**LIMITS TO COMPUTATION SPEED**

**Devices:**
- Emitter
- Drain
- Gate
- Field-effect transistors
- Beyond scope of subject (e.g. 6.012)

**Interconnect:**
- Speed of light $= 3 \times 10^8$ meters/sec
- Say CPU and memory separated by 10 cm
- $2 \times 0.1/(3 \times 10^8) = 0.7$ nsec round-trip delay
- $\Rightarrow <1.5$ Gops without pipelining, but matters are worse

1) $c = (\varepsilon_0 \mu_0)^{0.5}$ where $c$ might be $\sim 2c_0$
2) Reflections may occur at changes in wire dimensions and at device junctions
3) Wire resistance can slow speeds, too

**SIMPLE INTERCONNECTIONS**

**Transverse EM Transmission Lines:**
- TEM: $E_z = H_z = 0$
- Coaxial cable
- Parallel plates
- Stripline
- Wires
- Arbitrary shape if cross-section not $= f(z)$
PARALLEL-PLATE TRANSMISSION LINE

Boundary Conditions:

\[ E_{||} = H_{\perp} = 0 \quad \text{at perfect conductors} \]

Uniform plane wave along \( z \)

satisfies boundary conditions

Wave Equation Solution:

Recall: \( \vec{E} = \hat{x}E_0 \cos(\omega t - kz) = \hat{x}E_0 \cos(\omega(t - z/c)) \) for an \( x \)-polarized wave

at \( \omega \) radians/sec propagating in the +z direction in free space

\( (k = \omega/c = 2\pi/\lambda) \)

\[ \vec{H} = \hat{y}(E_0/\mu_0) \cos(\omega t - kz) \] for the same wave

PARALLEL-PLATE TRANSMISSION LINE (2)

Currents in Plates:

\[ \int_C \vec{H} \cdot d\vec{s} = \int_A \vec{J} \cdot d\vec{a} = I(z) \]

= \( H_w \) independent of path

Surface Currents \( K_s \) (a m\(^{-1}\)):

Boundary conditions:

\[ K_s(z) = \hat{n} \times H(z) \quad \text{amperes/meter} \]

[since \( K_s = |\nabla \times H = 0 \) (from above); \( H_{AC} = 0 \) in conductor]
PARALLEL-PLATE TRANSMISSION LINE (3)

Volatges Across Plate:

\[ \int \mathbf{E} \cdot d\mathbf{s} = \Phi_1 - \Phi_2 = V(z) \]

Since all \( \int_c \mathbf{E} \cdot d\mathbf{s} = 0 \) at fixed \( z \) because \( H_2 = 0 \),

Therefore, \( \int_1^2 \mathbf{E} \cdot d\mathbf{s} = V(z) \) and \( V(z) \) is uniquely defined.

Surface Charges (Coulombs/m²):

Boundary conditions: \( \mathbf{E} \cdot \mathbf{n}(z) = \sigma_S(z) \)

since \( \nabla \cdot \mathbf{D} = \rho \) and \( \int_S \mathbf{E} \cdot \mathbf{n} \, da = \int_V \rho \, dv = A \sigma_S \) for \( A \) (surface area).

VOLTAGE AND CURRENT ON TEM LINES

Integrate \( \mathbf{E}, \mathbf{H} \) to find \( v(z,t), i(z,t) \)

Voltage \( v(z) \) on TEM Lines:

Recall:

\( \mathbf{E} = \hat{x} E_0 \cos(\omega t - kz) \) for an \( x \)-polarized wave

at \( \omega \) radians/sec propagating in the +z direction

\( v(z) = \int_1^2 \mathbf{E} \cdot d\mathbf{s} = E_0 d \cos(\omega t - kz) \) for our example

Currents \( i(z) \) on TEM Lines:

Recall: \( \mathbf{H} = \hat{y} (E_0/\eta_0) \cos(\omega t - kz) \) for the same wave

\( i(z) = \int_c \mathbf{H} \cdot d\mathbf{s} = (E_0 w/\eta_0) \cos(\omega t - kz) \)

Note: \( v(z) \) violates KVL, and \( i(z) \) violates KCL, why?

[Note \( \partial \mathbf{B}/\partial t \) through \( \mathbf{E} \) loop, and \( \partial \mathbf{D}/\partial t \) into plates]

Note: \( v(z,t)/i(z,t) = \eta_0 d/w \) ohms for +z wave alone
TELEGRAPHER’S EQUATIONS

Equivalent Circuit:

\[ \begin{align*}
&L \Delta z + C \Delta z = i(t, z) + i(t, z + \Delta z) \\
&C \Delta z + C \Delta z = v(t, z) + v(t, z + \Delta z)
\end{align*} \]

\[ E \cdot \Delta z = H \cdot \Delta z = 0 \]

Difference Equations:

\[ \begin{align*}
v(z + \Delta z) - v(z) &= -L \Delta z \frac{di(z)}{dt} \\
i(z + \Delta z) - i(z) &= -C \Delta z \frac{dv(z)}{dt}
\end{align*} \]  

\[ [Q = CV] \]

Limit as \( \Delta z \to 0 \):

\[ \begin{align*}
&dv(z)/dz = -L \frac{di(z)}{dt} \\
di(z)/dz = -C \frac{dv(z)}{dt}
\end{align*} \]  

\[ \implies \text{Wave Equation} \]

\[ d^2v/dz^2 = LC \frac{dv}{dt}^2 \]

\[ d^2i/dz^2 = LC \frac{di}{dt}^2 \]

SOLUTION: TELEGRAPHER’S EQUATIONS

Wave Equation:  
\[ d^2v/dz^2 = LC \frac{dv}{dt}^2 \]

Solution:

\[ v(z,t) = f_+(t - z/c) + f_-(t + z/c) \]

\( f_+ \) and \( f_- \) are arbitrary functions

Substituting Into Wave Equation:

\[ \frac{1}{c^2}[f_+^{''}(t - z/c) + f_-^{''}(t + z/c)] = LC[f_+^{''}(t - z/c) + f_-^{''}(t + z/c)] \]

Therefore

\[ c = 1/\sqrt{LC} = 1/\sqrt{\mu_0} \]

Current \( i(z,t) \)

Recall:

\[ di(z)/dz = -C \frac{dv(z)}{dt} = -C[f_+(t - z/c) + f_-^{'}(t + z/c)] \]

Therefore:

\[ i(z,t) = cC[f_+(t - z/c) - f_-(t + z/c)] \]

where

\[ cC = C/\sqrt{LC} = \sqrt{C/L} = Y_0 = 1/Z_0 \]  

"characteristic admittance"

And therefore:

\[ i(z,t) = Y_0[f_+(t - z/c) - f_-(t + z/c)] \]