Lecture 1, Slide #1

6.02 Fall 2009
Lecture #11

• Summary of progress
• The problem of multiplexing
• Eternal Signals and LTI Systems
• Discrete-Time Sines and Cosines
So Far - Low BER Transmission on Wires

- **Problems and Analysis Techniques:**
  - Intersymbol Interference (ISI)
    - LTI Systems and Unit Sample Responses
    - Eye Diagrams
  - Noise
    - Probability Density and Cumulative Distribution Functions
    - Normal (Gaussian) random variables

- **Solution Approaches**
  - Techniques based on LTI models
    - Deconvolution and Decision Feedback Equalization
  - Error Detection and Correction Codes
    - Parity bits, Reed-Solomon Codes, Viterbi Algorithm
New Problem - Resource Sharing

- Two Approaches
  - Time Division Multiplexing (TDM)
    - Each Xmit-Rcvr pair gets a time slot (how to decide?)
    - Used by wired internet
  - Frequency Division Multiplexing (FDM)
    - Each Xmit-Rcvr gets part of the spectrum (explained later)
    - Like Broadcast TV and Radio, many wireless devices

- Next Two Weeks on FDM
Sinusoids and LTI Systems

Three sinusoids of different frequency

Summed then scaled and shifted sinusoids
Sinusoids and LTI Systems

Summed then scaled and shifted sinusoids

IR System Output from Summed sinusoids
Sinusoids and LTI Systems

Summed then scaled and shifted sinusoids

IR System (off ceiling) Output from Summed sinusoids
IR System Response to Different Sines
Sinusoids in LTI Systems

• Sinusoids $\rightarrow$ LTI Systems $\rightarrow$ Sinusoids
  – Different Frequencies do not seem to mix
    • Xmit: encode data on different frequencies (Modulation)
    • Rcvr: decode data from different frequencies (Filtering)
  – Different Frequencies have different “Gain”
    • But in the IR Channel:
      – higher frequencies $\rightarrow$ lower “gain”.

• Analyzing Sinusoids in LTI systems
  – Eternal Signals
  – Response to $x[n] = z^n$
  – Representing

\[
\cos \Omega = \frac{e^{j\Omega} + e^{-j\Omega}}{2}
\]
Consider Samples for $-\infty < n < \infty$

$x[n] = 1^n u[n]$

$x[n] = 1^n$

$x[n] = (-1)^n u[n]$

$x[n] = (-1)^n$

Eternal Signals
LTI Systems for Eternal Signals

- Convolution Changes a little

\[ y[n] = \sum_{m=-\infty}^{\infty} h[m] x[n - m] \]

- \( x[n] \) may be eternal
- \( h[m] \) may not be causal

\[ h[n] = 0.7^{|n|} \]
Suppose $x[n] = z^n$, $-\infty < n < \infty$

- Find y using convolution

$$y[n] = \sum_{m=-\infty}^{\infty} h[m]z^{n-m}$$

Reorganizing

$$y[n] = \left( \sum_{m=-\infty}^{\infty} h[m]z^{-m} \right) z^n$$

Just a number if sum converges = $H(z)$

$$y[n] = H(z)z^n$$

$y[n]$ has the same form as $x[n]$!
So What

• Suppose \( x[n] = A_1 z_1^n + A_2 z_2^n \)

\[
y[n] = (\sum_{m=-\infty}^{\infty} h[m] z_1^{-m}) A_1 z_1^n + (\sum_{m=-\infty}^{\infty} h[m] z_2^{-m}) A_2 z_2^n
\]

\[
y[n] = H(z_1) A_1 z_1^n + H(z_2) A_2 z_2^n
\]

• If we can separate \( z_1^n \) part from \( z_2^n \) part in \( y[n] \)
  – Can recover two messages

• Could then use lots of \( z_i \)'s
  – Multiple channels!
A problem with eternal signals

$|z_i| < 1 \quad z_i^n \text{ blows up } n \to -\infty$

$|z_i| > 1 \quad z_i^n \text{ blows up } n \to \infty$
Use $|z_i| = 1$

- How many $z$’s are there with unit magnitude?
  - $z = 1, z = -1$, any others?

- Can generate more $z$’s using complex numbers

$$z = e^{j\Omega} = \cos \Omega + j \sin \Omega$$

- Magnitude is one for any frequency

$$|z| = |e^{j\Omega}| = |\cos \Omega + j \sin \Omega| = \sqrt{\cos^2 \Omega + \sin^2 \Omega} = 1$$

- Know how to evaluate $z^n$

$$z^n = e^{j\Omega n} = \cos \Omega n + j \sin \Omega n$$
Example: \( z^n = e^{0.4jn} = \cos 0.4n + j \sin 0.4n \)
Frequency Response

- From convolution

\[ y[n] = \sum_{m=-\infty}^{\infty} h[m] e^{j\omega(n-m)} \]

Reorganizing

\[ y[n] = \left( \sum_{m=-\infty}^{\infty} h[m] e^{-j\omega m} \right) e^{j\omega n} \]

A complex number if the sum converges

\[ y[n] = H(e^{j\omega}) e^{j\omega n} \]

\[ H(e^{j\omega}), \ -\pi < \omega < \pi, \ is \ the \ frequency \ response \]
Non-eternal Example: Cosine starts at zero

\[ x[n] = \cos(\Omega n)u[n] \quad \Omega = \left\{ \frac{\pi}{10}, \frac{2\pi}{10}, \frac{3\pi}{10} \right\} \]
Summary and Next Time

• Frequency Division Multiplexing
  – Eternal $z^n$’s do not mix when input to LTI systems
  – Need many eternal $z^n$’s that do not blow up, $|z| = 1$
  – Use $z_i = e^{j\Omega_i}$, $-\pi < \Omega_i \leq \pi$

• Next Week
  – How do we separate the different frequencies in $y[n]$
    • Filters and Fourier Analysis

• Two Weeks
  – Encoding Information using different frequencies
    • What happens when we use
      \[ x[n] = A_1[n]e^{j\Omega_1 n} + A_2[n]e^{j\Omega_2 n} \]
    • Do the modulated complex exponentials still stay separated?