Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
6.012
Microelectronic Devices and Circuits
Spring 2007
Homework #6
Out: 04/13/2007 Due: 04/20/2007

Problem 1

An npn transistor with area A_E = 2.5 µm x 2.5 µm is biased in the forward active region, with the collector current I_C = 50 µA. The emitter, base and collector dimensions and doping are:
$N_{dE} = 10^{19}$ cm^{-3}, W_E = 0.3 µm, $N_{aB} = 10^{17}$ cm^{-3}, W_B = 0.25 µm, and $N_{dC} = 10^{16}$ cm^{-3}, W_C = 1.5 µm.

A) Draw a picture of the minority carrier concentration in the emitter and base (identify the minority carrier concentration at the base and emitter edges).

From page 36 of Professor Sodini's book:

$$\mu_{nB} = 775cm^2/Vs \quad \text{Thus} \quad D_{nB} = 20.03cm^2/s$$
$$\mu_{pE} = 75cm^2/Vs \qquad D_{pE} = 1.939cm^2/s$$

$$\beta_F = \frac{N_{dE}D_{nB}W_E}{N_{aB}D_{pE}W_B} = 1240 \text{ And } I_B = \frac{I_C}{\beta_F} = 40.323nA \text{ Also } I_B = qA_E\frac{D_{pE}}{W_E}\frac{n_i^2}{N_{dE}}e^{V_{BS}/V_{th}}$$

Therefore V_{BE} = 0.7616V

$$\text{Thus} \quad p_{nE}(0^-) = 6.23\cdot10^{13}cm^{-3} \quad \text{and} \quad n_{pB}(0^+) = 6.23\cdot10^{15}cm^{-3}$$
$$p_{nE}(-W_E) = 10cm^{-3} \qquad n_{pB}(W_B) = 0cm^{-3}$$

B) Find the base-emitter bias $V_{BE.}$

V_{BE} = 0.7616V

C) Find the base current $I_{B.}$

$$I_B = \frac{I_C}{\beta_F} = 40.323nA$$

D) For the npn BJT biased as above, given that V_{An} = 25 V, find the transconductance g_m, the input resistance r_π, and the output resistance r_o.

$$g_m = \frac{I_C}{Vth} = 1.93mS\ ;\quad r_\pi = \frac{\beta_F}{g_m} = 641k\Omega\ ;\quad r_0 = \frac{V_A}{I_C} = 500k\Omega$$

E) For the npn BJT biased as above, given that the emitter-base depletion region width is x_{BE} = 0.05 µm, what is the minority electron charge storage in the base Q_{NB}(VBE) at this operating point?

$$|Q_{nB}| = qA\int_{Base} n(x)dx = 1/2qAn_{pB}(0^-)(W_B - x_{BE}) = 6.24\cdot 10^{-16}C$$

F) What is C_π at this operating point?

$$C_\pi = A\frac{\varepsilon_{Si}}{x_d} + g_m\frac{w_B{}^2}{2D_{nB}} = 12.9\,fF + 19.3\,fF = 32.2\,fF$$

Problem 2

In this problem we will consider an important development of the late 1980s, the SiGe alloy base BJT. This Hetero Bipolar Transistor (HBT) is usually fabricated as an npn BJT with a base made of SiGe to increase the intrinsic carrier concentration in the base and with Si collector and emitter. The emitter, base, and collector dimensions are:
N_{dE} = 5x10^{19} cm^{-3}, W_E = 0.25 µm, N_{aB} = 10^{18} cm^{-3}, W_B = 0.25 µm, and N_{dC} = 10^{17} cm^{-3}, W_C = 1.5 µm. Note that at room temperature the intrinsic carrier concentration of SiGe is n_{iSiGe} = 5x10^{10} cm^{-3}.

For this problem assume that the concentration of Ge is low thus the mobility, dielectric constant of the SiGe base film remain unchanged from that of Si.

A) Find α_F and the forward active current gain β_F for the npn SiGe HBT (SiGe Base Transistor) and npn BJT (Si Base) at room temperature.

$$\mu_{nB} = 325cm^2/Vs \qquad D_{nB} = 8.4cm^2/s$$
Thus
$$\mu_{pE} = 50cm^2/Vs \qquad D_{pE} = 1.29cm^2/s$$

$$\beta_{FSiGe} = \frac{N_{dE}D_{nB}n_{iSiGe}{}^2W_E}{N_{aB}D_{pE}n_{iSi}{}^2W_B} = 8125 \quad \text{And} \quad \alpha_{FSiGe} = \frac{\beta_{FSiGe}}{1+\beta_{FSiGe}} = 0.99988$$

$$\beta_{FSi} = \frac{N_{dE}D_{nB}W_E}{N_{aB}D_{pE}W_B} = 325 \quad \text{And} \quad \alpha_{FSi} = \frac{\beta_{FSi}}{1+\beta_{FSi}} = 0.9969$$

B) What is the ratio between forward active current gains for the npn SiGe HBT and the corresponding npn BJT?

$$\frac{\beta_{FSiGe}}{\beta_{FSi}} = \frac{n_{iSiGe}^{\ 2}}{n_{iSi}^{\ 2}} = 25$$

C) Determine the base doping of the npn BJT that will yield the same value of β_F as in the npn SiGe HBT. (Note that the mobility depend on the doping, thus changing the doping would change the mobility. You should converge to the solution through few iterations)

For $N_{aB} = 9.2 \cdot 10^{16} cm^{-3}$ the mobility is $\mu_{nB} = 750 cm^2 / Vs$

$$\beta_{FSi} = \beta_{FSiGe} = \frac{N_{dE} D_{nB} W_E}{N_{aB} D_{pE} W_B} = 8150$$

Problem 3

A p^+np bipolar transistor has the geometry and doping profile described below. For all the following questions the BJT is operating in a common-emitter mode in the forward active region.

BJT Data:

D_{pB} = 5 cm^2/s; D_{nE} = 10 cm^2/s; W_E = 500 nm; A = 25 µm^2; N_{aE} = 10^{19} cm^{-3}; N_{dB} = 10^{17}cm^{-3}; N_{aC} = 10^{16}cm^{-3}.

A) We want the current gain β_F to be 100, what should be the value for the base thickness W_B? Neglect depletion region widths.

$$\beta_F = \frac{N_{aE} D_{pB} W_E}{N_{dB} D_{nE} W_B} \quad \text{Therefore} \quad W_B = \frac{N_{aE} D_{pB} W_E}{N_{dB} D_{nE} \beta_F} = 250nm$$

B) What is the saturation current I_S for the emitter-base p-n diode?

$$I_E = qA_E \frac{D_{pB}}{W_B} \frac{n_i^2}{N_{aB}} e^{V_{EB}/Vth} = I_s e^{V_{EB}/Vth} \quad \text{Therefore} \quad I_s = qA_E \frac{D_{pB}}{W_B} \frac{n_i^2}{N_{dB}} = 8 \cdot 10^{-18} A$$

C) What should be the EB voltage to obtain a collector current of I_C = 100 µA?

$$I_C = I_s e^{V_{EB}/Vth} \quad \text{Therefore} \quad V_{EB} = Vth \cdot \ln\left(I_C / I_s \right) = -0.78V$$

D) What is the transconductance at I_C = 100 µA?

$$g_m = \frac{I_C}{Vth} = 3.87mS$$

E) What is the capacitance C_π at I_C = 100 µA?

$$C_\pi = \sqrt{2} C_{j0} A_E + g_m \frac{W_B{}^2}{2D_{pB}} = 32.7\,fF + 242\,fF = 275\,fF$$

$$C_{j0} = 92.5nF/cm^2$$

F) What is the input resistance at I_C = 100 µA?

$$Rin = R_\pi = \frac{\beta_F}{g_m} = 25.8k\Omega$$

G) What is the output resistance at I_C = 100 µA given an Early Voltage V_A = 30V?

$$Ro = \frac{V_A}{I_C} = \frac{30V}{100\mu A} = 300k\Omega$$

H) In forward active regime find the frequency limit set by the base diffusion transit time?

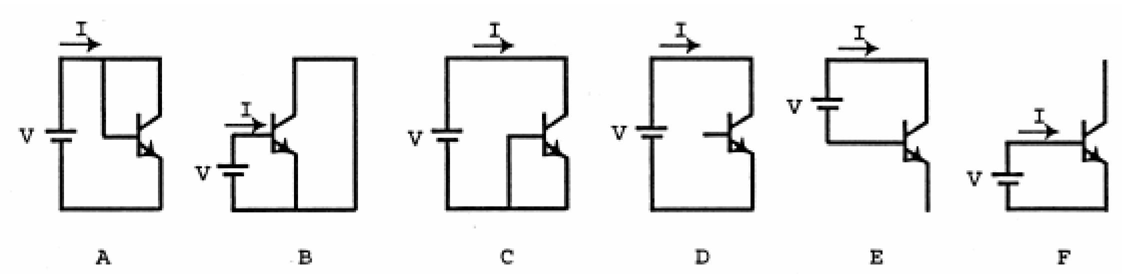
The BJT needs to be in the forward active region.

$V_{BE} < 0$ and $V_{BC} > 0$

$$f_T = \frac{1}{\tau_F} = \frac{1}{\frac{W_B^{\ 2}}{2D_{pB}}} = 16GHz$$

Problem 4

The figure below shows six possible ways of connecting an npn bipolar transistor that may yield a diode-like behavior. Using the Ideal Non-Linear Hybrid-π Model, write the I-V characteristics of the two-terminal device in each configuration. Express your results as a function of I_S, β_F, and β_R.



Configuration A

$V_{BE} = V$ and $V_{BC} = 0$

$$I = \frac{I_S}{\beta}\left(e^{V_{BE}/Vth} - 1\right) + I_S\left(e^{V_{BE}/Vth} - 1\right) \text{ Therefore } I = I_S\frac{1+\beta_F}{\beta_F}\left(e^{V/Vth} - 1\right)$$

For $\beta_F >> 1$ $\quad I \cong I_S\left(e^{V_{BE}/Vth} - 1\right)$

Configuration B

$V_{BE} = V$ and $V_{BC} = V$ (because there is a short circuit between E and C)

$$I = \frac{I_S}{\beta_F}\left(e^{V_{BE}/Vth} - 1\right) + \frac{I_S}{\beta_R}\left(e^{V_{BE}/Vth} - 1\right) \text{ Therefore } I = \frac{I_S}{\beta_F}\left(e^{V/Vth} - 1\right) + \frac{I_S}{\beta_R}\left(e^{V/Vth} - 1\right)$$

For $\beta_F >> \beta_R$

$$I \cong \frac{I_S}{\beta_R}\left(e^{V_{BE}/Vth} - 1\right)$$

Configuration C

$V_{BE} = 0$ and $V_{BC} = -V$

$$I = -\frac{I_S}{\beta_R}\left(e^{-V/Vth} - 1\right) + I_S\left(1 - e^{-V/Vth}\right) \text{ Therefore } I = -I_S \frac{\beta_R + 1}{\beta_R}\left(e^{-V/Vth} - 1\right)$$

Configuration D

$V = V_{BC} - V_{BC}$

$$\frac{I_S}{\beta_R}\left(e^{V_{BC}/Vth} - 1\right) + \frac{I_S}{\beta_F}\left(e^{V_{BE}/Vth} - 1\right) \text{ Therefore } \frac{I_S}{\beta_R}\left(e^{V_{BE}-V/Vth} - 1\right) + \frac{I_S}{\beta_F}\left(e^{V_{BE}/Vth} - 1\right)$$

$$\text{Therefore } e^{V_{BE}/Vth} = \frac{\frac{1}{\beta_R} - \frac{1}{\beta_F}}{\frac{e^{-V/Vth}}{\beta_R} - \frac{1}{\beta_F}} \text{ Therefore } I = \left(\frac{\frac{1}{\beta_R} - \frac{1}{\beta_F}}{\frac{e^{-V/Vth}}{\beta_R} - \frac{1}{\beta_F}}\right)\left(1 - e^{-V/Vth} + \frac{1}{\beta_F}\right) - \frac{1}{\beta_F}$$

Configuration E

$V = -V_{BC}$ and $V_{BE} = \phi_{BE}$

No net current flow through the base/emitter junction since the emitter is open. The BJT is working as a diode.

$$I = \frac{I_S}{\beta_R}\left(e^{-V/Vth} - 1\right)$$

Configuration E

$V = V_{BE}$ and $V_{BC} = \phi_{BC}$
No net current flow through the base/collector junction since the emitter is open. The BJT is working as a diode.

$$I = \frac{I_S}{\beta_F}\left(e^{V/V_{th}} - 1\right)$$