6.02 Fall 2010
Lecture #14

• Frequency Domain Sharing
• Spectrum and the Fourier Series
• Fourier Series Examples
• Rise Time and Spectrum
Frequency Domain Sharing - Big Picture

Questions

- What is Modulation (Mod in figure)?
- What do typical X’s look like?
- What should the Demodulator be?
- What is the relation between $x_i[n]$ and $y_i[n]$?
Cosine Modulation

\[ x[n] = \sum_{i=1}^{P} x_i[n] \cos i \pi \]

\[ x_1[n] \]

\[ x_P[n] \]

\[ \cos 1 \pi \]

\[ \cos P \pi \]
Ideal Channel Case \( Y = X \)

\[
y[n] = \sum_{i=1}^{P} x_i[n] \cos \Omega_i n
\]

\( x_i[n] \)

\( x_i[n] \cos 0.5n \)
\[ x[n] = \cos \frac{\pi}{3} n = \frac{1}{2} e^{j \frac{\pi}{3} n} + \frac{1}{2} e^{-j \frac{\pi}{3} n} \]
Sine in Time

\[ x[n] = \sin \frac{\pi}{3} n = \frac{-j}{2} e^{j \frac{\pi}{3} n} + \frac{j}{2} e^{-j \frac{\pi}{3} n} \]
Sine Fourier Series, 2 Imaginary values

\[ x[n] = \sum_{k=-\frac{N}{2}}^{\frac{N}{2}-1} X[k] e^{j \frac{2\pi}{N} kn} = -\frac{j}{2} e^{j \frac{\pi}{3} n} + \frac{j}{2} e^{-j \frac{\pi}{3} n} \]

Note horizontal axis ranges over \(-\pi\) to \(\pi\)!
Fast Rise Pulse

\[ x[n] \]

\[ n \]
Spectrum of Fast Rise Pulse

\[ x[n] = \sum_{k = -\frac{N}{2}}^{\frac{N}{2} - 1} X[k] e^{j \frac{2\pi}{N} kn} \]

\[ X[k] \]
Slow Rise Pulse

$x[n]$
Spectrum Slow Rise Pulse

\[ x[n] = \sum_{k=-\frac{N}{2}}^{\frac{N}{2}-1} X[k] e^{j \frac{2\pi}{N} kn} \]
Key Fourier Series Advantage

\[ x[n] = \sum_{k=-N/2}^{N/2-1} X[k] e^{j2\pi kn} \]

\[ y[n] = \sum_{m=0}^{n} h[m] x[n-m] \]

OR

\[ y[n] = \sum_{k=-N/2}^{N/2-1} H(e^{j2\pi k/N}) X[k] e^{j2\pi kn} \]

\[ H(e^{j\omega}) = \sum_{m=0}^{L} h[m] e^{-j\omega m} \]