^{-\frac{V}{V_{th}}} \right) \] Therefore \[ I = -\frac{I_s \beta_R + 1}{\beta_R} \left( e^{-\frac{V}{V_{th}}} - 1 \right) \]

Configuration D

\( V = V_{BC} - V_{BC} \)

\[ \frac{I_s}{\beta_R} \left( e^{\frac{V_{ae}}{V_{th}}} - 1 \right) + \frac{I_s}{\beta_F} \left( e^{\frac{V_{ae}}{V_{th}}} - 1 \right) \] Therefore \[ \frac{I_s}{\beta_R} \left( e^{\frac{V_{ae}-V}{V_{th}}} - 1 \right) + \frac{I_s}{\beta_F} \left( e^{\frac{V_{ae}}{V_{th}}} - 1 \right) \]

Therefore \[ e^{\frac{V_{ae}}{V_{th}}} = \frac{1 - 1}{\beta_R} \frac{e^{-\frac{V}{V_{th}}} - 1}{\beta_R - 1} \] Therefore \[ I = \left( \frac{1}{\beta_R} - \frac{1}{\beta_F} \right) \left( e^{-\frac{V}{V_{th}}} + 1 \right) - \frac{1}{\beta_F} \]

Configuration E

\( V = -V_{BC} \) and \( V_{BE} = \phi_{BE} \)

No net current flow through the base/emitter junction since the emitter is open. The BJT is working as a diode.

\[ I = \frac{I_s}{\beta_R} \left( e^{-\frac{V}{V_{th}}} - 1 \right) \]
Configuration E

$V = V_{BE}$ and $V_{BC} = \phi_{BC}$

No net current flow through the base/collector junction since the emitter is open. The BJT is working as a diode.

$$I = \frac{I_S}{\beta_F} \left( e^{V_{th}/V_{fb}} - 1 \right)$$