6.014 Lecture 20:
Micro-ElectroMechanical Systems (MEMS)

A. Overview

One of the current major revolutions in electrical engineering involves the extension of integrated circuit technology to fabrication of micro-electromechanical systems (MEMS) as independent components and also as elements integrated on the same substrate as the integrated circuits with which they interoperate. Significant advances in cost and/or functionality have already been achieved with MEMS incorporating optical switches, radio-frequency switches, microphones, accelerometers, thermometers, pressure sensors, chemical sensors, micro-fluidic systems, electrostatic and magnetic motors, biological sensors, and others. MEMS devices have been used in everything from video projectors to automobile air bag triggers and mechanical digital memories for hot environments. Figure L20-1 illustrates two common configurations—a cantilever actuator or sensor, and a rotary electrostatic motor.

The size of MEMS devices ranges from the micron or sub-micron scale up to one or more millimeters, although the basic electromagnetic principles apply to devices of any scale. Recent advances in micro-fabrication techniques, such as new lithography and etching techniques, precision micro-molds, and laser cutting and chipping tools, have simplified exploiting MEMS technology.

Efficient motors and actuator configurations also generally can work as sensors. The first example explored below is that of a parallel-plate capacitor where the charged plates are electrically attracted and can do work as they squeeze together. Conversely, the same plates can be mechanically forced apart so as to charge a battery. Two different ways to calculate these forces are explored below: use of the Lorentz force equation and use of system-energy derivatives with respect to distance.

Lorentz Forces

The Lorentz force law expresses the force vector $\overline{f}$ acting on a charge q [Coulombs]. The force is a function of the local electric field $\overline{E}$, magnetic field $\overline{H}$, and the charge velocity vector $\overline{v}$ [ms^{-1}]:

$$\overline{f} = q(\overline{E} + \overline{v} \times \mu_o \overline{H}) \text{ [Newtons]} \tag{1}$$

A cathode-ray tube (CRT) provides a simple example of this law, as illustrated in Figure L20-1, where electrons thermally excited by a heated cathode escape at low energy into a vacuum. There the electric field $\overline{E}$ between anode and cathode draw it toward the anode at acceleration $\overline{a}$ [ms^{-2}], as governed by Newton's law:

$$\overline{f} = m\,\overline{a} \tag{2}$$

where m is the mass of the accelerating electron. The kinetic energy of the electron w_k is the accumulated work done on it by the electric field $\overline{E}$. That is, the increased energy of the electron is the product of the force f acting on it and the distance D the electron moved while experiencing that force. If D is the separation between anode and cathode, then:

$$w_k = fD = (eV/D)D = eV \text{ [J]} \tag{3}$$

Thus the kinetic energy acquired by the electron in moving through the potential difference V is eV Joules. If V = 1 volt, then w_k is one "electron volt", or "e" Joules, where $e \cong 1.6 \times 10^{-19}$ Coulombs. Typical values for V in television CRT's are 25,000 volts or more. This voltage is limited in part by the desire not to generate excessive x-rays as the electrons impact the phosphors on the CRT faceplate, which is often leaded glass designed to reduce x-ray release.

Figure L20-2b illustrates how lateral electric fields can deflect electron beams in CRT's so as to scan the beam across the faceplate, "painting" the image to be displayed. At higher tube voltages the electrons move so quickly that they do not sufficiently experience the lateral forces, and magnetic deflection is used instead because the lateral magnetic forces increase with v and are then greater.

Static electric forces on capacitor plates can also be calculated using the Lorentz force equation (1), which becomes $\overline{f} = q\,\overline{E}$. To compute the total attractive force on each plate we must sum the forces acting on all the charges. If we assume the surface charge on the plates is distributed over some infinitesimal depth δ, as illustrated in Figure L20-3b, then the electric field $\overline{E}$ diminishes to zero at that depth δ. $\overline{E}$ must go to zero when ρ = 0 because there can be no $\overline{E}$ in a perfect conductor unless $\nabla \bullet \varepsilon\,\overline{E} = \rho \neq 0$. It is easy to see that if the charge density is uniform over the depth δ, then E declines linearly with depth to zero. The average electric field acting on each charge then has half the maximum field strength E, and the total force pulling on the plate is therefore QE/2, where Q is the total charge on the plate.[1]

The force f on each capacitor plate,

$$f = QE/2 \text{ [N]} \tag{4}$$

can be expressed in terms of E alone, using the boundary condition at a perfect conductor:

$$\varepsilon E = \rho_s \text{ [Coulombs m}^{-2}\text{]} \tag{5}$$

which follows from $\nabla \bullet \varepsilon\,\overline{E} = \rho$. Multiplying (5) by A we obtain εEA = Q, and from (4) we obtain $f = \varepsilon E^2 A/2$, and a force density F Newtons per square meter:

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[1] This can be shown for any charge distribution ρ(z) varying with depth z.

$$F = \varepsilon E^2/2 \quad [Nm^{-2}] \tag{6}$$

Thus the force density (pressure, Nm^{-2}) pulling on a charged conductor is the same as the energy density [Jm^{-3}]. These dimensions are actually the same, because J = Nm.

The maximum achievable E thus limits the maximum achievable pressure. In MEMS devices with micron-level plate separations, E can approach 3×10^8 [Vm^{-1}] before electric breakdown occurs, so $F = \varepsilon E^2/2 \cong 40$ N/cm^2, or ~10 lb/cm^2. For plate separations larger than a few mean-free paths for free electrons, E_{max} can be two orders of magnitude smaller. Larger gaps are more vulnerable to breakdown because any free electrons accelerated to sufficient energies before a collision can produce additional free electrons that repeat the process several times in a cascade (breakdown) before they all strike the wall.

If the two plates are each charged with the same Q, instead of oppositely, the surface charges repel each other and reside on the outer surfaces of the two plates, away from each other. Since there is now no E between the plates, it can apply no force. However, the charges Q on the outside are associated with $E = Q/\varepsilon A$, which pulls the plates apart with the same force density $F = \varepsilon E^2/2$.

Calculating Forces using Energy Derivatives

Mechanics teaches that a force f pushing an object a distance dz expends energy dw = f dz [J], so:

$$f_{on\ object} = dw/dz \tag{7}$$

The force f required to separate two capacitor plates oppositely charged with Q is therefore:

$$f = dw/dz = d(Q^2\delta/2\varepsilon A)/d\delta = Q^2/2\varepsilon A \quad [N] \tag{8}$$

where the energy stored in a capacitor C is $CV^2/2 = Q^2/2C = Q^2\delta/2\varepsilon A$ (we recall Q = CV and $C = \varepsilon A/\delta$). We can put (8) in a more familiar form for force density F by noting F = f/A and $Q = \varepsilon EA$:

$$F = Q^2/2\varepsilon A^2 = \varepsilon E^2/2 \quad [Nm^{-2}] \tag{9}$$

which is the same expression as (6). Note that (8) could be easily evaluated because Q remains constant as the plates separate. If (8) had been expressed instead as $d(CV^2/2)/d\delta$, the derivative would have been more difficult to evaluate because both C and V depend on δ. Therefore we always seek to express energy in terms of parameters that remain constant as dz changes.

This static attractive force remains the same if the plates are connected to a battery of voltage V. A more awkward way to calculate the same force (9) is to assume (unnecessarily) that a battery is connected and that V remains constant. In this case Q must vary with dz, and dQ flows into the battery, increasing its energy by VdQ. Since dw in the force expression (7) is the total system energy, the changes in both battery and electric field energy must be calculated to yield the correct energy when we use (8). As noted above, this complexity can be avoided by carefully restating the problem without the source.

The power of the energy method (8) is much more evident if we wish to calculate the force $\overline{f}$ needed to pull two capacitor plates apart laterally, as illustrated in Figure L20-6a. The Lorentz force law requires knowledge of $\overline{E}$, the lateral components of which are responsible for the force of interest and are not readily determined. Since energy derivatives can be computed accurately and easily, it is the preferred method in this case.

In this case we shall (needlessly, except for illustration) approach the energy problem the more difficult way by including the change in battery energy. The capacitance $C = \varepsilon A/\delta = \varepsilon LW/\delta$, where the plate overlap is L meters and the width is W. The force f pulling the plates apart is:

$$f = dw_T/dz = -dw_T/dL = -(d/dL)(\varepsilon WLV^2/2\delta - VQ) \qquad (10)$$

where w_T is the total energy and the two terms reflect the energy changes in the capacitor and battery respectively. In (10) only L and Q vary with L, where $Q = CV = \varepsilon WLV/\delta$. Thus (10) becomes:

$$f = -(d/dL)(\varepsilon WLV^2/2\delta - \varepsilon WLV^2/\delta) = -(d/dL)(-\varepsilon WLV^2/2\delta) = \varepsilon WV^2/2\delta \text{ [N]} \qquad (11)$$

Had we noted that the static force f would be the same whether or not the battery were connected, we could have more easily set Q = constant and taken the derivative only with respect to the energy in the electric fields, yielding the same answer (11).

A simple example using (11) is the following. Let W = 10 cm, δ = 10 microns in air, and V = 10 volts. Then $f = \varepsilon_o WV^2/2\delta = 8.854\times10^{-12} \times 0.1 \times 10^2/2\times10^{-5} \cong 4.4\times10^{-6}$ Newtons, or ~one micro-pound of force. Even if we decrease δ to one micron and increase V to 100 volts, the force still is only ~one milli-pound. This force does not depend on L but can be boosted by increasing W, for example, by increasing the number of teeth in the electrodes, as illustrated in Figure L20-6b.

Rotary Electrostatic Motors

One way to boost power levels of a motor with limited force f or torque is to increase the velocity at which it moves since power P = fv, where v is velocity. Consider the ideal 4-segment rotary electrostatic motor illustrated in Figure L20-7. It has radius R, plate separation δ, total plate overlap $A = R^2\theta$ [m^2], and operating voltage V. Stator plates occupy two quadrants of the motor and a rotating pair of quadrant plates (the rotor) can rotate to yield an overlap with the stator varying from zero to perfect as θ increases from zero to $\pi/2$. If the voltage V is applied across the plates, a torque T is produced[2], where:

$$T = -dw_T/d\theta \quad [\mathrm{Nm}] \tag{12}$$

and dw_T is the increment by which the total system energy (fields plus battery) is increased as a result of the motion θ. This difference in sign between (10) and (12) is due to the fact that the force in (10) is applied to the system, and that in (12) is applied by the system to its environment.

The total energy increase is:

$$\begin{aligned} dw_T &= d(CV^2/2) - VdQ \quad [\mathrm{J}] = d(A\varepsilon_o V^2/2\delta) - V^2 dC \\ &= R^2 d\theta\varepsilon_o V^2/2\delta - V^2R^2 d\theta\varepsilon_o/\delta = -V^2R^2 d\theta\varepsilon_o/2\delta \end{aligned} \tag{13}$$

Therefore the torque T from (12) is:

$$T = -dw_T/d\theta = V^2R^2\varepsilon_o/2\delta \quad [\mathrm{Nm}] \tag{14}$$

If we assume $R = 10^{-3}$, $\delta = 10^{-6}$, $V = 300$ volts; then $T = 300^2\times10^{-6}\times8.8\times10^{-12}/2\times10^{-6} \cong 4\times10^{-7}$. A single such ideal motor can then deliver $T\omega$ watts, where we might assume the tip velocity v of the rotor is 300 ms^{-1}, slightly less than the speed of sound. The corresponding angular velocity ω is $v/R = 300/10^{-3} = 3\times10^5$ radians s^{-1}, and the available power is $\sim4\times10^{-7} \times 3\times10^5 = 0.12$ watts if we neglect all losses[3]. In principle one might pack ~2500 motors into one cubic centimeter if each motor were 10 microns thick, yielding ~300 W/cm^3. This can be compared to a 300-hp automobile engine that delivers 300×746 watts[4], and would therefore occupy 746 cm^3, or roughly the volume of a softball. Such densities are impractical here because of the difficulties in removing heat and torque from such a package of over a million tiny motors. They have great potential, however, for extremely low power applications where torque extraction can be efficient. For example, the torque could drive a gas turbine at high speeds. The field of MEMS motors is still young, so its full potential remains unknown.

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[2] Torque equals the force on a lever times its length. Therefore $T\theta$ is work performed by the torque, where θ is the angle (radians) through which the lever rotates about its pivot at one end. Power is $Td\theta/dt = T\omega$ [W].

[3] The slide L20-7 over estimates power by a factor of ~1000 by stating $A = R\theta$, not $R^2\theta$.

[4] There are 746 watts per horsepower.

MEMS Sensors

A common type of MEMS sensor electrostatically measures small displacements of lever arms due to temperature, pressure, acceleration, chemistry, or other changes. Figure 20-8a portrays a standard configuration that illustrates the basic principles, where the capacitor plates of area A are separated by the distance d, and the voltage V is determined in part by the voltage divider formed by the source resistance R_s and the sensor output resistance R. V_s is the source voltage.

The circuit response to an increase δ in the plate separation d is to increase the capacitor voltage V above its normal equilibrium value $\overline{V}$ determined by the voltage divider, where $\overline{V} = V_s(R/[R+R_s])$. The capacitor then discharges exponentially toward $\overline{V}$ with a time constant ~R'C. If $R_s >> R$ then $R' \cong R$; otherwise $R' = R//R_s$. If $R_s >> R$ and R represents a sensor similar to the best of those used for communications systems, then that sensor can detect as little as $\sim 4\times10^{-20}$ joules per "bit" of information. This can be compared to the incremental increase in capacitor energy dw due to the displacement $\delta << d$ as C decreases to C'.

$$dw = (C - C')V^2/2 = V^2\varepsilon_o A(d^{-1} - [d+\delta]^{-1})/2 \cong V^2\varepsilon_o A\delta/2d^2 \text{ [J]} \quad (15)$$

A simple example illustrates the extreme potential sensitivity of such a sensor. Assume the plate separation d is one micron, A is 1-mm square (10^{-6}), and V = 300. Then the minimum detectable δ given by (15), assuming $dw = 4\times10^{-20}$, is:

$$\delta_{min} = dw \times 2d^2/V^2\varepsilon_o A \cong 4\times10^{-20}\, 2(10^{-6})^2/(300^2 \times 8.8\times10^{-12} \times 10^{-6}) \cong 10^{-19} \text{ meters} \quad (16)$$

At this level of sensitivity we are limited instead by thermal and mechanical noise due to the Brownian motion of air molecules and conduction electrons. A more practical set of parameters might involve a less sensitive detector ($dw \cong 4\times10^{-16}$), and lower voltages ($V \cong 10$); then $\delta_{min} \cong 1$ angstrom (very roughly an atomic diameter). The dynamic range of such a sensor would be enormously greater, of course.