MICRO-ELECTROMECHANICAL SYSTEMS (MEMS)

Micro-, Meso-, and Mega-Electromechanical Systems:

Micro: Micro-fabrication in IC fabs; micro-molds possible (<1 micron)
Possible integration with circuits on same chip
Mass production and low cost even for complex options

Meso: Meso-molds, laser cutting, meso-machine fabrication, miniature parts

Examples of MEMS:
- Microphones
- Accelerometers
- Video projectors
- Microfluidics
- Motors (electrostatic)
- Chemical, thermal, pressure sensors
- Mechanical “computers” (hot environments)
- Mechanical memories (hot environments, static)

LORENTZ FORCES

Lorentz Force Law:
\[ \mathbf{f} = q (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \text{ Newtons} \]

q = electric charge (Coulombs)
\( \mathbf{v} \) = velocity vector (m s\(^{-1}\))

Example: electric forces on an electron beam:

Longitudinal forces:
- \( \mathbf{f} = ma \), \( \mathbf{v} = at \), \( z = vt = at^2/2 \)
- Kinetic energy = \( m v^2/2 = eV \) Joules
- So an electron moving through 1 volt gains 1 electron volt
  = “e” = 1.6 \times 10^{-19} Joules
- Anode, phosphors

Lateral forces:
- Same results as longitudinal forces, but laterally; e.g. cathode ray tubes
**Electric Forces on Capacitor Plates**

Electric forces calculated from Lorentz equation:

- Capacitor equations:
  - \( Q = CV \)
  - \( C = \varepsilon A/d \)
  - \( E = V/d \ [V \cdot m^{-1}] \)
  - \( \omega_e = CV/2 \ [J] \)

- Forces attraction capacitor plates:
  - Average electric field acting on charges: \( E/2 \)
  - \( E = \sigma_x / \varepsilon_0 = Q/A \varepsilon_0 \)

- Force: \( f = QE/2 = QV/2d = CV^2/2d = \varepsilon A (V/d)^2/2 = f = A \varepsilon E^2/2 \) Newtons

- Force density: \( \varepsilon E^2/2 \ [N \cdot m^{-2}] \) [J] Maximize by maximizing \( E \), limited by breakdown

For \( d \approx \) atomic mean-free-path between collisions \( < \sim 1 \) micron; \( V/d \rightarrow E_{\text{max}} \approx 10^6 \)

- Maximum MEMS electric force density: \( [N \cdot m^{-2}] \approx \varepsilon_0 E_{\text{max}}^2/2 \approx 4.4 \times 10^6 \) [J m\(^{-3}\)]

Alternatively, \( f = QE/2 = Q^2/2A \varepsilon_0 \) Newtons \( \Rightarrow \) Force density \( = Q^2/2A \varepsilon_0 \) [N m\(^{-2}\)]

**Repulsive Forces between Parallel Metal Plates:**

- Force density \( = Q^2/2A \varepsilon_0 \)

**Computing Forces from Energy Derivatives**

**Force, work, and energy:**

- Work is: \( f \rightarrow dz \) Box increases in kinetic energy

\[ \text{Force on box [Newton]} \times \text{Distance [m]} = \text{Increase in box energy [J]} \]

\[ dw = f_{\text{on box}} dz \] (\( dw \) is negative if the velocity and force are opposite)

Therefore:

\[ f_{\text{on box}} = \frac{dw}{dz} \]

**Work required to separate charged capacitor plates:**

- Example: Two plates with \( \pm Q \), open-circuit. Therefore
  - \( \omega_e = CV/2 = Q^2/2C \ [J] \)
  - \( \omega_e = \varepsilon A/d, \) so
  - \( \omega_e = Q^2d/2\varepsilon_0 A \) \( \rightarrow \infty \) as \( d \rightarrow \infty \), therefore
  - \( f = \delta \omega_e / \delta z = Q^2/2\varepsilon_0 A \) [Newtons]

(same answer as on L20-3; \( z = d \) here)

Note that the operand \([Q^2d/2\varepsilon_0 A]\) for \( \delta \omega_e / \delta z \) has no elements (like \( V,E \)) that vary with \( d \).

If we connect \( C \) to a circuit, we must compute \( \delta \omega_e / \delta z \) for the total system energy, not just for \( C \).
ELECTROSTATIC ENERGY GENERATORS

Mechanical work separating charged plates increases electric energy:

Reversibility: Mechanical energy $\leftrightarrow$ Electrical energy

Battery charger:

Since $Q = CV = \varepsilon AV/d$,
Separating plates (increasing d) reduces Q on plates and puts i into battery

Generated energy stored in rechargeable battery = $V \Delta Q_{\text{into battery}} = \varepsilon AV^2 (d_1^{-1} - d_2^{-1})$

Mechanical work on capacitor plates using battery = $V \Delta Q_{\text{from battery}} = \varepsilon AV^2 (d_1^{-1} - d_2^{-1})$

Lateral mechanical forces:

Electrical energy: $w_v = Q^2d/2\varepsilon_oA = Q^2d/2\varepsilon_oWL$ [J]

Lateral force: $f = \partial w_v/\partial z = -\varepsilon_oAV^2/2d + V \partial Q/\partial d$ where

$Q = CV = \varepsilon_oWL/d$, so $\partial Q/\partial d = \varepsilon_oWL/d$, therefore

$f = -\varepsilon_oAV^2/2d + \varepsilon_oAV^2/d = \varepsilon_oAV^2/2d$ [Newtons]

(Note: f is independent of L, as is $\partial w_v/\partial L$) $f$ is force exerted on plates by environment

Example:

Let: $W = 10$ cm, $d = 10$ $\mu$m, $V = 10$ volts (10 kV/cm)

Then: $f = \varepsilon_oAV^2/2d = 8.854 \times 10^{-12} \times 0.1 \times 10^9/2 \times 10^{-6} = 4.4 \times 10^{-6}$ Newtons (~1 millipound)

Try: $d = 1$ $\mu$m, $V = 100$ volts (1MV/cm) $\Rightarrow 4.4 \times 10^{-3}$ Newtons (~1 millipound)

Multi-segment actuator:

$N$ teeth in length $L$:

Force is proportional to $W$, so $N$ edges boost force $\times N$
(N is limited by $d << L/N$)
ELECTROSTATIC ROTARY MOTOR

Example, ideal 4-segment motor:

Radius R, plate separation d, ε₀, E_{max} = 10^{10} [V m^{-1}]
Plate overlap = A = Rθ [m^2]

\[ \Delta \text{stored electrical energy (from L20-6)} \]
\[ dw = dCV^2/2 - VdQ [J] = dA \varepsilon_0 V^2/2d - V^2dC = 2Rd\theta \varepsilon_0 V^2/2d - V^22Rd\theta \varepsilon_0 V^2/d [J] \]

Lateral torque T [Newton meters] on rotor:

\[ T = -dw/d\theta = R \varepsilon_0 V^2/d [N m] \]

Cadillac Engine

~year 2040

300 hp x 746W/160W = 1400 motors
\[ \Rightarrow \sim 0.22 \text{ cm}^3! \ P = VI, I = 746A, \text{ Heat} \]

Motor power P [Watts] at \( \omega \) radians/sec:

\[ P = T\omega [W] \]

Example for R = 1 mm, d = 1 micron, V = 300 volts, \( v_{max} = \sim 200 \text{ ms}^{-1} \):

\[ \omega = V/R = 200/10^{-6} = 2 \times 10^8 \text{ radians/sec} (\sim 530,000 \text{ rpm}) \]

\[ P = \omega T = \omega R \varepsilon_0 V^2/2 \approx 2 \times 10^8 \times 10^{-6} \times 8.854 \times 10^{-12} \times 2 \times 10^{-6} \approx 160 \text{ Watts}! \]

(could lift 70-kg person [f = ma = 70 \times 9.8 = 686 N] at \sim 23 \text{ cm/sec} [P = Fv])

Back-off mode: 10 volts = 180 mW. At 4 mm^2, 10-micron thick = 6250 motors/cm^3

MEMS SENSORS

Measuring displacement:

Microphone, accelerometer, barometer, strain gauge, etc.

Cantilevered capacitor C, stressed at V volts

Circuit response to displacement \( \delta \):

Assume \( R_s >> R \); impulse increase in separation d is \( \delta \ll d \)

Mechanical energy \( \delta \) first increases \( w_s \) in C as \( V \rightarrow V+dV \)

Then voltage relaxes back to \( V = R/(R + R_s) \) as current flows mostly through R releasing \( dw[J] \)

Recall \( C = \varepsilon_0 A/d \)

\[ dw \equiv (C - C')V^2/2 = \varepsilon_0 A(d^{-1} - [d+\delta]^{-1})/2 = (V^2\varepsilon_0 A/2d)(1 - [1+d/\delta]^{-1}) \equiv V^2\varepsilon_0 A \delta^2/2d [J] \]

Example:

Let \( dw = 4 \times 10^{-20} \text{ Joules} \) (energy needed to convey one bit of information to sensor)

Plate separation \( d = 1 \text{ micron}, A = 1\text{-mm square (}10^{-6}) \)

\( V = 300; \text{ solve for minimum } \delta \)

Then:

\( \delta = dw 2d/V^2\varepsilon_0 A = 4 \times 10^{-23} \times 2 \times 10^{-12}/300^2 \times 8.8 \times 10^{-12} \times 10^{-6} = 10^{-15} \text{ meters} = 10^{-9} \text{ A} \)

or:

\( d = 4 \times 10^{-19} \times 2 \times 10^{-12}/(300,000 \times 8.8 \times 10^{-12} \times 10^{-6}) \approx 10^{-19} \text{ meters} = 1 \text{ A} \)