Problem 1 (25 points)

a) The integral of \( H \) around the toroid equals the current flowing through it, or \( 3 \times 5 \) amperes. But this integral \( \approx -\mu_0 H_g d = dB/\mu + dB/\mu_0 \) where \( \mu >> \mu_0 \), so that the first term is negligible and the integral \( \approx dB/\mu_0 = dH_g \), and \( H_g = 15/d \) (a m\(^{-1}\)). Note that \( B \) is continuous across the gap.

b) \( L = \Delta I \) where \( \Delta = \mu H_g A^3 \) since \( B \) is continuous across the gap boundaries and is conserved within the toroid as it threads the \( 3I \) amperes. Therefore \( L = \mu H_g A/5 = 9 \mu A/d \) (henries).

Problem 2 (35 points)

c) The Thevenin equivalent source at the end of the TEM line is twice the forward wave, or a 0.4 volt step function, delayed by \( D/c \) seconds. This charges the capacitor with \( \tau = RC = 4Z_0 x D/8cZ_0 = D/2c \) (seconds). The voltage \( V(z=D) \) across the load rises instantly to \( 3/4 \) of 0.4 volts, since the \( C \) voltage is initially 0. Then it rises further exponentially (\( \tau = D/c \)) to the open circuit value of 0.4 volts, which is:

\[ V(z = D) = 0.3 \]

The reflected wave, \( v_r(t) \) is given by: \( v_r(t) = 0.4 - v_r(t) \).
Problem 3.

a) \[ w_e = CV^2/2 = \epsilon_0 A V^2/2 d \] [J]

b) \[ f = \partial w_e/\partial d = \partial \{CV^2/2\}/\partial d. \] But \( V = f(d) \), so try \( f = \partial (Q^2/2C)/\partial d \) \[ = \partial (dQ^2/2\epsilon_0 A)/\partial d = Q^2/2\epsilon_0 A = C^2V^2/2\epsilon_0 A = \epsilon_0 A V^2/2d^2 \] [N]

Problem 4.

a) \[ Z_{LA} = -j2, \quad Z_{L\alpha} = 1/Z_{LA} \] (use Smith chart or eqn), \( Z_A = Z_{cd}/Z_{LA} = jZ_0/2 \)

b) \[ Z_B = Z_\circ = Z_A + 1/j\omega C. \] So \( Z_{L\alpha} = \frac{1}{1+j} \) since \( C = 1/\omega Z_\circ \)

\[ Z_{L\alpha'} = 1/Z_{L\alpha}, \quad Z_{L'} = Z_\circ(1/(1+j)) = Z_\circ(1-j)/2 \]

Problem 5. (40 points)

a) \[ I = G 100/4\pi R^2 \] where \( G = A4\pi/\lambda^2 \), so \( I = 100(D/\lambda R)^2 \) [W/m²]

b) \[ A = G \lambda^2/4\pi, \quad G = 3/2, \] and \( P_\tau = AI = 37.5(D/R)^2/\pi \) [W]

c) \[ \theta_\tau = \lambda/D \] radians

d) \( P/\hbar f = \) photons/sec @ momentum = \( \hbar f/c. \) Force = momentum/sec = \( P/c = 100/c \)

\[ F = 33 \times 10^4 \] Newtons in the -z direction.

Problem 6. (20 points)

\[ \nabla \times (\nabla \times Q) = a(\partial^2 / \partial t^2) R = -ab(\partial^2 / \partial t^2) = -\nabla^2 Q \]

a) \( (\nabla^2 - ab \partial^2 / \partial t^2) Q = 0 \)

b) \( c_Q = (ab)^{-0.5} \)

Problem 7. (10 points)

\[ \theta = 60^\circ \] top view