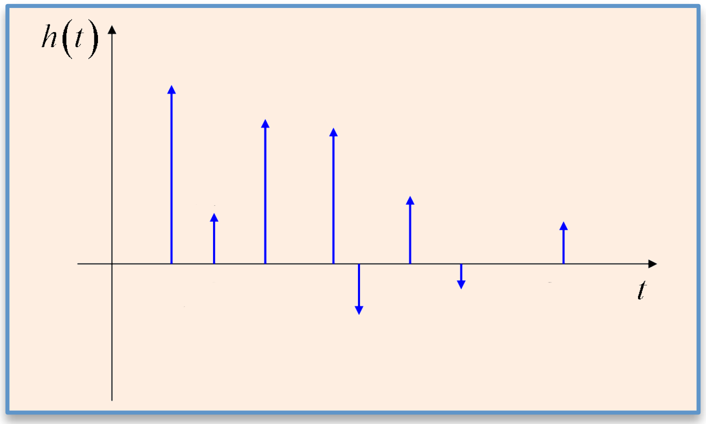
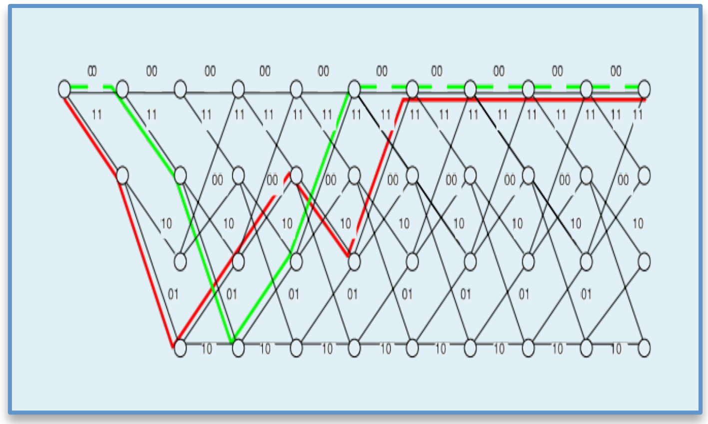
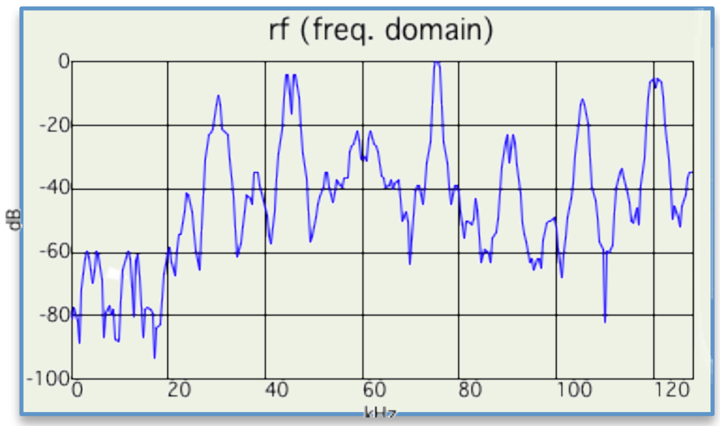
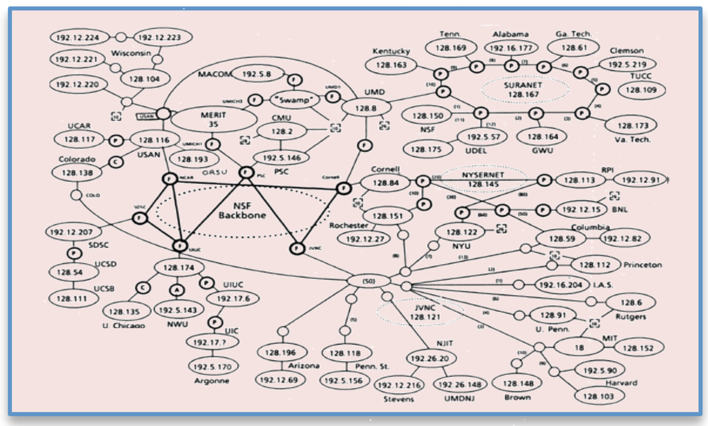








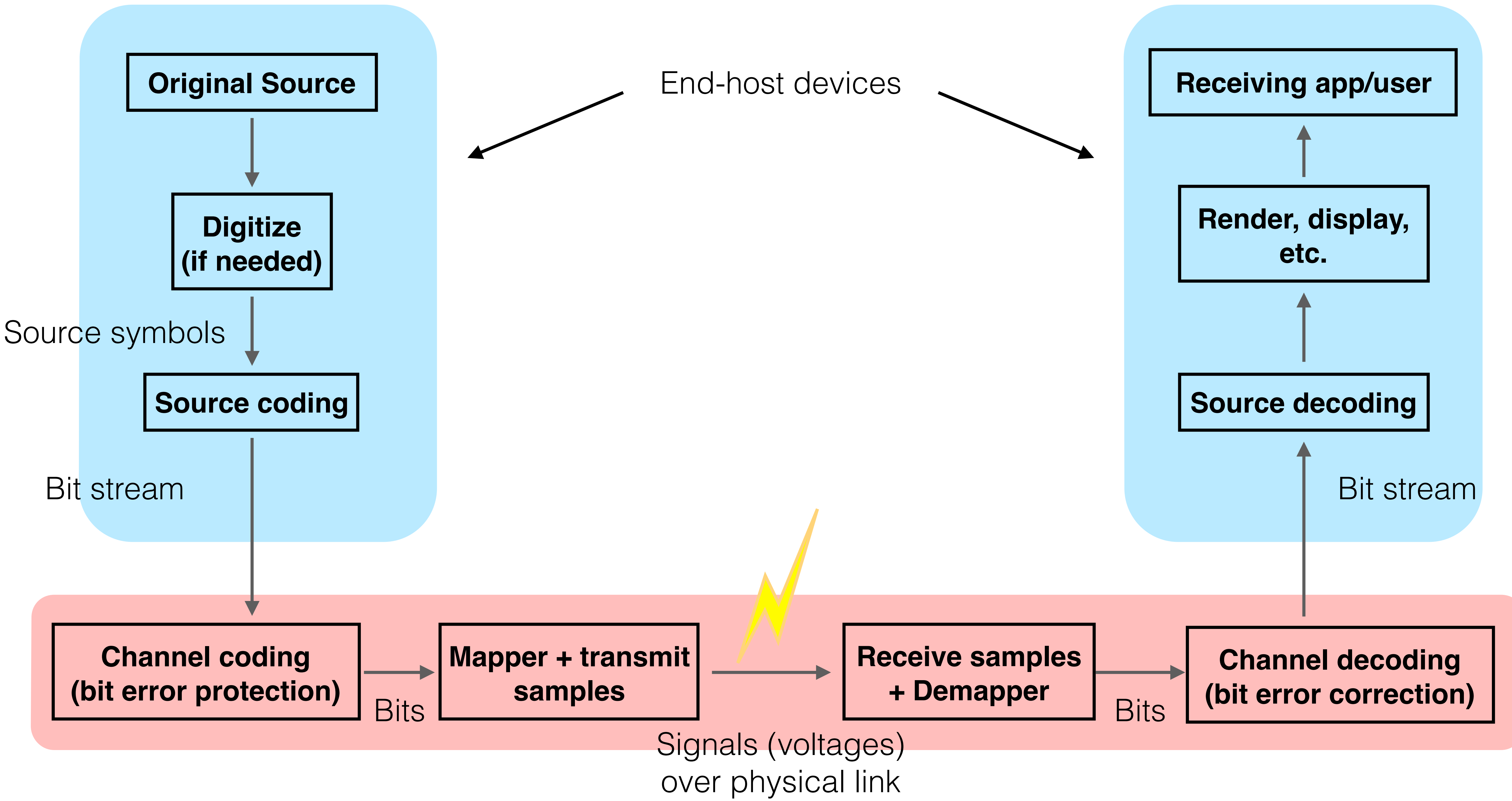
INTRODUCTION TO EECS II

DIGITAL COMMUNICATION SYSTEMS

6.02 Fall 2014
Lecture #4

- Review channel coding, Hamming distance
- Linear codes
- Relationship between number of parity bits and size of the message
- Hamming codes

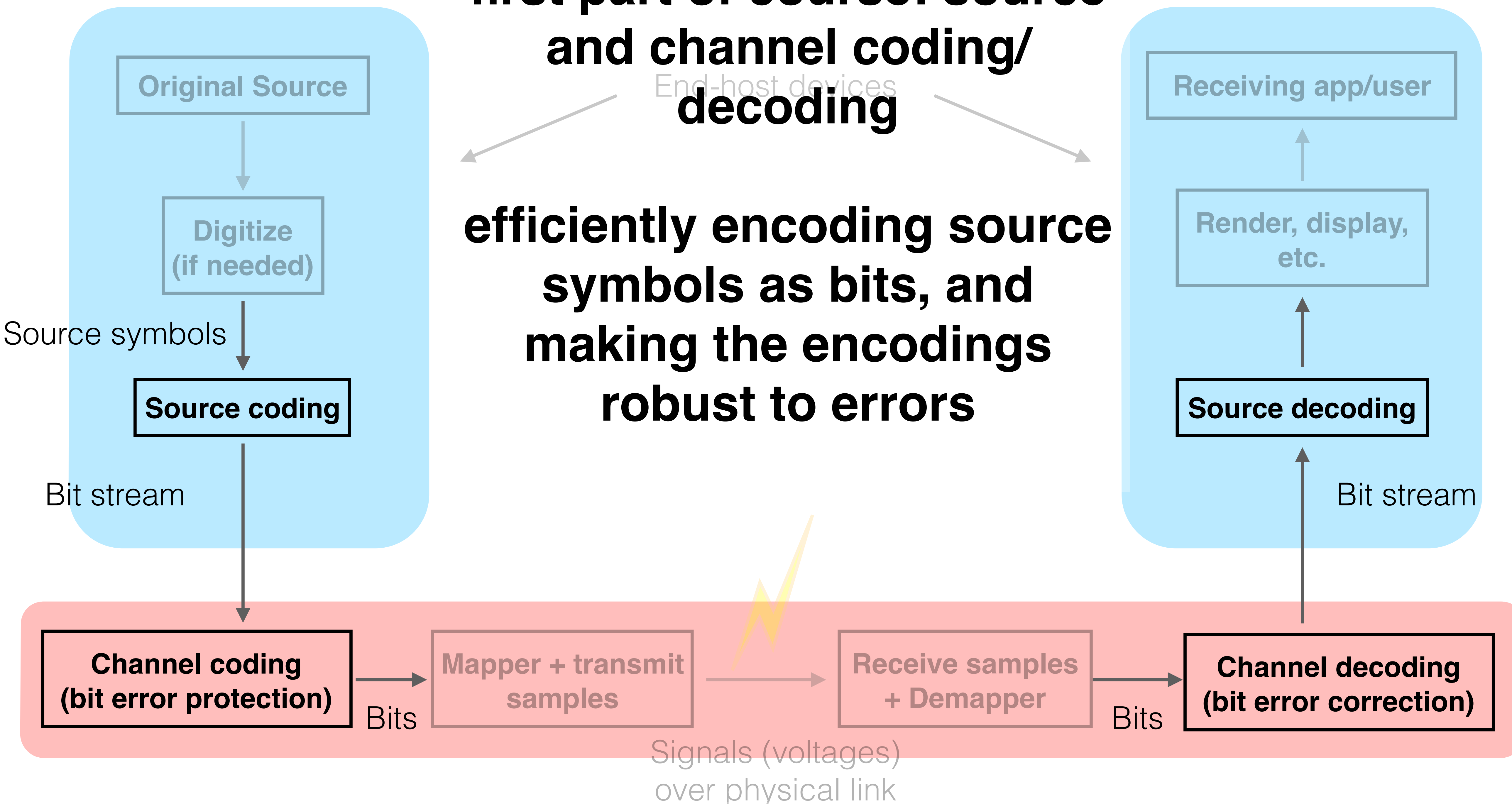
Single Link Communication Model



Single Link Communication Model

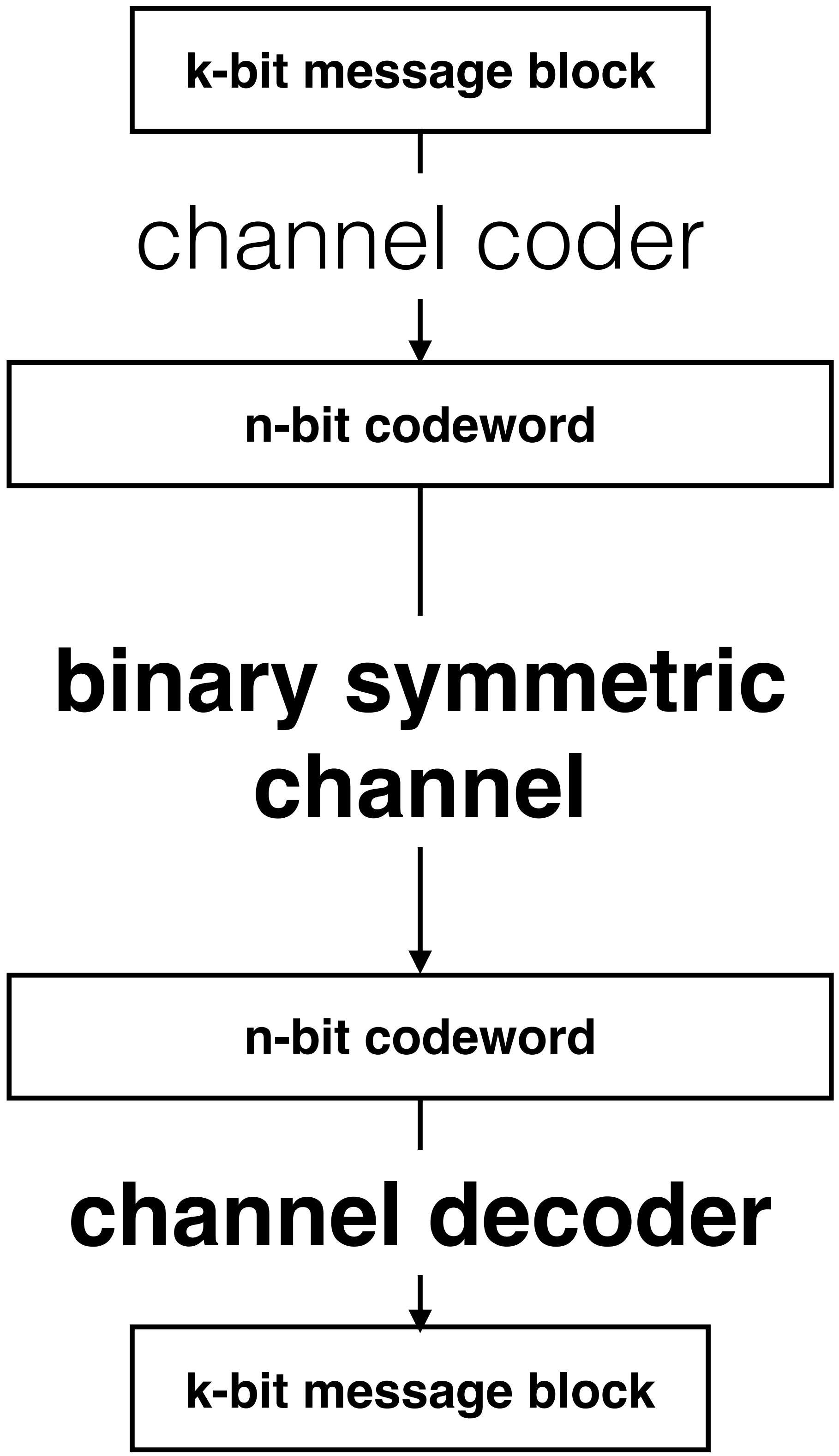
first part of course: source and channel coding/ decoding

efficiently encoding source symbols as bits, and making the encodings robust to errors



today's goal: develop codes that can correct single-bit errors

Channel Coding



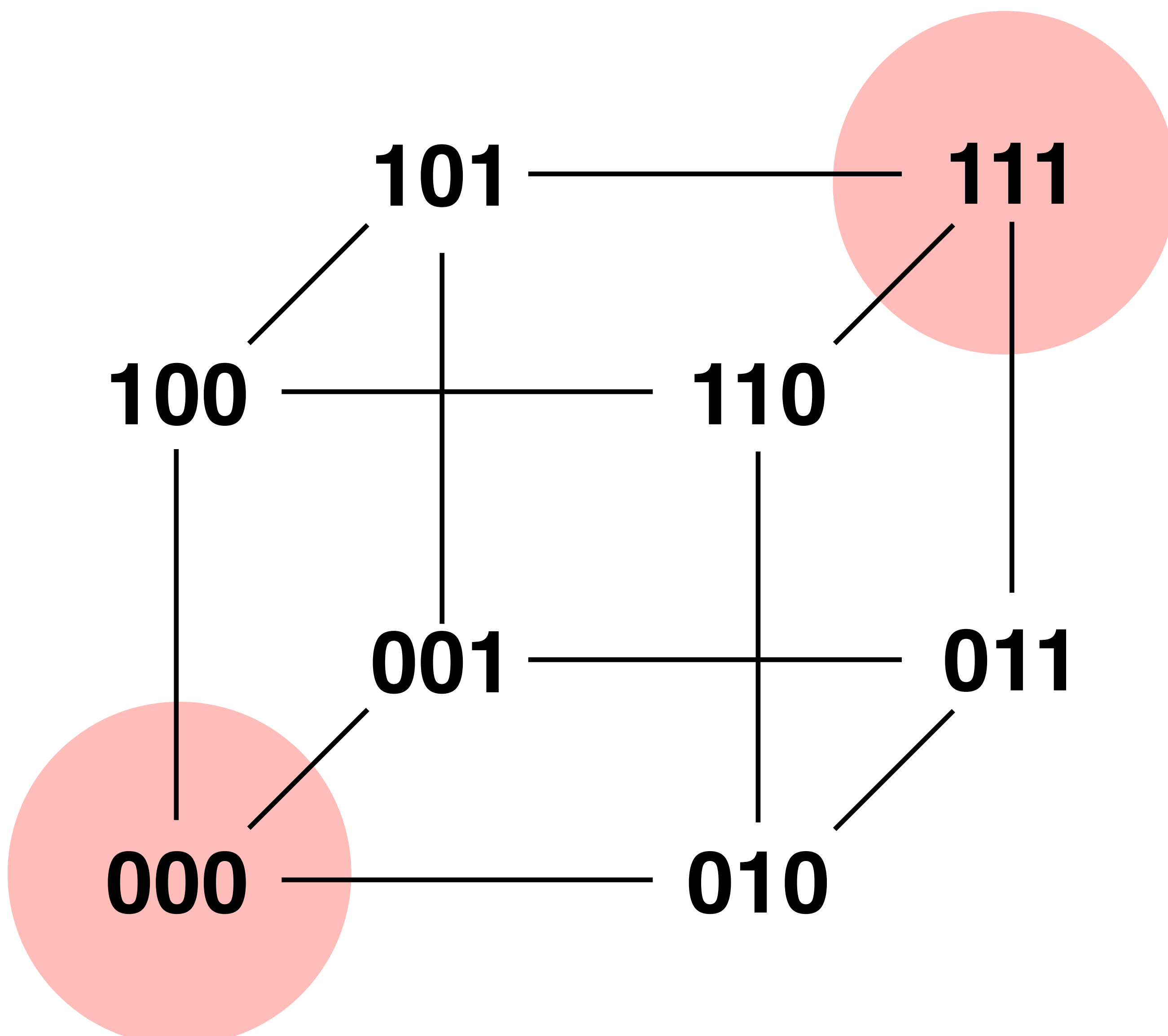
$n \geq k$, so there are 2^k codewords used out of the 2^n possibilities

bits flipped with probability $p < .5$

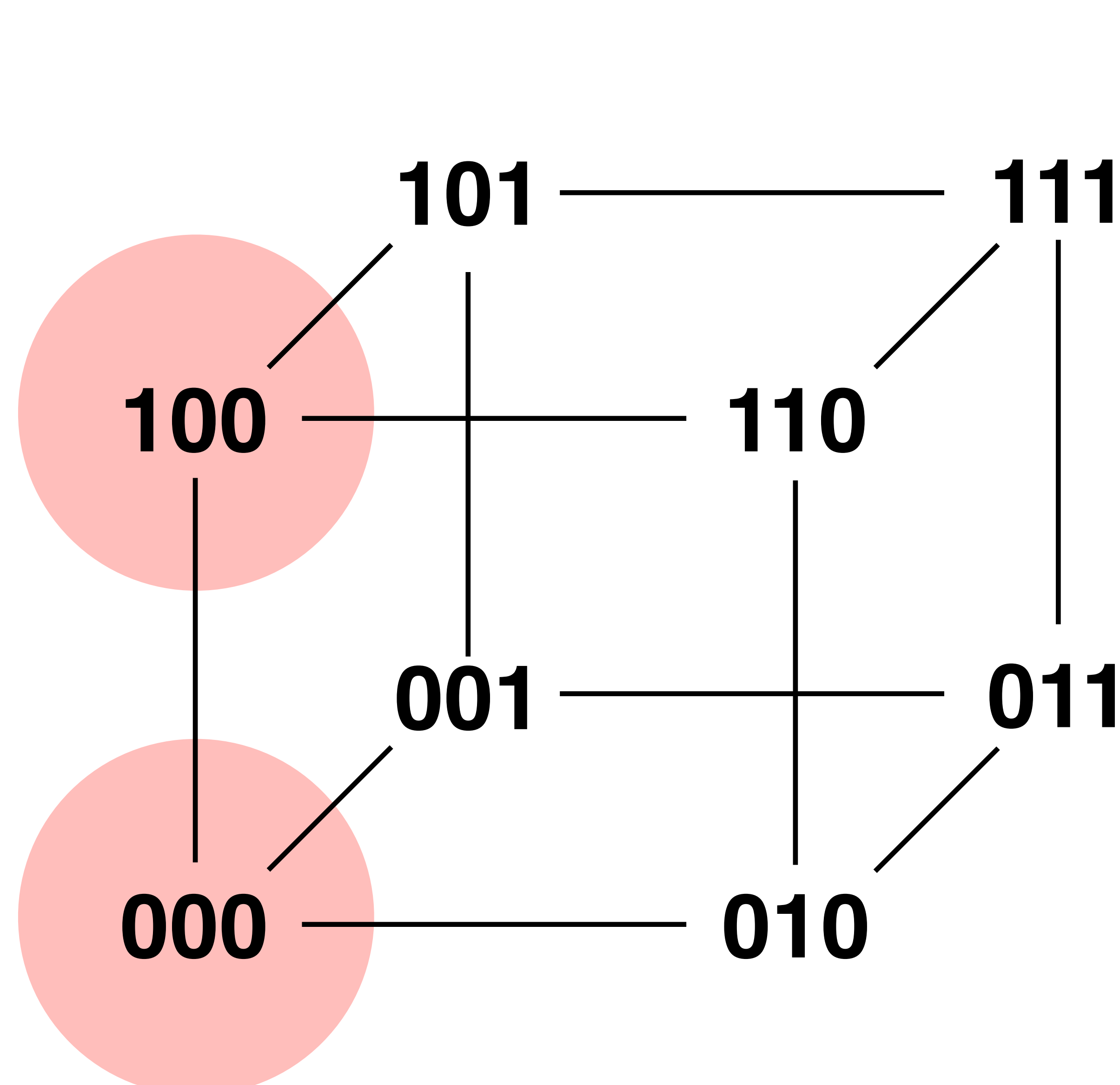
decode received codeword as its nearest neighbor, where nearest is based on Hamming distance

Choosing Codewords with Structural Separation

connected codewords have hamming distance 1



0 → 000
1 → 111



0 → 000
1 → 100

bad: not a lot of structural separation

Error Detection and Correction

1. We can **detect** all patterns of up to t bit errors iff $d \geq t+1$

2. We can **correct** all patterns of up to t bit errors iff $d \geq 2t+1$

3. We can detect all patterns of up to t_D while **simultaneously** correcting all patters of up to t_C ($t_C \leq t_D$) iff $d \geq t_D + t_C + 1$

Goal: to correct all single-bit errors

Goal: to correct all single-bit errors while using as few parity bits as possible

Parity bits for (7,4,3)

	1	2	3	4	5	6	7
	001	010	011	100	101	110	111
	P1	P2	D1	P3	D2	D3	D4
P1	X		X		X		X
P2		X	X			X	X
P3				X	X	X	X

P1 = D1 + D2 + D4

P2 = D1 + D3 + D4

P3 = D2 + D3 + D4

Parity bits for (15,11,3)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	0001	0010	0011	0100	0101	0110	0111	1000	1001	1010	1011	1100	1101	1110	1111
	P1	P2	D1	P3	D2	D3	D4	P4	D5	D6	D7	D8	D9	D10	D11
P1	X		X		X		X		X		X		X		X
P2		X	X			X	X			X	X			X	X
P3				X	X	X	X					X	X	X	X
P4								X	X	X	X	X	X	X	X

```
P1 = D1 + D2 + D4 + D5 + D7 + D9  + D11
P2 = D1 + D3 + D4 + D6 + D7 + D10 + D11
P3 = D2 + D3 + D4 + D8 + D9 + D10 + D11
P4 = D5 + D6 + D7 + D8 + D9 + D10 + D11
```

in general, we can construct $(2^m-1,\ 2^m-1-m,\ 3)$ codes

- **Review channel coding, Hamming distance**
  Minimum hamming distance d tells us what types of errors we can detect and/or correct

- **Linear codes**
  Powerful and efficient (we'll see more Wednesday).
  Parity check provides no error correction; rectangular codes do, but with high overhead

- **Relationship between number of parity bits and size of the message**
  $n \leq 2^{n-k} - 1$

- **Hamming codes**
  A type of linear code that corrects single-bit errors with the minimum number of parity bits