Massachusetts Institute of Technology
Dept. of Electrical Engineering and Computer Science
Fall Semester, 2006
6.082 Introduction to EECS 2

Modulation and Demodulation

Introduction

A communication system that sends information between two locations consists of a *transmitter*, *channel*, and *receiver* as illustrated in Figure 1. The channel refers to the physical medium carrying the *information signal* (voice, video, data etc.) from one location to another. The physical medium can be free space or a variety of waveguides (wires, optical fibers, etc.) that direct the energy across the channel to the receiver. The *transmitted signal* carrying the information through the channel can be electromagnetic, optical, acoustic, or other forms of energy radiation. Cell phones and wireless networks send information across free space using electromagnetic waves.

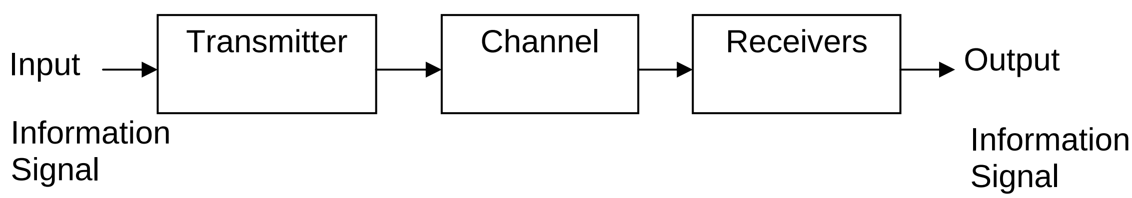

Figure 1: Communication System Block Diagram

In order to send these electromagnetic waves across free space the frequency of the transmitted signal must be quite high compared to the frequency of the information signal. For example the information signal in a cell phone is a voice signal with a bandwidth on the order of 4kHz. The typical frequency of the transmitted and received signal is on the order of 900MHz. Figure 2 illustrates how the spectrum of voice signals falls outside the frequency range of the transmission channel; the figure is not drawn to scale.

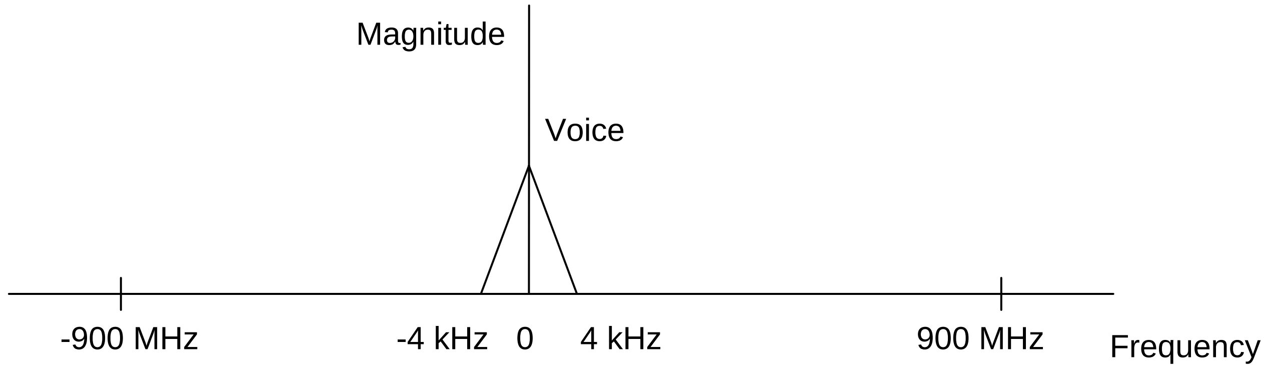

Figure 2

The main reason the transmission frequency is so high is that the wavelength of the electromagnetic wave is proportional to the reciprocal of the frequency. For example the wavelength of a 1 GHz electromagnetic wave in free space is 30cm whereas a 1kHz electromagnetic wave is one million times larger or 30km. As you will see when we study antennas, the size of the antenna and other components are related to the wavelength. For small portable devices, higher frequency transmission is a requirement.

To transmit signals with frequencies required by the communication channel, *the transmitter centers the spectrum of the information signal at the transmission frequency*. This process of shifting the frequency spectrum of a signal is called *modulation.* As an example human voice spans a 4 kHz range or *bandwidth*, and is centered at 0 kHz. In order to transmit human voice over a cell phone, the transmitter shifts the voice signal so that it has a 4 kHz bandwidth but is now centered at the transmission frequency, 900MHz as illustrated in Figure 3.

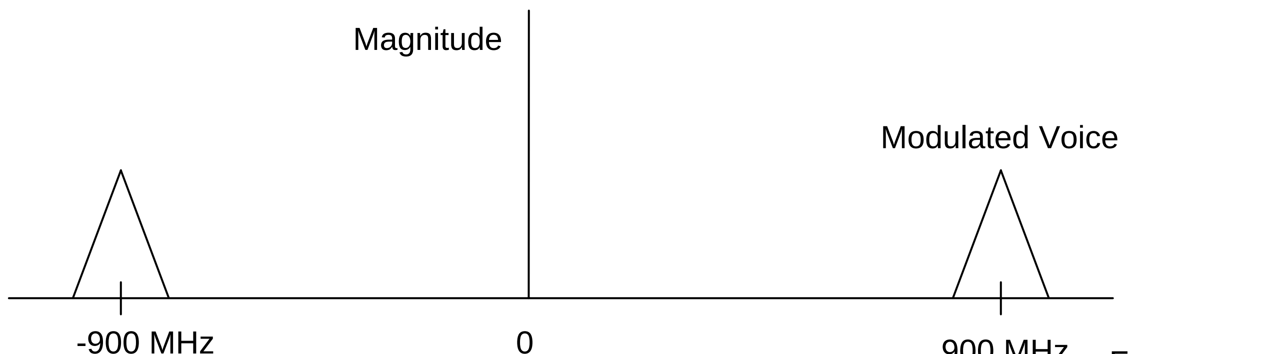

Figure 3

In the receiver the reverse process takes place. The receiver *centers the spectrum of the received signal at the original center frequency of the information signal*; we refer to this process as *demodulation*. In the case of human voice transmission, the receiver shifts the spectrum of the received signal (the received signal had a spectrum centered at 900 MHz) so that it is centered at 0 kHz as shown in Figure 4.

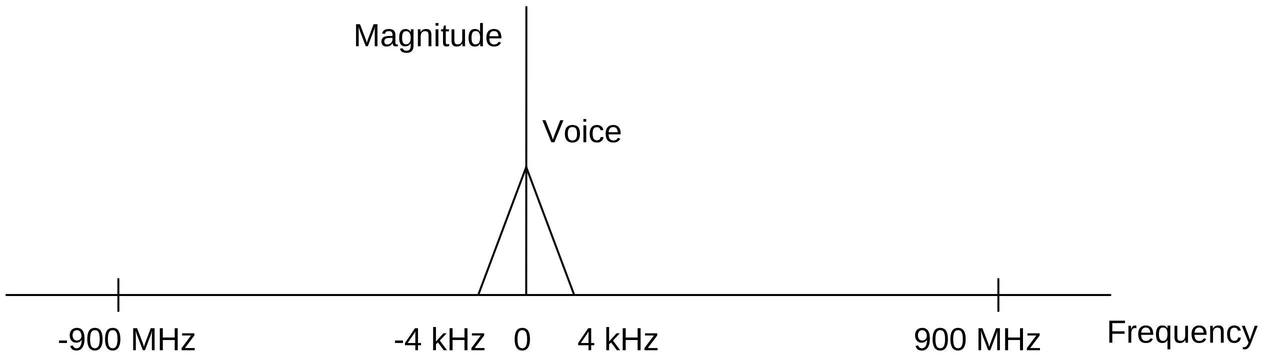

Figure 4

Modulation

We take advantage of the trigonometric identity shown below in the implementation of signal modulation. The identity shows that the product of cosines with frequencies f_1 and f_2 results in cosines with frequencies f_1+f_2 and f_1-f_2. *In other words, multiplication by the f_2 cosine shifts or modulates the f_1 cosine to the new frequencies f_1+f_2 and f_1-f_2.*

$$\cos(2\pi f_1 t) * \cos(2\pi f_2 t) = \frac{\cos\{2\pi(f_1 + f_2)t\} + \cos\{2\pi(f_1 - f_2)t\}}{2}$$

Let's study the effect of modulation in the time and frequency domain; assume f_1 = 1 Hz and f_2= 10 Hz. Figure 5 and 6 show the time-domain plots of the 1 Hz and 10 Hz cosines, and Figure 7 shows the time-domain plot of the product of these two cosines. Notice how the 1 Hz cosine appears as the *envelope* that shapes the 10 Hz cosine.

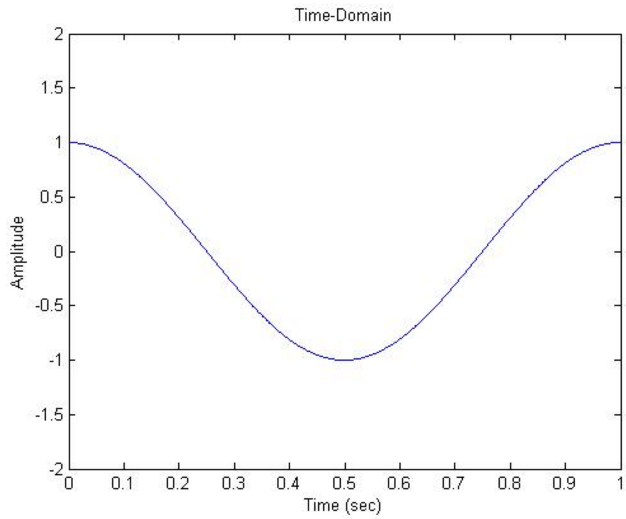

Figure 5

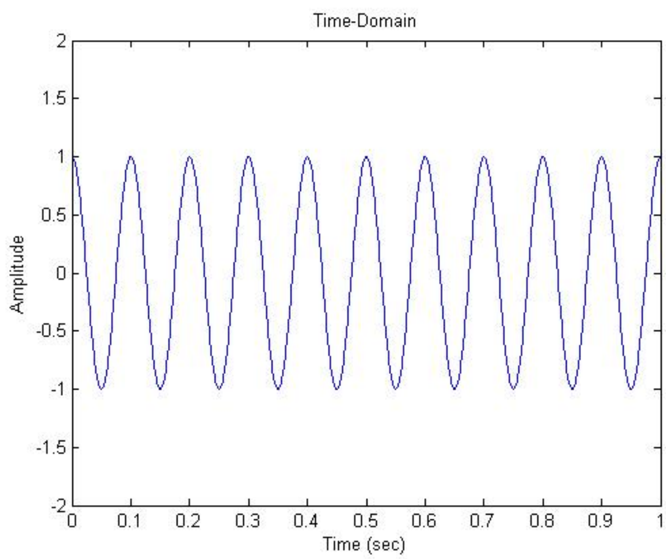

Figure 6

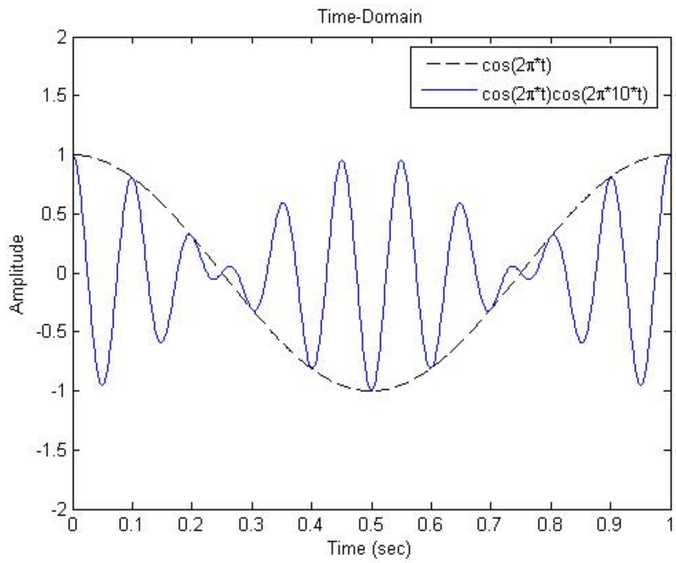

Figure 7

In the frequency domain the 1 Hz and 10 Hz cosines appear as illustrated in Figures 8 and 9 respectively; recall that spectrum of a real signal has even magnitude, which is why you see spectral peaks at +/- 1 Hz and +/- 10 Hz.

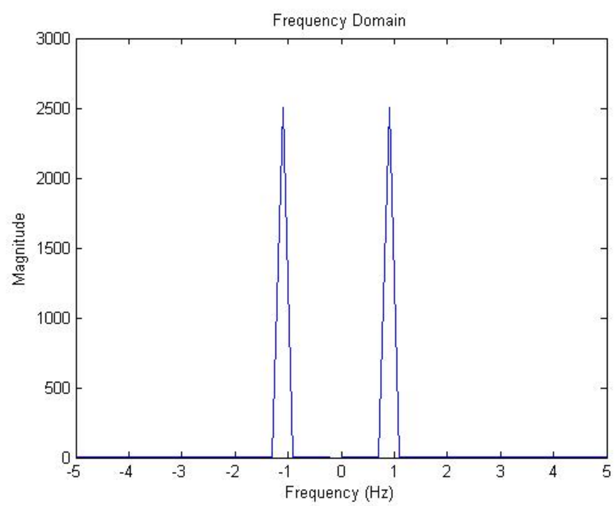

Figure 8

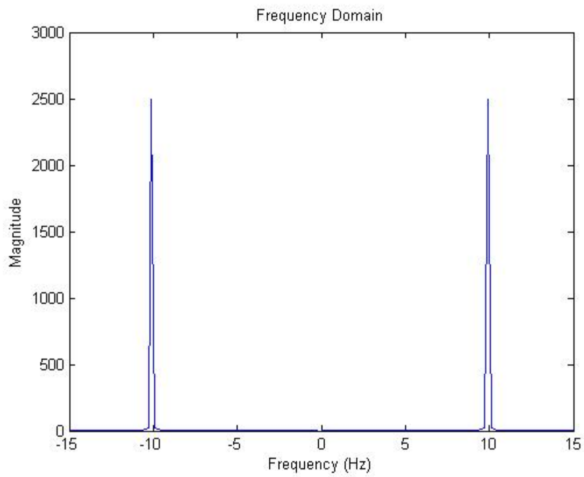

Figure 9

Figure 10 illustrates the spectrum of the product of the 1 and 10 Hz cosines; note how the spectrum has four peaks. The two spectral peaks at +/- 11 Hz correspond to the f_1+f_2 cosine while the two peaks at +/- 9Hz correspond to the f_1-f_2 cosine. At this point, we have modulated a 1 Hz cosine up to 9 and 11 Hz. Of course we could have also said that we modulated the 10 Hz signal to 9 and 11 Hz, but it is customary to think of the lower frequency signal as the information signal that we modulate up to the carrier (higher) frequency signal.

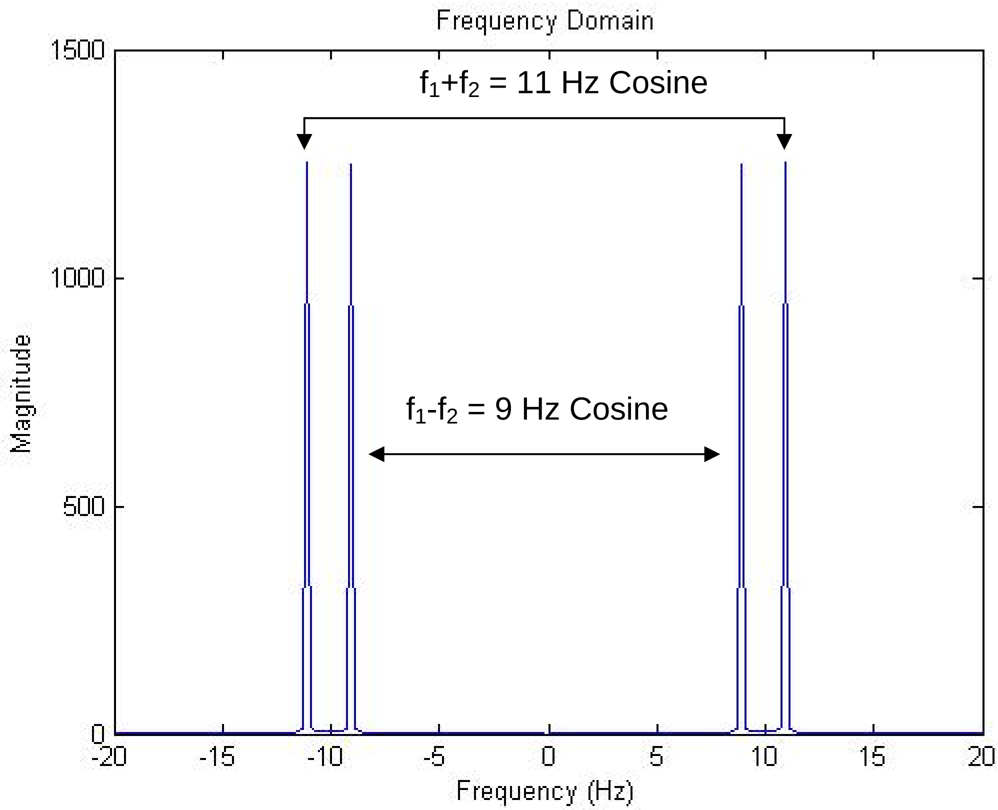

Figure 10

Demodulation

We know that demodulation involves shifting spectra, and that shifting spectra involves multiplication by cosines. Let's multiply the modulated signal from the previous section (the signal containing the f_1+f_2 and f_1-f_2 cosines) by another cosine with a frequency of f_2. The hope is that this operation will allow us to recover the f_1 cosine.

$$\frac{1}{2}\left\{\cos(2\pi(f_1+f_2)t)+\cos(2\pi(f_1-f_2)t)\right\}*\cos(2\pi f_2 t)$$

$$=\frac{1}{2}\left\{\cos(2\pi(f_1+f_2)t)*\cos(2\pi f_2 t)+\cos(2\pi(f_1-f_2)t)*\cos(2\pi f_2 t)\right\}$$

$$=\frac{1}{2}\left\{\frac{\cos(2\pi(f_1+2f_2)t)+\cos(2\pi f_1 t)}{2}+\frac{\cos(2\pi f_1 t)+\cos(2\pi(f_1-2f_2)t)}{2}\right\}$$

$$=\frac{1}{4}\cos(2\pi(f_1-2f_2)t)+\frac{1}{2}\cos(2\pi f_1 t)+\frac{1}{4}\cos(2\pi(f_1+2f_2)t)$$

From the above analysis, we see that demodulating by multipyling by a cosine with frequency f_2 results in a signal that contains the f_1 cosine (our target signal) as well as cosines at f_1+2f_2 and f_1-2f_2. We need further processing of the demodulated signal to remove the high frequency cosines (f_1+2f_2 and f_1-2f_2) and retain the f_1 cosine. Before discussing these steps, lets study the time and frequency domain plots of the demodulated signal. Figure 11 illustrates the time-domain plot of the demodulated signal.

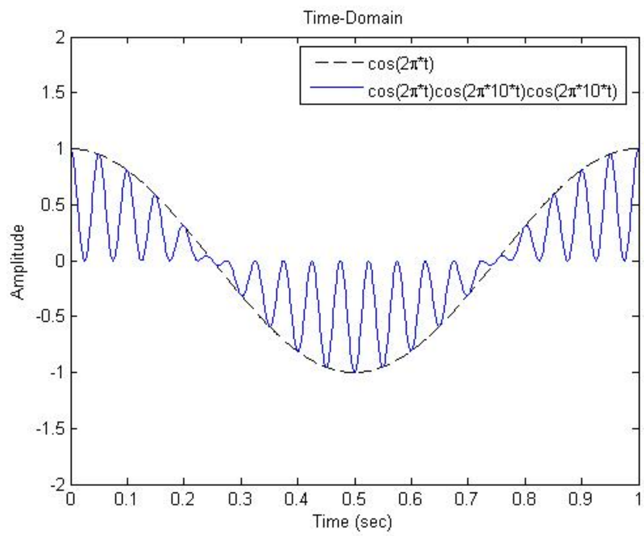

Figure 11

Figure 12 illustrates the spectrum of the demodulated signal; note how the spectrum has six peaks. The two spectral peaks at +/- 21 Hz correspond to the f_1+2f_2 cosine; the two peaks at +/- 19 Hz correspond to the f_1-2f_2 cosine; and the two spectral peaks at +/- 1 Hz correspond to the f_1 cosine.

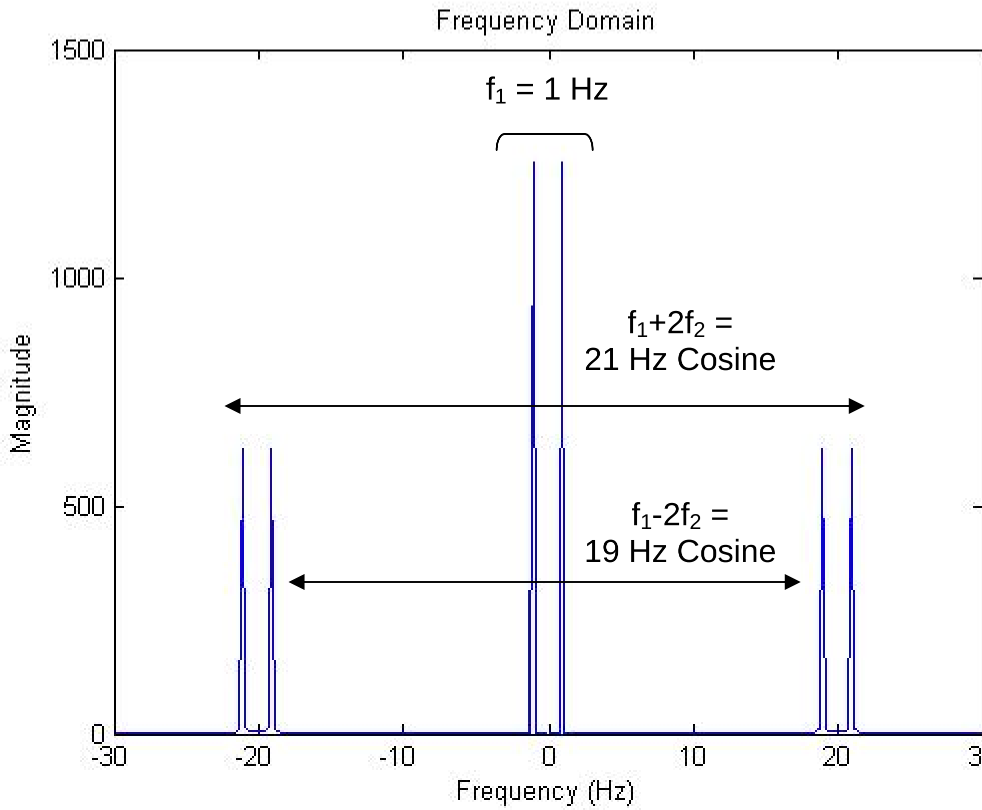

Figure 12

To remove the higher frequency cosines (f_1+2f_2 and f_1-$2f_2$) and only retain the f_1 cosine we will apply a low frequency filter with cutoff frequency f_c = 10 Hz. We will learn about filtering in the next lecture. Figure 13 and 14 illustrate the time and frequency plots of the demodulated signal after the low pass filter; note all that is left is the f_1 cosine (target signal).

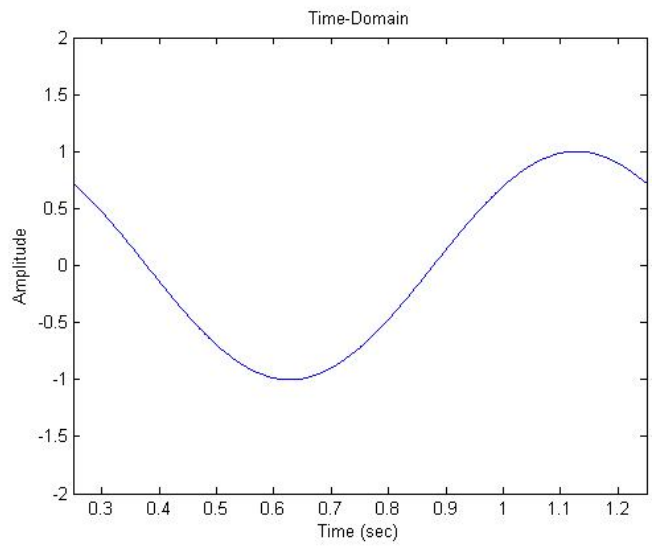

Figure 13

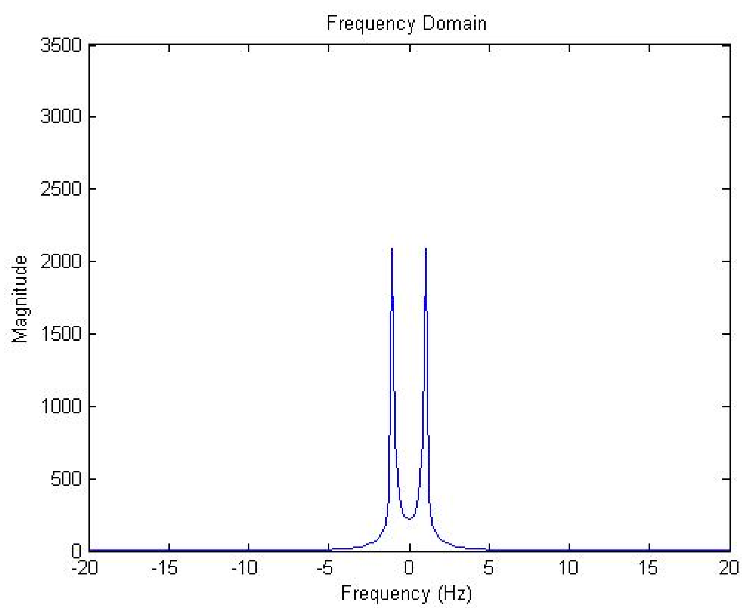

Figure 14

Modulating Signals With Nonzero Bandwidth

So far we have been dealing with the modulation of sinusoids; these signals have a single frequency component and zero bandwidth. Nonetheless signals with non-zero bandwidths, such as the voice signal whose time and frequency domain representations are shown in Figures 15 and 16, are also modulated by multiplying the signal by a cosine. The reason this works is that these signals are made up of sine and cosines (recall Fourier series and transform), and modulation simply moves each of those sine/cosine components (and therefore the entire signal) up/down the spectrum.

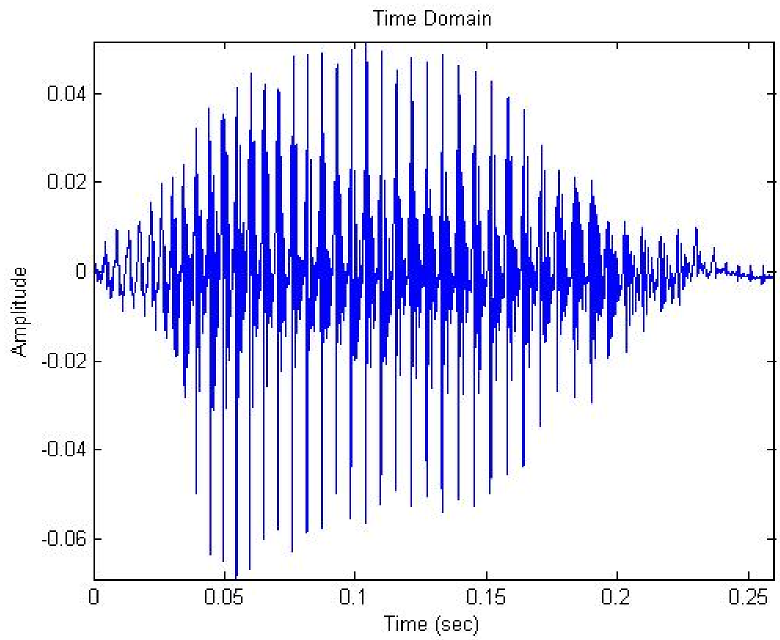

Figure 15

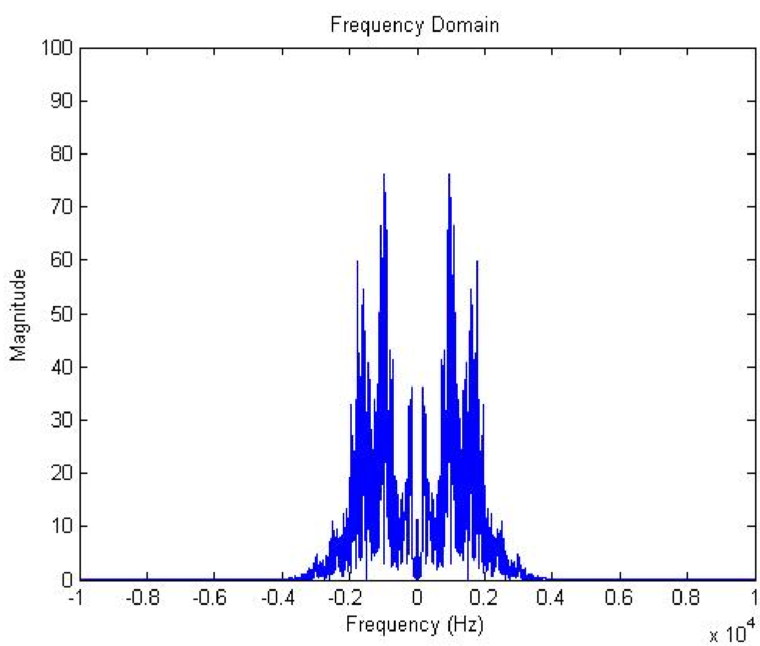

Figure 16

As an example, Figure 17 illustrates the spectrum of the voice signal in Figure 15 after it is modulated by a 10 kHz cosine. Note how the spectrum in Figure 16 is centered at +/- 10kHz; the plot is analogous to Figure 10.

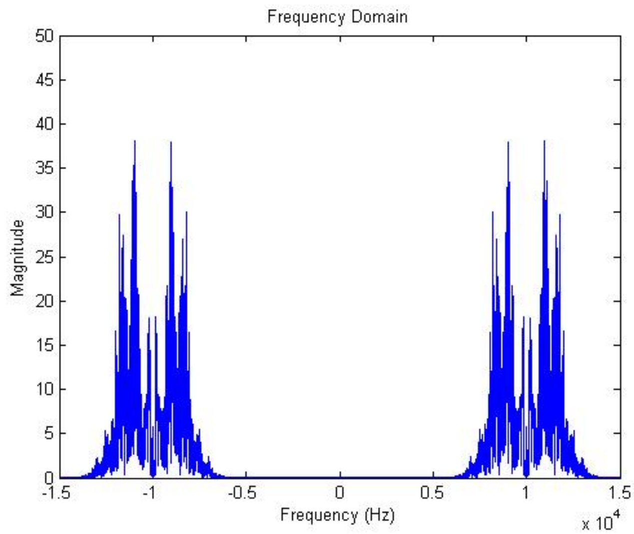

Figure 17

Now we will demodulate the voice signal back to DC (0 Hz). Figure 18 illustrates the spectrum of the voice signal after demodulation; multiplying the modulated signal (Figure 17) by a 10kHz cosine. Note that we have one copy of the voice signal spectrum at 0Hz and two copies at +/- 20kHz, which is analogous to Figure 12. Once again, to remove the higher frequency copies at +/-20kHz we need to apply a low frequency filter.

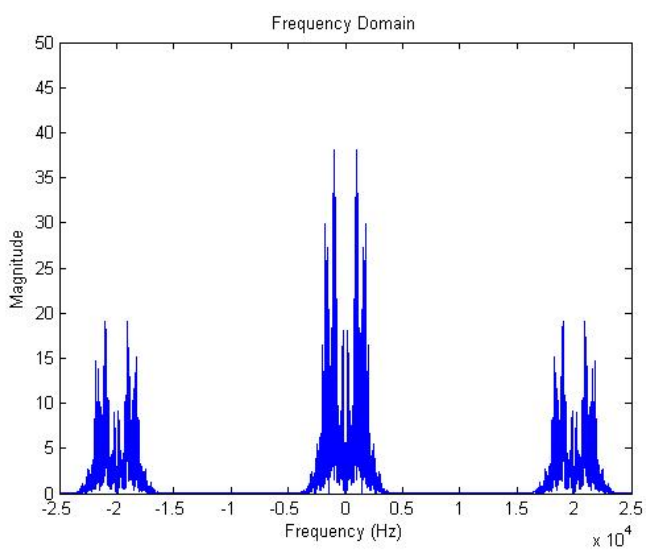

Figure 18

Modulation and Demodulation Summary

Figure 19 is a block diagram that summarizes the steps involved in modulating and demodulating a signal by a frequency f_2. Modulation involves multiplying the input signal x(t) by a cosine with a frequency f_2. Demodulation involves multiplying the modulated signal again by a cosine with a frequency f_2, and then applying a low pass filter with cutoff frequency f_2. The low pass filter removes the high frequency components centered about $2f_2$.

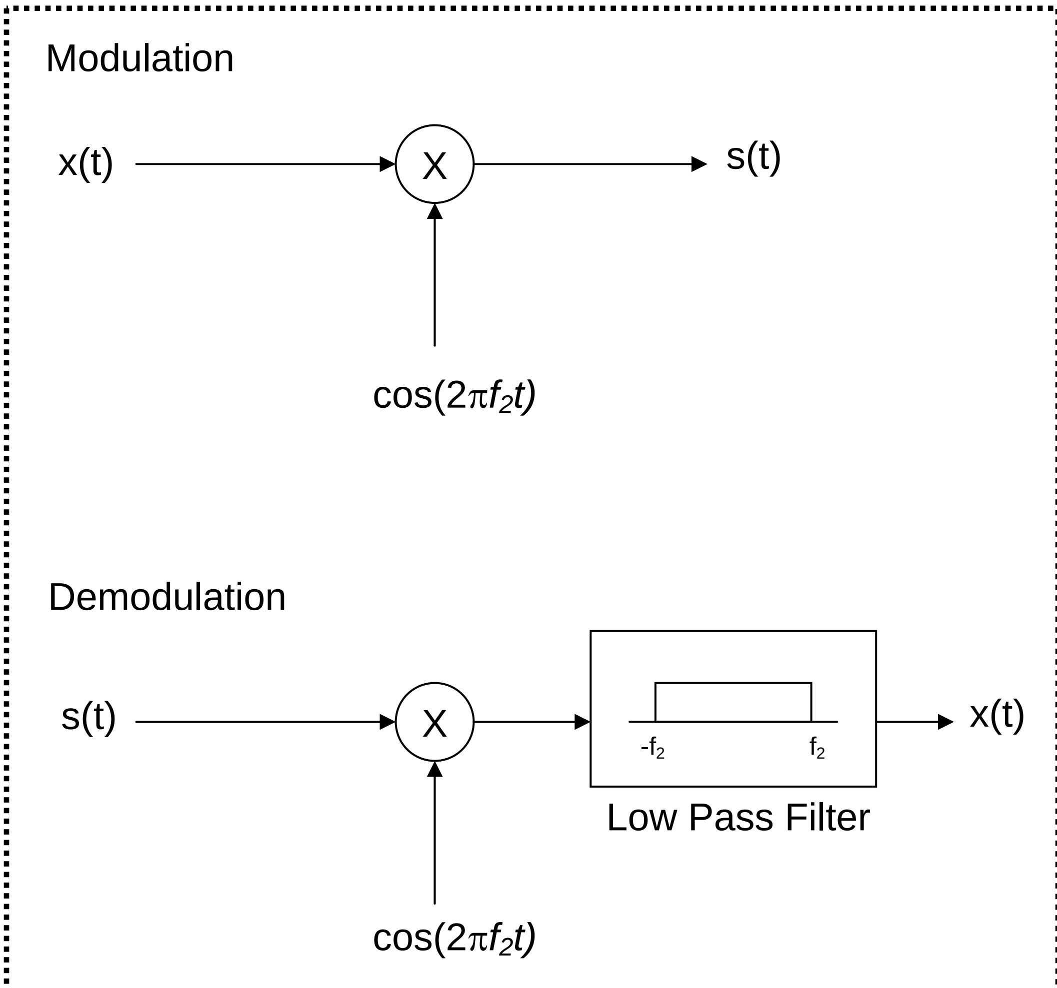

Figure 19