

6.02 Digital Communication Systems—Fall 2009

Quiz 1 - 7:30-9:30pm (Two Hours)

Tuesday, October 6th, 2009

Check your section	Section	Time	Room	Rec. Instr.
<input type="checkbox"/>	1	10-11	36-112	Vladimir Stojanovic
<input type="checkbox"/>	2	11-12	36-112	Vladimir Stojanovic
<input type="checkbox"/>	3	12-1	36-112	Bert Shi
<input type="checkbox"/>	4	1-2	36-112	Bert Shi
<input type="checkbox"/>	5	2-3	26-168	Fred Chen
<input type="checkbox"/>	6	3-4	26-168	Fred Chen

Directions: The exam consists of 5 problems on 13 pages. Please make sure you have all the pages. **Enter all your work and your answers directly in the spaces provided on the printed pages of this exam. Please make sure your name is on all sheets. DO IT NOW!** All sketches must be adequately labeled. Unless indicated otherwise, **answers must be derived or explained in the space provided**, not just simply written down. This examination is closed book, but students may use one $8\frac{1}{2} \times 11$ sheet of paper for reference. Calculators may not be used.

The probability density function for a zero-mean unit standard deviation Normal(Gaussian) random variable is:

$$f_X(x) \equiv \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

The cumulative distribution function for a zero-mean unit standard deviation Normal(Gaussian) random variable is:

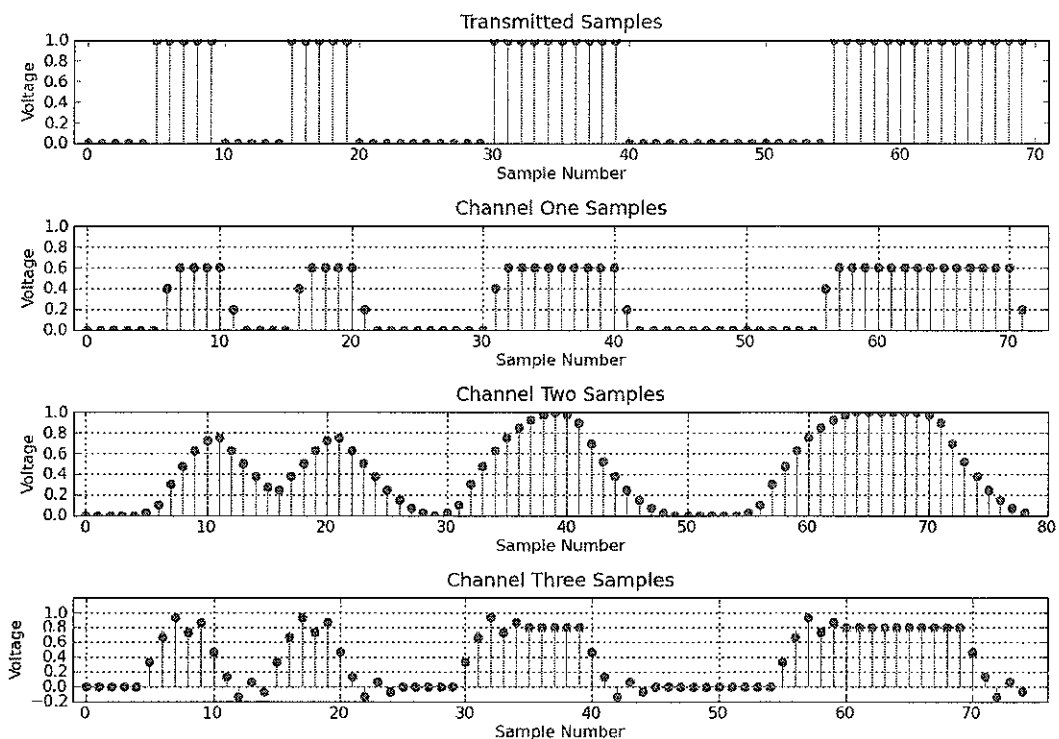
$$\Phi(x) \equiv \int_{-\infty}^x \frac{1}{\sqrt{2\pi}} e^{-\frac{\hat{x}^2}{2}} d\hat{x}$$

Please leave the rest of this page blank for use by the graders:

Problem	No. of points	Score	Grader
1	25		
2	15		
3	25		
4	15		
5	20		
Total	100		

Problem 1 (25 points)

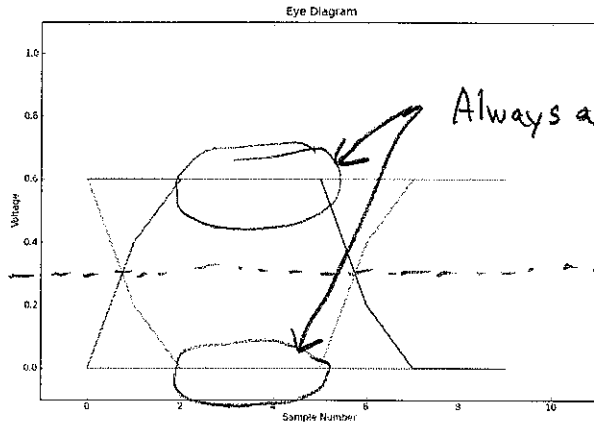
For this problem, please consider three linear and time-invariant channels, denoted, perhaps unoriginally, as *channel one*, *channel two*, and *channel three*. The response of each of these three channels to the transmitted voltage sample sequence, given in the first plot below, are shown in the second, third and fourth plot below (Note: the voltage samples of channel one's response never rise above 0.6 volts; there are several cases where channel three's response is ≈ 0.8 volts for several samples in a row). Please use these plots to answer all the parts of this question, and please assume that *five voltage samples are used to represent each bit*.



1A. (2 points) Noting again that a sequence of five voltage samples are used to represent each bit, what is the *bit* sequence being transmitted?

Bit Sequence = $[0, 1, 0, 1, 0, 0, 1, 1, 0, 0, 0, 1, 1, 1]$

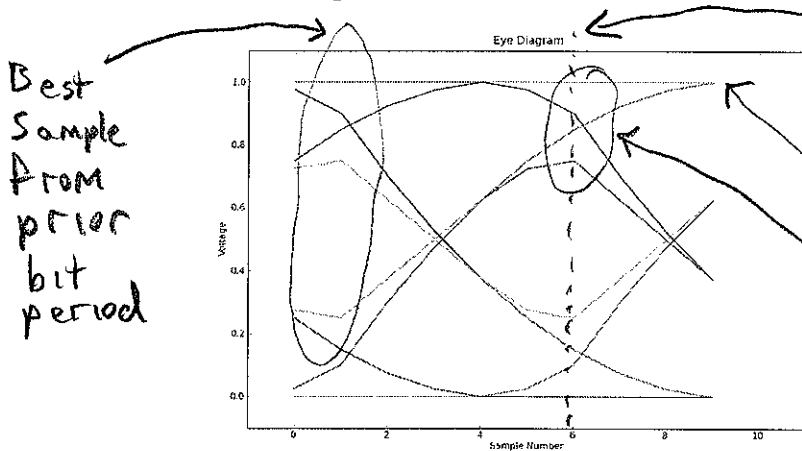
1B. (4 points) Which channel (one, two or three) has the following eye diagram, and what would be the best threshold voltage to use to differentiate between a transmitted zero bit and a transmitted one bit.



Channel Number = One

Threshold voltage = 0.3V

1C. (6 points) Which channel (one, two or three) has the following eye diagram, and based on the eye diagram, what is the best sample number to use to determine the value of the 20th transmitted bit (hint: since the transmitter only begins sending the 20th bit starting with sample 100, your answer should be greater than 100).



Best sample is sample six

1) Peak = 1V

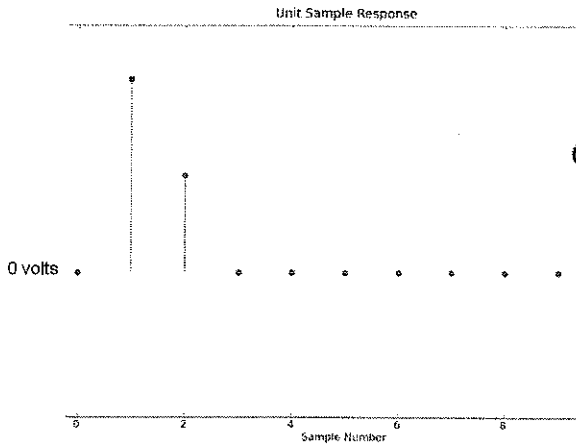
2) Four different voltage values when sending 1
3) No oscillations.

Must be channel 2

Channel Number = Two

Sample number to use for determining the 20th bit = 100 + 6 = 106

1D. (7 points) Which channel (one, two or three) has the following unit sample response, $h[n]$, and what are the numerical values of $h[1]$ and $h[2]$?



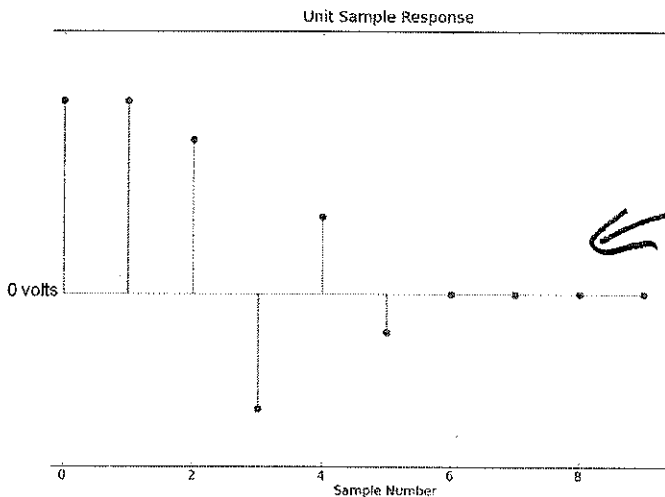
← Rise or fall completes in three samples, Must be channel One.

Channel Number = One

$h[1] = \underline{0.4}$ $h[2] = \underline{0.6 - 0.4 = 0.2}$

Channel one step response (From Graph) 0 1 2

1E. (6 points) Which channel (one, two or three) has the following unit sample response, and what is the numerical value of the sum $\sum_{n=0}^{\infty} h[n]$?



← Non monotonically increasing step response (oscillations) Must be Three

Channel Number = Three

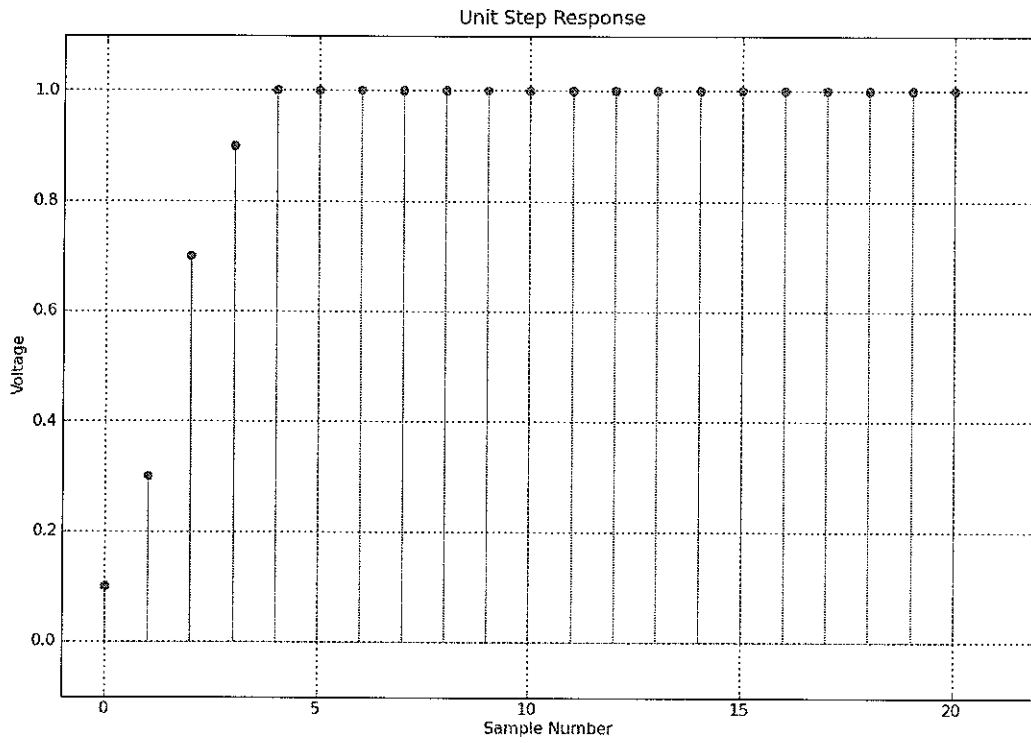
$\sum_{n=0}^{\infty} h[n] = \underline{0.8}$

← Step response eventually $= \sum_{n=0}^{\infty} h[n]$, and samples

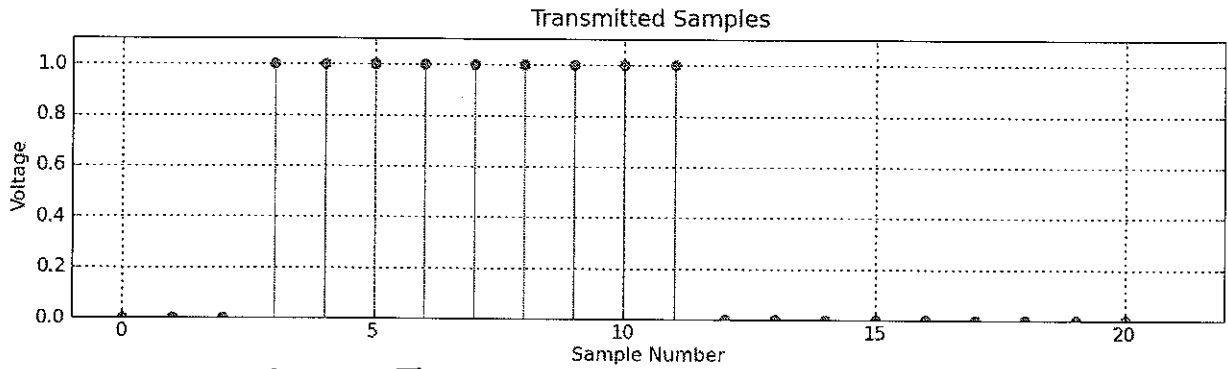
60 → 70 are enough to be close enough to eventually and = 0.8

Problem 2 (15 points)

Suppose a linear time-invariant channel has the following step response:

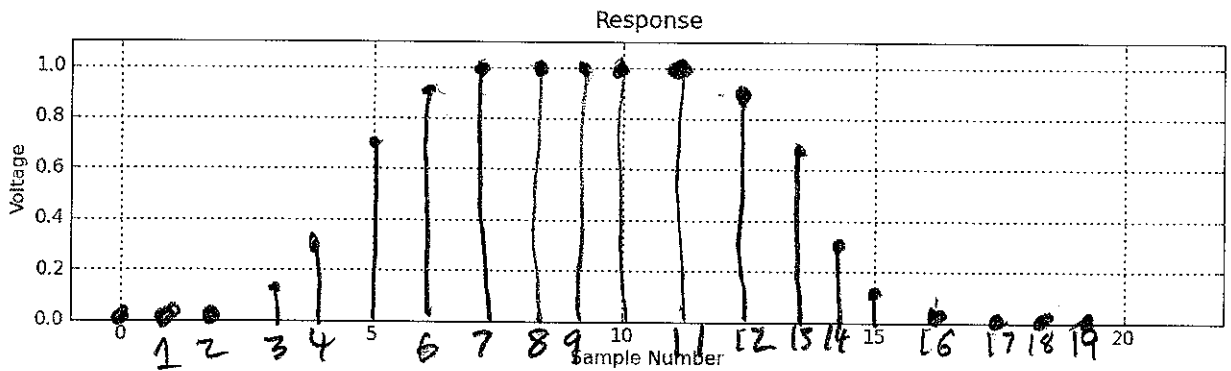


2A. (6 points) Given the channel has the above step response, plot the channel response to the following transmitted samples. Please use the axes provided just underneath the transmitted samples.

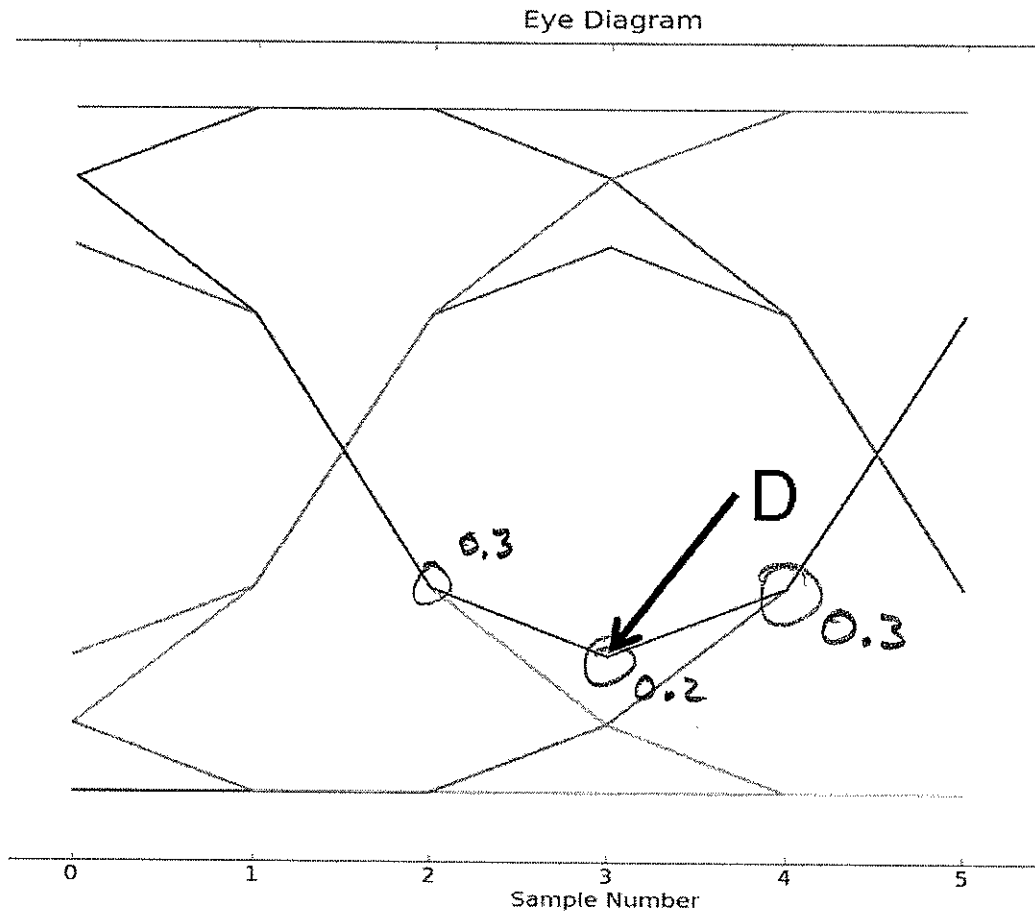


$$s \rightarrow S[n-3] - S[n-11] \approx \text{response}$$

$$s \equiv \text{step response}$$



2B. (8 points) For the case where three voltage samples are used for each bit, the channel described by the above step response has the following eye diagram. What is the value of D in the eye diagram?



Value of D = 0.2

D must correspond to 1 1 1 0 1 1 bit sequence. or

$$\min_n (1 - S[n] + S[n-3])$$

$$\text{at } n=3 \quad S[3] = 0.9 \quad S[n-3] = S[0] = 0.1$$

$$1 - 0.9 + 0.1 = 0.2$$

$$\text{at } n=2 \quad 1 - 0.7 = 0.3$$

$$\text{at } n=4 \quad 1 - 1 + 0.3 = 0.3$$

Problem 3 (25 points)

For this problem we ask you to consider two transmission systems, cleverly denoted *system one* and *system two*.

For system one, the channel has effectively no intersymbol interference, and the voltage sampled by the receiver of system one is:

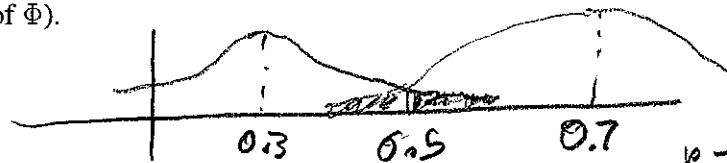
- 0.7 + *noise* volts whenever the transmitter sends a one,
- 0.3 + *noise* volts when the transmitter sends a zero.

For system two, there is intersymbol interference. The voltage sampled by the receiver in system two is:

- 1.0 + *noise* volts when the transmitter sends a one preceded by a one,
- 0.6 + *noise* volts when the transmitter sends a one preceded by a zero,
- 0.0 + *noise* volts when the transmitter sends a zero preceded by a zero,
- 0.4 + *noise* volts when the transmitter sends a zero preceded by a one.

In answering the following parts, please assume that the transmitters in both systems are equally likely to send zeros or ones, and that the receivers in both systems use 0.5 volts as the threshold for deciding the bit value.

3A.(6 points) Suppose *noise* in system one is a zero-mean Normal(Gaussian) random variable with standard deviation σ . Give an expression for the probability of a bit error in terms of the zero-mean unit standard deviation Normal cumulative distribution function, Φ , and σ (see cover page for definition of Φ).



Expression for the probability of a bit error = _____

$$\rightarrow \frac{1}{2} \left(1 - \Phi\left(\frac{0.2}{\sigma}\right) + \Phi\left(\frac{-0.2}{\sigma}\right) \right)$$

or

$$\Phi\left(\frac{-0.2}{\sigma}\right) + \Phi\left(\frac{-0.2}{\sigma}\right)$$

or

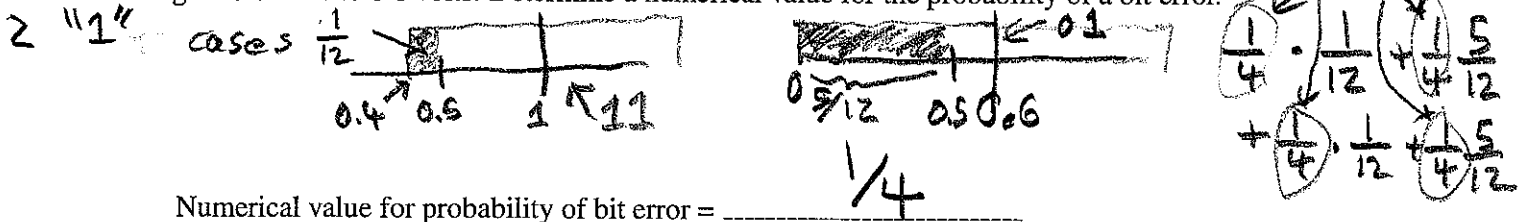
$$1 - \Phi\left(\frac{0.2}{\sigma}\right)$$

Prob error

$$\frac{1}{2} \left(1 - \Phi\left(\frac{0.2}{\sigma}\right) \right) + \frac{1}{2} \Phi\left(\frac{-0.2}{\sigma}\right)$$

Prob of "1"

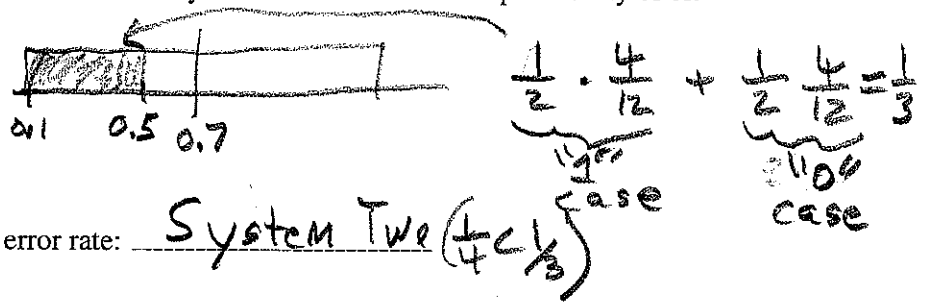
3B.(7 points) Suppose *noise* in system two is a uniformly distributed random variable over the range -0.6 volts to 0.6 volts. Determine a numerical value for the probability of a bit error.



Numerical value for probability of bit error = $\frac{1}{4}$

3C.(6 points) Suppose *noise* in both system one and system two is a uniformly distributed random variable over the range -0.6 volts to 0.6 volts. Which system will have the lower probability of bit error? Please justify your answer.

System one



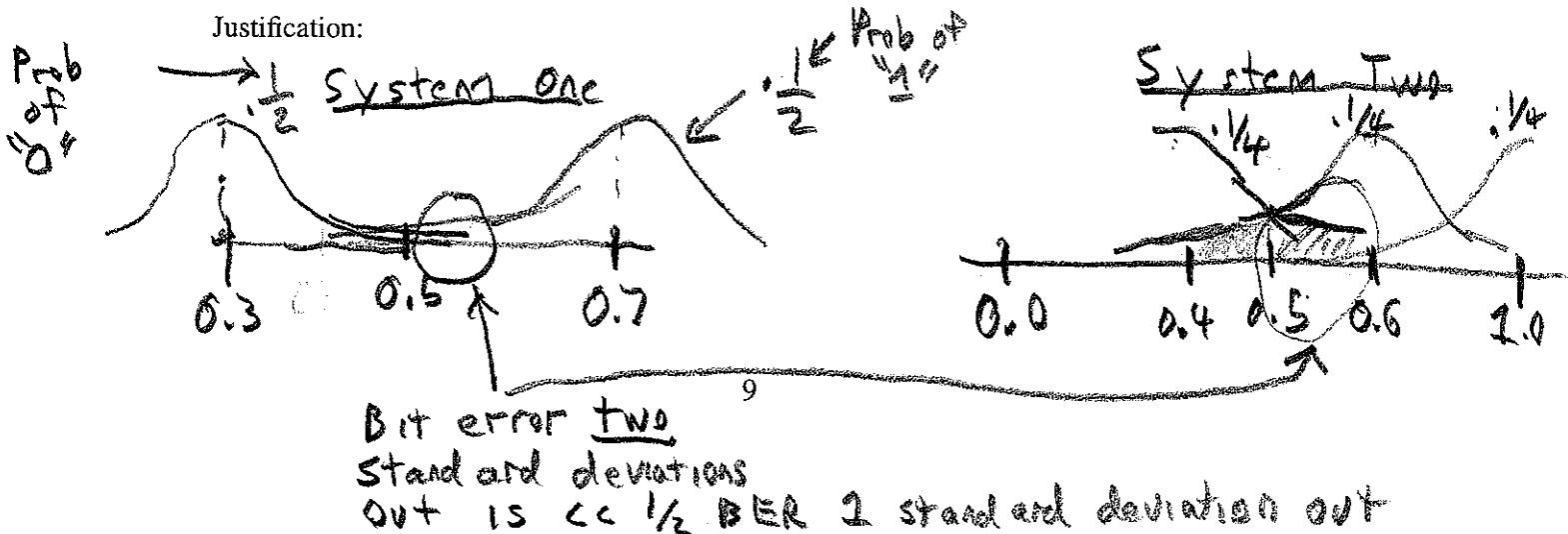
Which system will have a lower bit error rate: System Two ($\frac{1}{4} < \frac{2}{3}$)

Justification:

3D.(6 points) Suppose *noise* in both system one and system two is a zero-mean Normal(Gaussian) random variable with standard deviation 0.1 . Which system will have the lower probability of bit error? Please justify your answer.

Which system will have a lower bit error rate: System One ($(1 - \Phi(2)) < \frac{1}{2}(1 - \Phi(1))$)

Justification:



Problem 4 (15 points)

In this problem we will again consider a linear time-invariant channel and, as usual, denote the transmitted samples by x , the channel response by y , and the channel's unit sample response by h . In addition we will assume that $x[n] = 0$ and $h[n] = 0$ for $n < 0$, and therefore

$$y[n] = \sum_{m=0}^n h[m]x[n-m].$$

4A.(5 points) Suppose the sample sequence $w[n]$ is generated by solving a deconvolving difference equation given by

$$w[n] = \frac{1}{\alpha} (y[n] + \beta w[n-2] + \gamma w[n-3]).$$

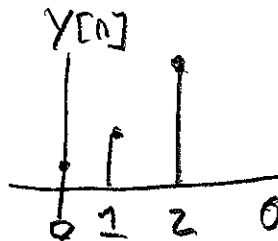
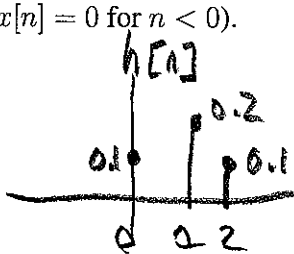
If the deconvolved samples, w , match the transmitted samples x (assuming no noise), then what must the nonzero values of the unit sample response, h , be? Please specify your answer in terms of the non-zero parameters α , β and γ , and please be careful to note the indices associated with w .

$$h[0]x[n] + h[1]x[n-1] + h[2]x[n-2] + h[3]x[n-3] = y[n]$$

$$\alpha w[n] - \beta w[n-2] - \gamma w[n-3] = y[n]$$

Nonzeros values of h $h[0] = \alpha, h[1] = 0, h[2] = -\beta, h[3] = -\gamma$

4B.(10 points) Now suppose that the only nonzero values of the unit sample response of the channel are given by $h[0] = 0.1$, $h[1] = 0.2$, and $h[2] = 0.1$. If the received samples are $y[0] = 0.01$, $y[1] = 0.07$, and $y[2] = 0.2$, what were the transmitted samples $x[0], x[1]$, and $x[2]$ (recall that $x[n] = 0$ for $n < 0$).



$$w[n] = 10 (y[n] - 2w[n-1] - w[n-2])$$

$$0.1 = x[0] = w[0] = 10 \cdot \overbrace{y[0]}^{0.01} \quad (w[n] = 0 \text{ for } n < 0)$$

$$0.5 = x[1] = w[1] = 10 (y[1] - 2(0.1))$$

$$0.9 = x[2] = w[2] = 10 (y[2] - 2(0.5) - 1(0.1))$$

$x[0] = \underline{0.1}$ $x[1] = \underline{0.5}$ $x[2] = \underline{0.9}$

Problem 5 (20 points)

Suppose the linear time-invariant channel considered in problem 4 has a known unit sample response whose nonzero values are given by $h[0] = 0.1$, $h[1] = 0.2$, and $h[2] = h[3] = h[4] = h[5] = h[6] = h[7] = h[8] = 0.1$.

For the four parts of this question, we ask you to consider two possible deconvolving systems given by the difference equations:

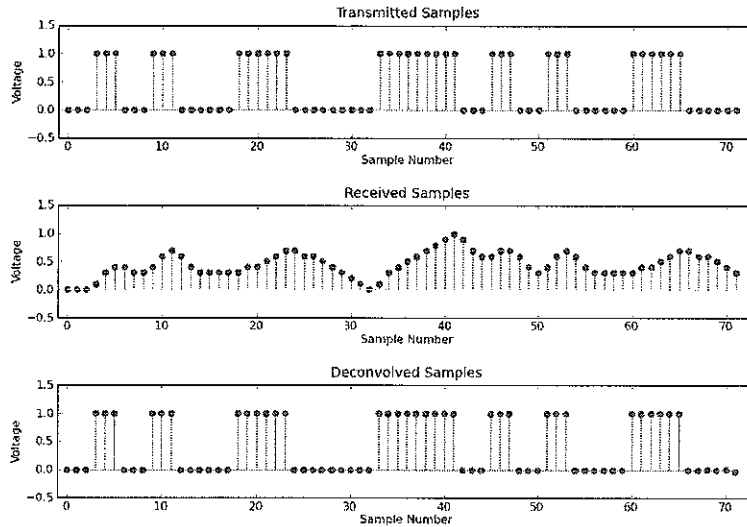
$$w_1[n] = 10y[n] - \left(2w_1[n-1] + \sum_{m=2}^8 w_1[n-m] \right)$$

with $w_1[n] = 0$ for $n < 0$ and

$$w_2[n-1] = \frac{1}{2} \left(10y[n] - \left(\sum_{m=2}^8 w_2[n-m] \right) \right)$$

with $w_2[n-1] = 0$ for $(n-1) < 0$. Please be careful to note the differences in how w is indexed on the left-hand side of the above two difference equations.

5A (5 points) Assuming that there is *no* noise, which of the above two deconvolving systems, 1 or 2, produced the following result? Please justify your answer.

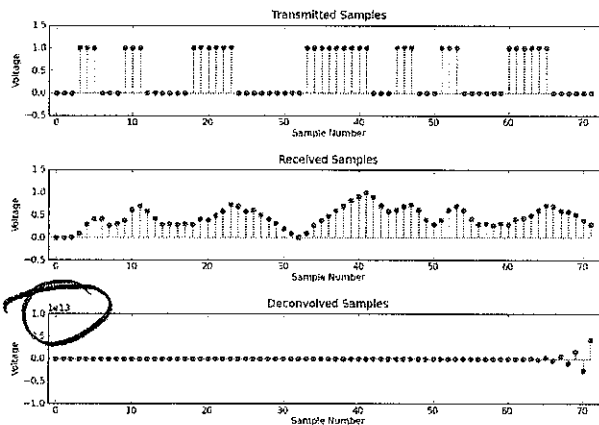


$$0.1 W_1[n] + 0.2 W_1[n-1] + 0.1 \sum_{m=2}^{\infty} W_1[n-m] = y[n]$$

Deconvolving System: System 1

Justification: $\sum h[m] W_1[n-m] = y[n]$ for system 1 so $W_1[n] = x[n]$ with no noise!

5B (5 points) If zero mean, 0.01 standard deviation Gaussian random noise is added to y before it is deconvolved, which of the above two deconvolving systems produced the following result (Note scale on third plot)? Please justify your answer.



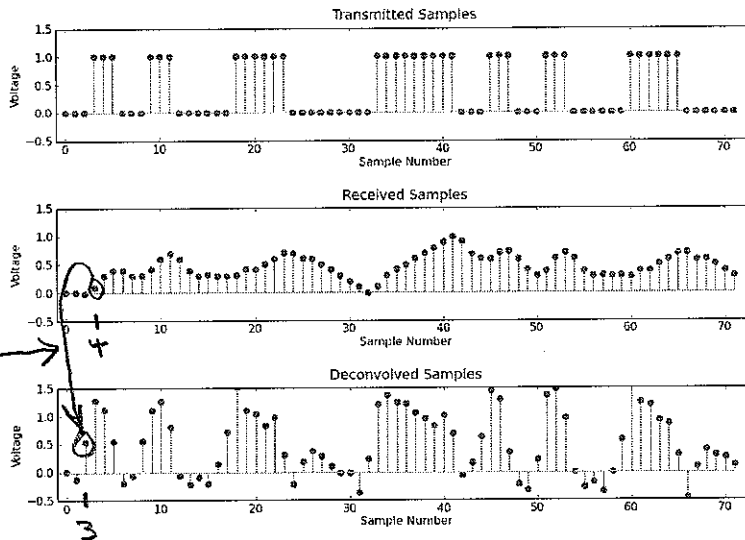
Deconvolving System: System One

Justification:

$$W_1[n] = 10 y[n] - 2 W_1[n-1] - \dots$$

> one so noise could keep being magnified and system blows up

5C (5 points) If zero mean, 0.01 standard deviation Gaussian random noise is added to y before it is deconvolved, which of the above two deconvolving systems produced the following result? Again please justify your answer.



Note Non-causality $W[n]$ depends on $y[n+1]$

$$W[n] = S y[n+1]$$

$$\left(\frac{1}{2} W[n-1] \right)$$

less than one, does not necessarily magnify noise. Also, system 2 is non-causal

Deconvolving System: System 2

Justification: By elimination and

5D (5 points) (Challenging Problem, save it for last) Determine the coefficients of a new difference equation that performs deconvolving better than either system 1 or system 2 for the case where noise is added to y before it is deconvolved. There are many reasonable answers to this question, please give a justification for your answer.

One Approach { System two is an ^{exact} deconvolver assuming $h_2[0] = 0$ $h_2[1] = 0.2$ $h_2[2] = \dots = h_2[8] = 0.1$ but $\sum h_2[i] \neq 1 = \sum h[i]$

A difference equation for a better deconvolving system:

Make $\sum h_3 = 1$

$$W_3[n-1] = \frac{9}{2} (y[n] - \sum_{m=2}^8 W_3[n-m])$$

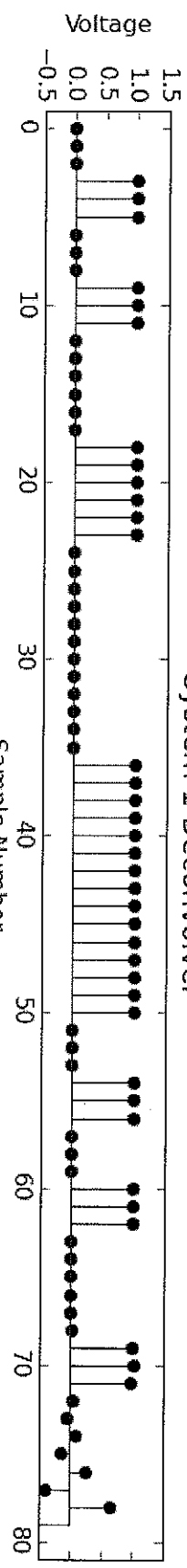
End of Quiz 1!

$$h_3 = [0, 0.2] + [0.1] * 8 \rightarrow \frac{1}{2} (10 y[n] - \sum_{m=2}^8 W_3[n-m])$$

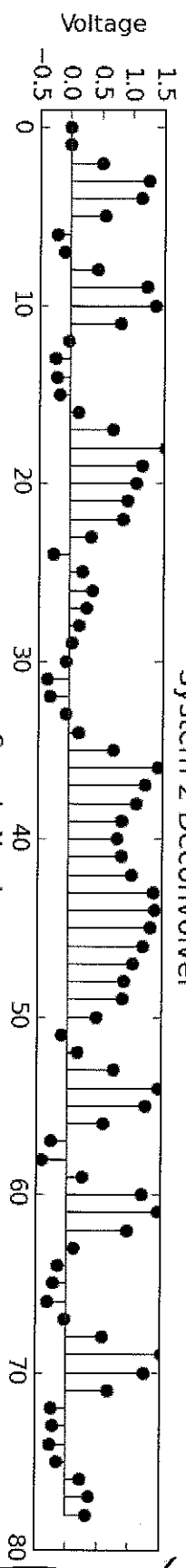
$$h_3 = [0, 0.3] + [0.1] * 7 \rightarrow \frac{1}{3} (10 y[n] - \sum_{m=2}^8 W_3[n-m])$$

SD Solutions: No Noise case

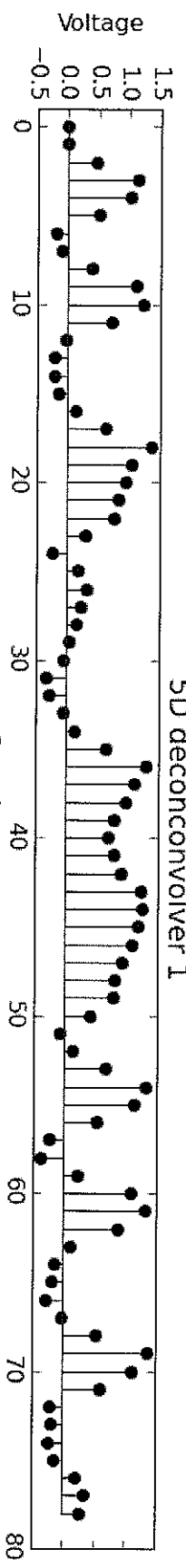
System 1 Deconvolver



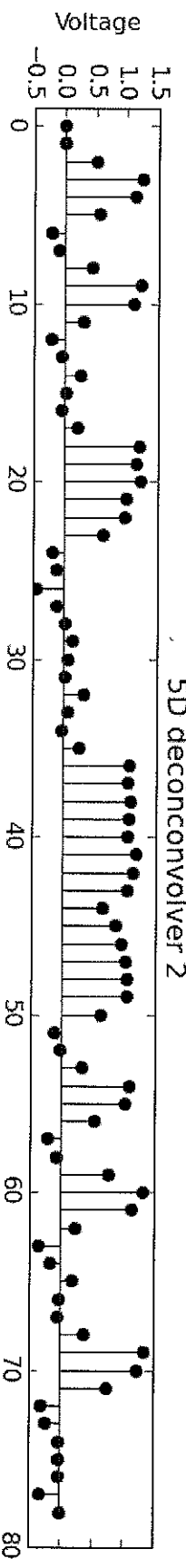
System 2 Deconvolver



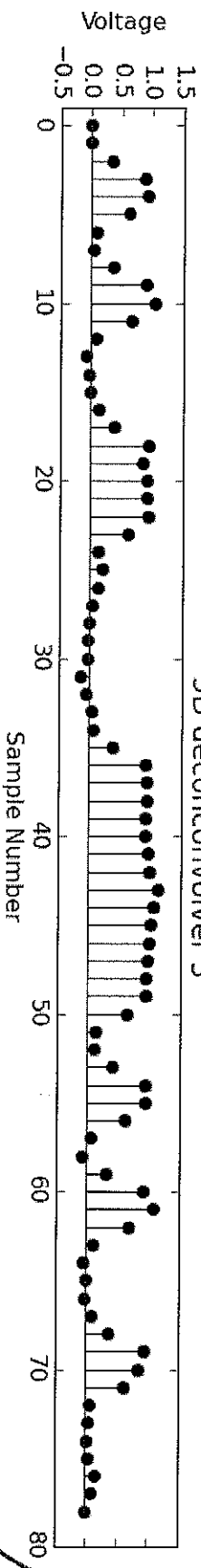
Sample Number
5D deconvolver 1



Sample Number
5D deconvolver 2



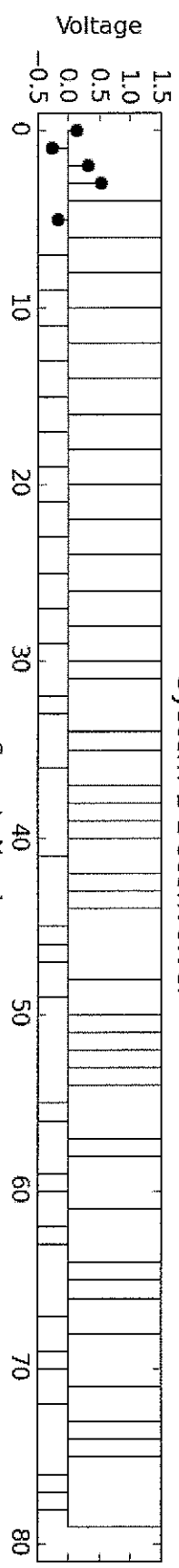
Sample Number
5D deconvolver 3



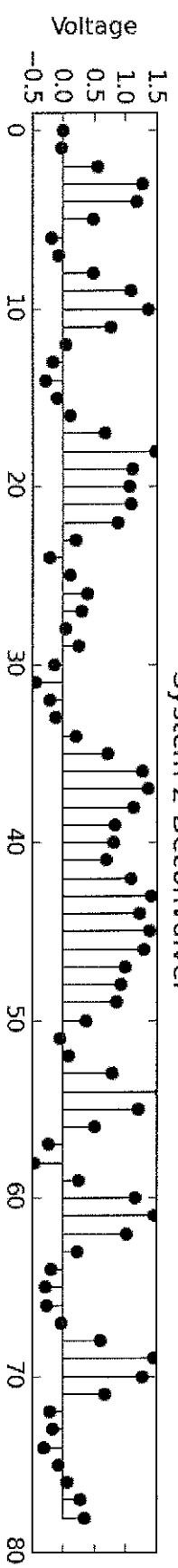
All
Approximate

5D Solutions: Noisy Case

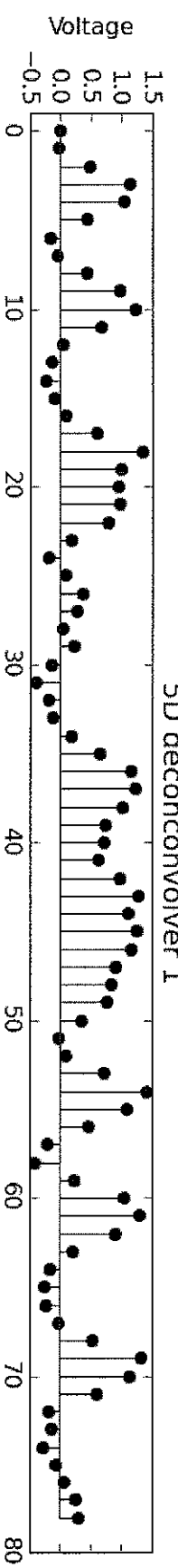
System 1 Deconvolver



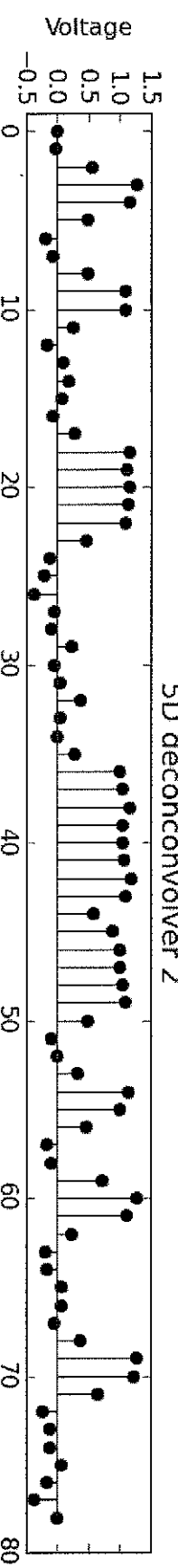
System 2 Deconvolver



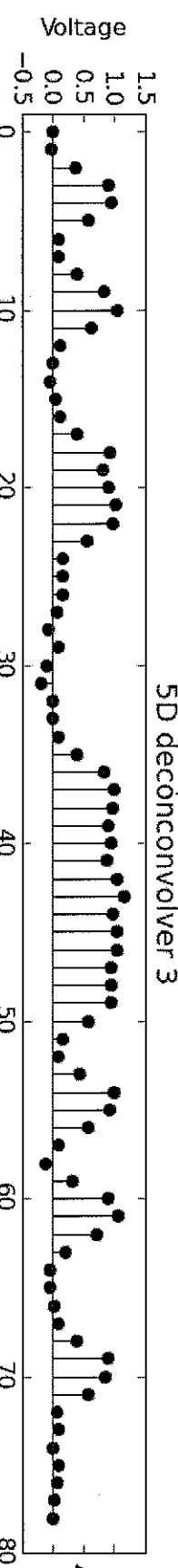
5D deconvolver 1



5D deconvolver 2



5D deconvolver 3



← System 1
blows
up!

← Best

$$h_3 = [0, 0.3] + [0.1] * 7$$