Problem 6.1

A parallel-plate structure has plate separation $d$ and area $A$. The medium between the plates is dry earth with permittivity $\varepsilon = 3\varepsilon_0 = 4.8 \times 10^{-19}$ and conductivity $\sigma = 10^{-14}$ [S m$^{-1}$].

a) What is the relaxation time $\tau$ of this capacitor (seconds)?

b) If $d = 1$ meter and $A = 10$ m$^2$, what is the resistance $R$ of the structure?

c) For the same $d$ and $A$, what is the capacitance $C$ of this structure?

d) If we put $N$ of these capacitors in parallel, what then is $\tau$?

e) If we put $M$ of these capacitors in series, what then is $\tau$?

Problem 6.2

Assume we wish to manufacture an inductor with the highest inductance $L$ we can using iron with $\mu = 10^5 \mu_0$, and with a fixed value for solenoid cross-sectional area $A$ [m$^2$] and number of turns $N$. Compare the two choices of a solenoid completely filled by permeability $\mu$, and a toroidal solenoid with the largest gap $d$ that satisfies the approximation that the magnetic energy stored in the gap comprises about half the total stored magnetic energy, i.e.: $(d\mu/\mu_0)^2 \approx (2\pi R)^2$ where we fix $R \equiv 2A^{0.5}$ [m]. (See slide L11-9). Which configuration has the higher inductance, and by approximately what ratio?
Problem 6.3

Using graphical field sketching techniques, sketch the approximate isopotentials and electric field lines for the two perfect conductors shown here in cross-section:
Let’s define $E_{\text{max}}$ and $E_{\text{min}}$ as the maximum and minimum electric field strengths at the inner conductor. What is the approximate ratio of $E_{\text{max}}/E_{\text{min}}$ (based on your sketch)? Assume this cross-section is constant down the length of this co-axial system.

Problem 6.4

A perfect conductor is shaped as shown, and linearly increasing electric potentials are applied along the remaining two sides of the rectangle. At these two edges they increase linearly from zero at the conductor to $V$ volts at the open corner, as illustrated:

a) Provide an expression for the electric potential in the rectangle.

b) Sketch the approximate electric fields within the rectangle.