Lecture 16
The pn Junction Diode (III)

Outline

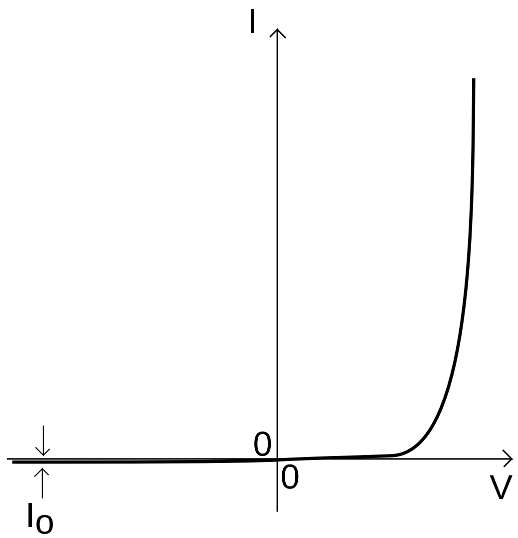
- Small-signal equivalent circuit model
- Carrier charge storage
  - –Diffusion capacitance

Reading Assignment:
Howe and Sodini; Chapter 6, Sections 6.4 - 6.5

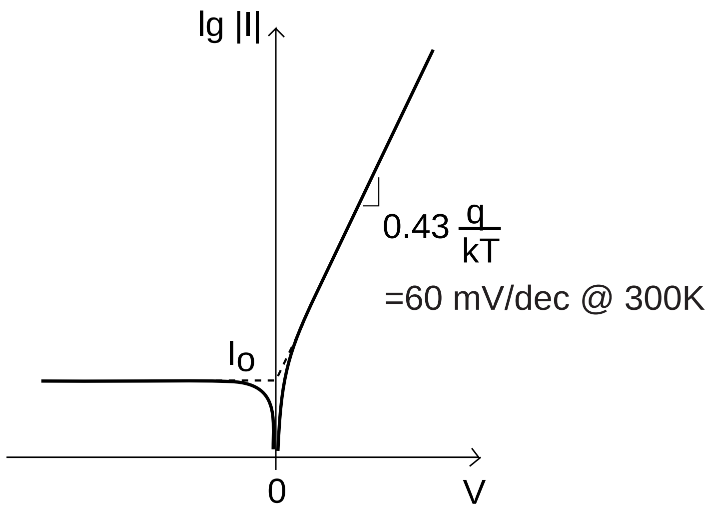
I-V Characteristics

Diode Current equation:

$$I = I_o \left[e^{\left(\frac{V}{V_{th}}\right)} - 1 \right]$$



linear scale



semilogarithmic scale

2. Small-signal equivalent circuit model

Examine effect of small signal adding to forward bias:

$$I + i = I_o \left[e^{\left(\frac{q(V+v)}{kT} \right)} - 1 \right] \approx I_o e^{\left(\frac{q(V+v)}{kT} \right)}$$

If v small enough, linearize exponential characteristics:

$$I + i \approx I_o \left[e^{\left(\frac{qV}{kT} \right)} e^{\left(\frac{qv}{kT} \right)} \right] \approx I_o \left[e^{\left(\frac{qV}{kT} \right)} \left(1 + \frac{qv}{kT} \right) \right]$$

$$= I_o e^{\left(\frac{qV}{kT} \right)} + I_o e^{\left(\frac{qV}{kT} \right)} \frac{qv}{kT}$$

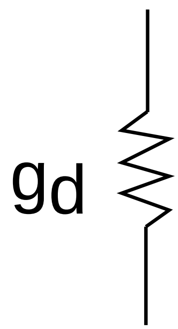
Then:

$$i = \frac{qI}{kT} \bullet v$$

From a small signal point of view. Diode behaves as ***conductance*** of value:

$$g_d = \frac{qI}{kT}$$

Small-signal equivalent circuit model

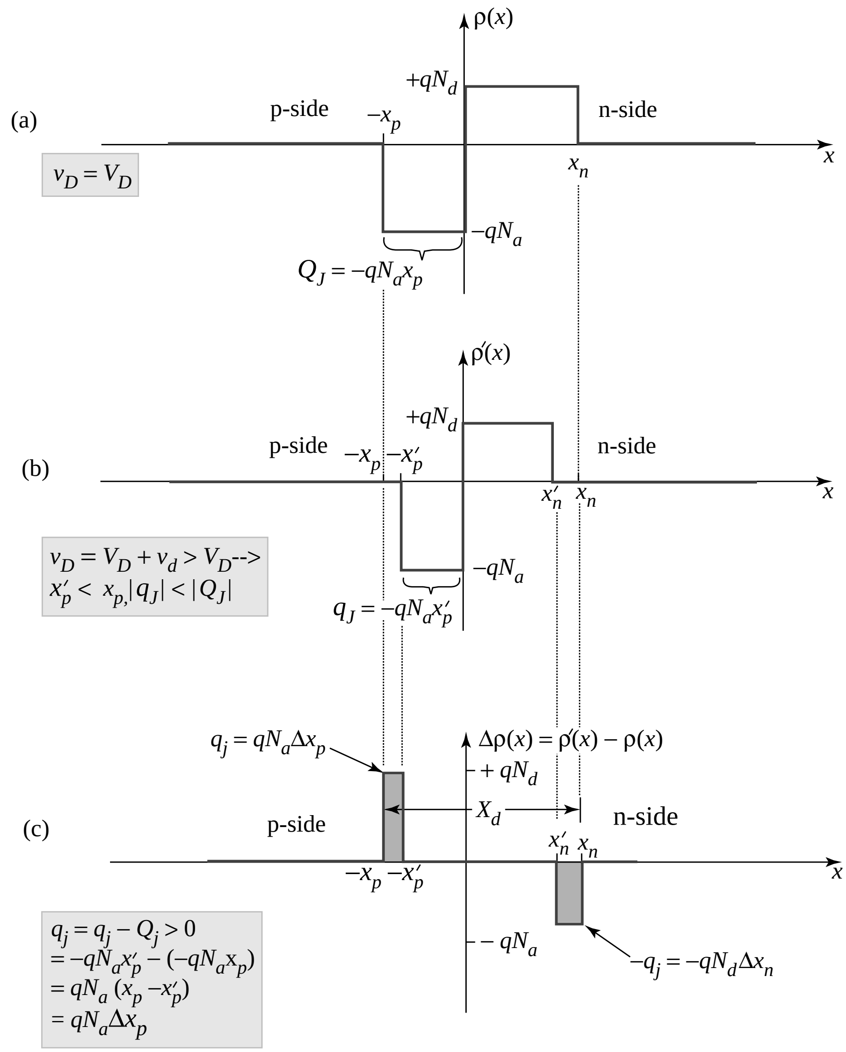


g_d depends on bias. In forward bias:

$$g_d = \frac{qI}{kT}$$

g_d is linear in diode current.

Capacitance associated with depletion region:

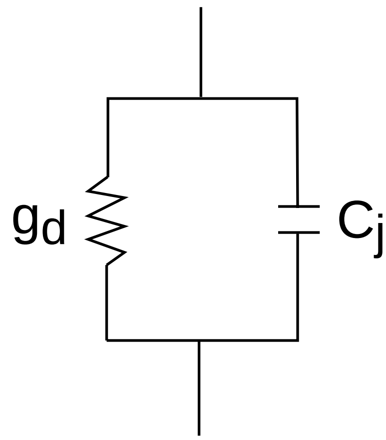


Depletion or junction capacitance:

$$C_j = C_j(V_D) = \left.\frac{dq_J}{dv_D}\right|_{V_D}$$

$$C_j = A\sqrt{\frac{q\varepsilon_s N_a N_d}{2(N_a + N_d)(\phi_B - V_D)}}$$

Small-signal equivalent circuit model



can rewrite as:

$$C_j = A\sqrt{\frac{q\varepsilon_s N_a N_d}{2(N_a + N_d)\phi_B}} \bullet \sqrt{\frac{\phi_B}{(\phi_B - V_D)}}$$

or,

$$C_j = \frac{C_{jo}}{\sqrt{1 - \frac{V_D}{\phi_B}}}$$

Under Forward Bias assume $V_D \approx \frac{\phi_B}{2}$

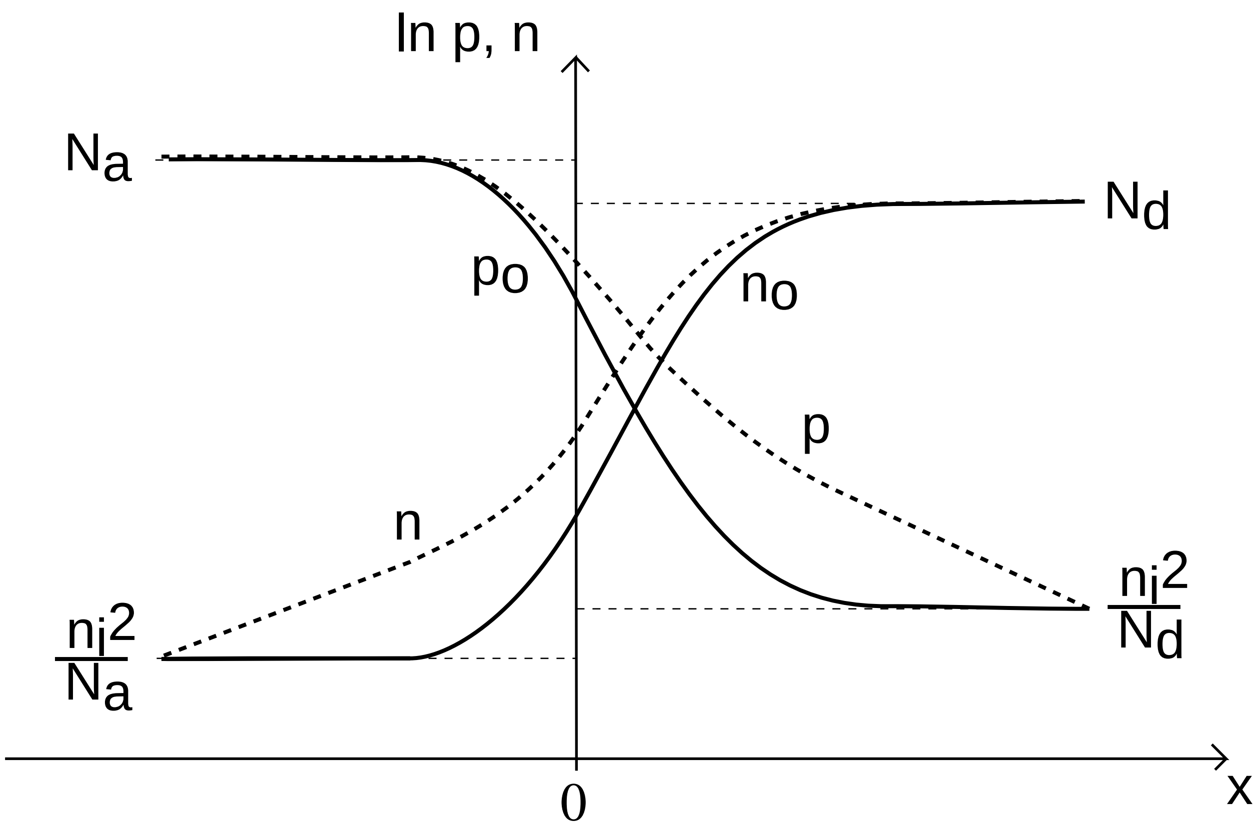
$$\boldsymbol{C_j = \sqrt{2}C_{jo}}$$

$C_{jo} \equiv$ ***zero-voltage junction capacitance***

3. Charge Carrier Storage:
diffusion capacitance

What happens to majority carriers?

Carrier picture thus far:

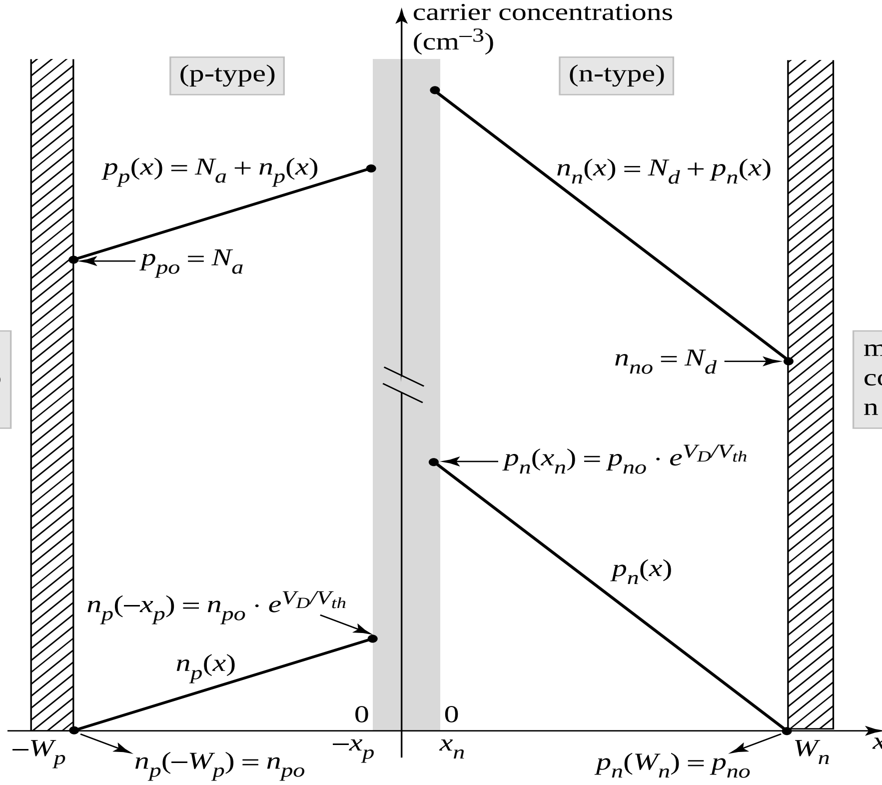


If QNR minority carrier concentration ↑ but majority carrier concentration unchanged? ⇒ quasi-neutrality is violated.

Quasi-neutrality demands that at every point in QNR:

excess minority carrier concentration

= excess majority carrier concentration



Mathematically:

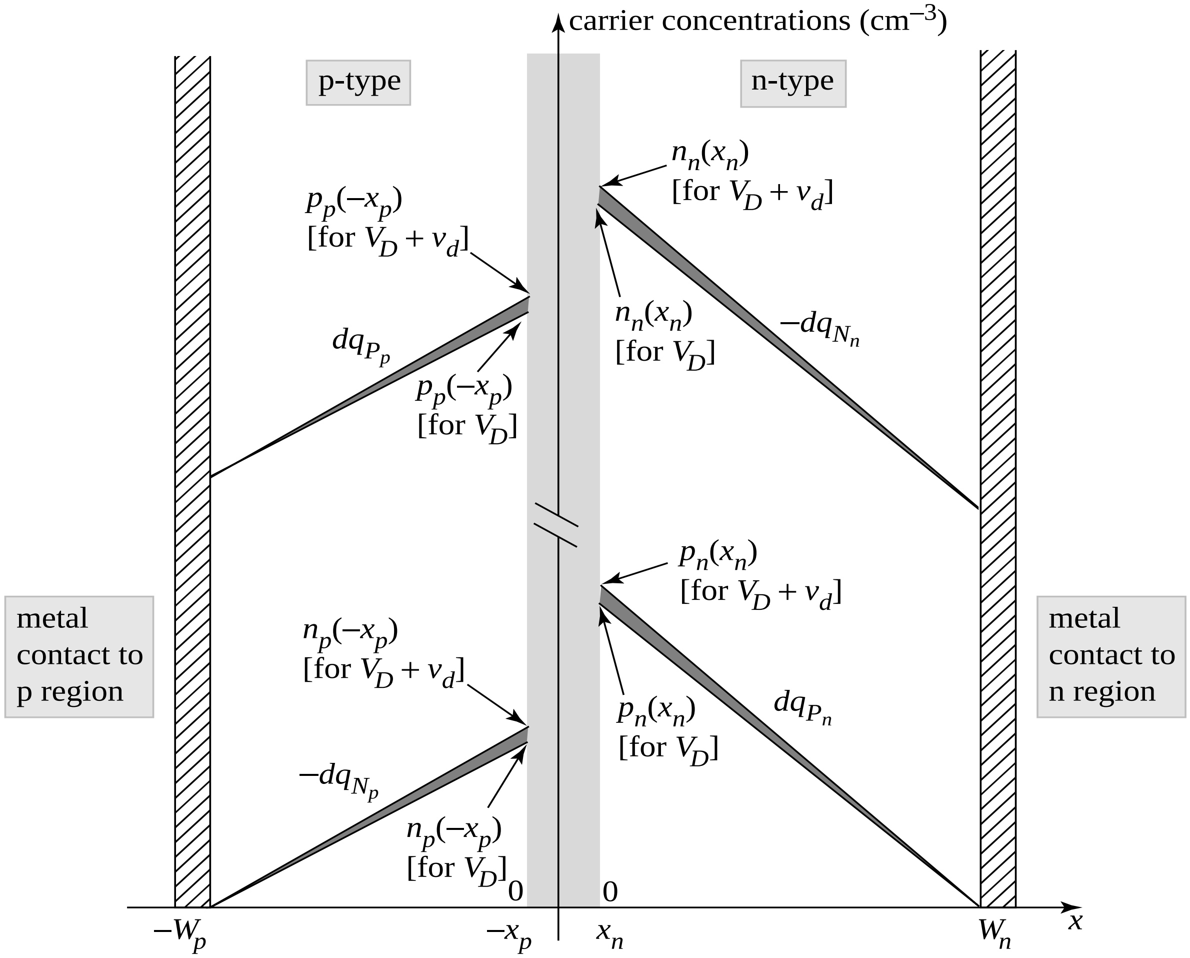
$$p_n(x) - p_{no} = n_n(x) - n_{no}$$

Define integrated carrier charge:

$$q_{Pn} = qA\frac{1}{2}\left(p_n(x_n) - p_{no}\right) \bullet \left(W_n - x_n\right)$$

$$= qA\frac{W_n - x_n}{2}\frac{n_i^2}{N_d}\left[e^{\frac{qV}{kT}} - 1\right] = -q_{Nn}$$

Now examine small increase in V:



Small increase in V ⇒ small increase in q_{Pn} ⇒ small increase in $|q_{Nn}|$

Behaves as capacitor of capacitance:

$$C_{dn} = \left.\frac{dq_{Pn}}{dV}\right|_{V_D} = qA\frac{W_n - x_n}{2}\frac{n_i^2}{N_d}\frac{q}{kT}e^{\left[\frac{qV_D}{kT}\right]}$$

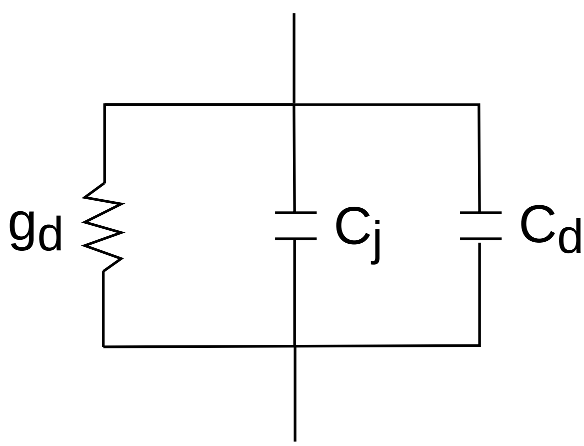
Similarly for p-QNR:

$$C_{dp} = \left. \frac{dq_{Np}}{dV} \right|_{V_D} = qA \frac{W_p - x_p}{2} \frac{n_i^2}{N_a} \frac{q}{kT} e^{\left[\frac{qV_D}{kT}\right]}$$

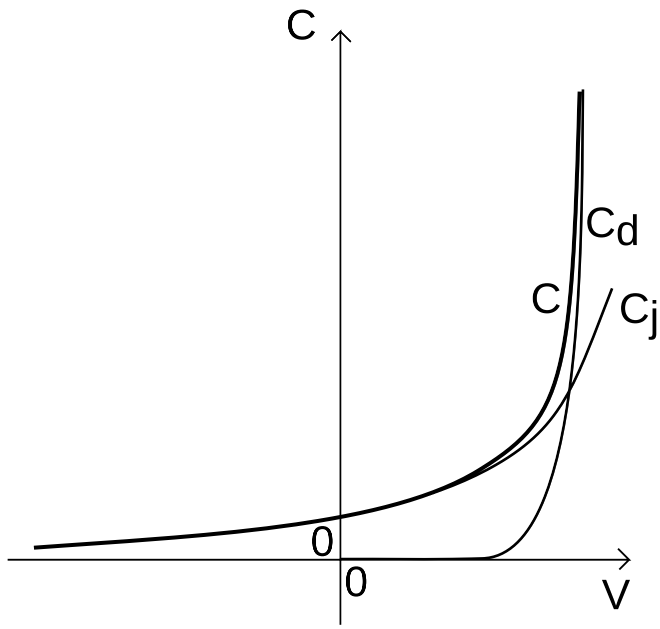
Both capacitors sit in ***parallel*** $\Rightarrow$ total diffusion capacitance:

$$\boldsymbol{C_d = C_{dn} + C_{dp}}$$

Complete small-signal equivalent circuit model for diode:



Bias dependence of C_j and C_d:



- C_j dominates in reverse bias and small forward bias

$$\propto \frac{1}{\sqrt{\phi_B - V}}$$

- C_d dominates in strong forward bias

$$\propto e^{\left[\frac{qV}{kT}\right]}$$

What did we learn today?

Summary of Key Concepts

Large and Small-signal behavior of diode:

- ***Diode Current:***

$$I = I_o\left(e^{\left[\frac{qV}{kT}\right]} - 1\right)$$

- ***Conductance***: associated with current-voltage characteristics
  - $g_d \propto I$ in forward bias,
  - g_d negligible in reverse bias

- ***Junction capacitance***: associated with charge modulation in depletion region

$$C_j \propto \frac{1}{\sqrt{\phi_B - V}}$$

- ***Diffusion capacitance***: associated with charge storage in QNRs to maintain quasi-neutrality.

$$C_d \propto e^{\left[\frac{qV}{kT}\right]}$$