Review problem 6

In this modulation problem you will be examining periodic signals and their associated discrete-time Fourier series (DTFS) coefficients. Recall that a periodic signal $x[n]$ with period N has DFTS coefficients given by

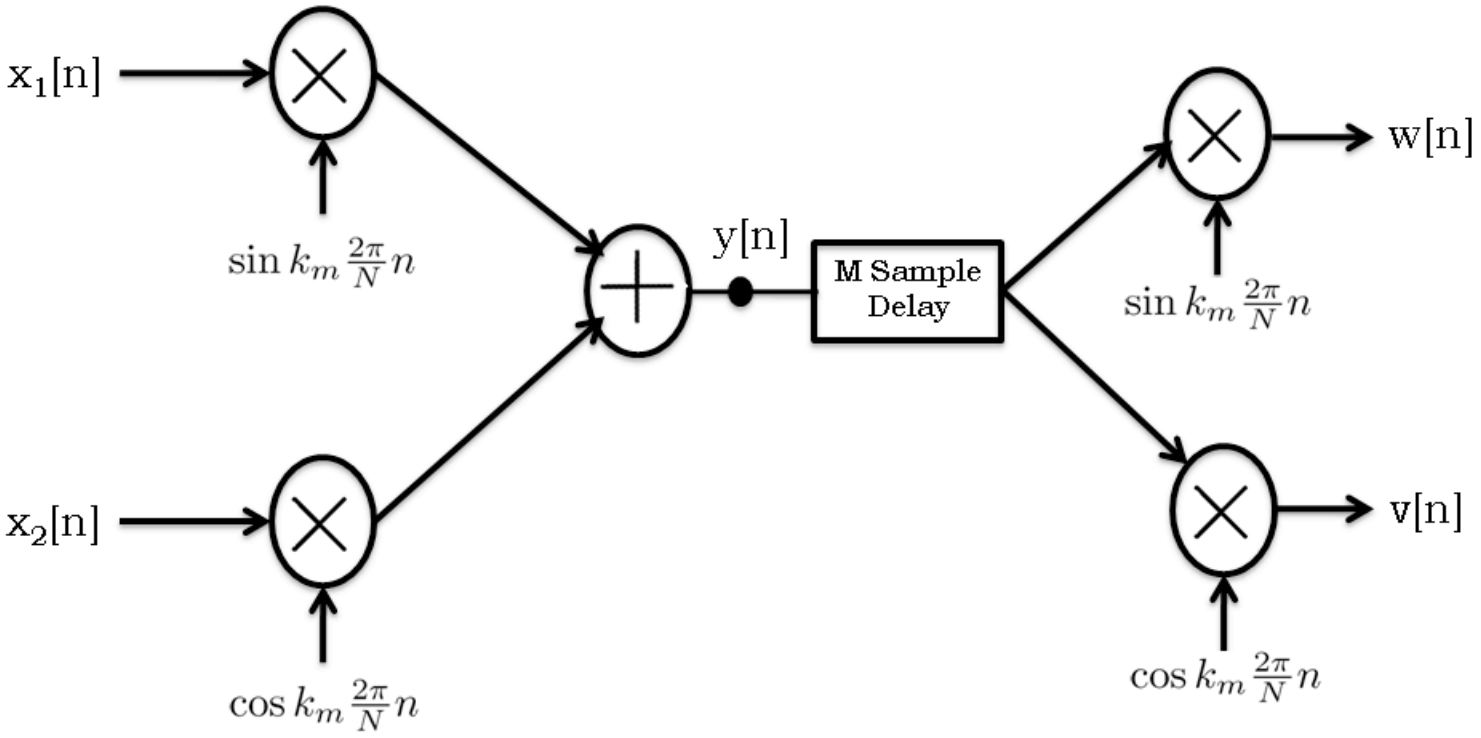
$$X[k] = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}kn}$$

and that the signal $x[n]$ can be reconstructed from the DFTS coefficients using

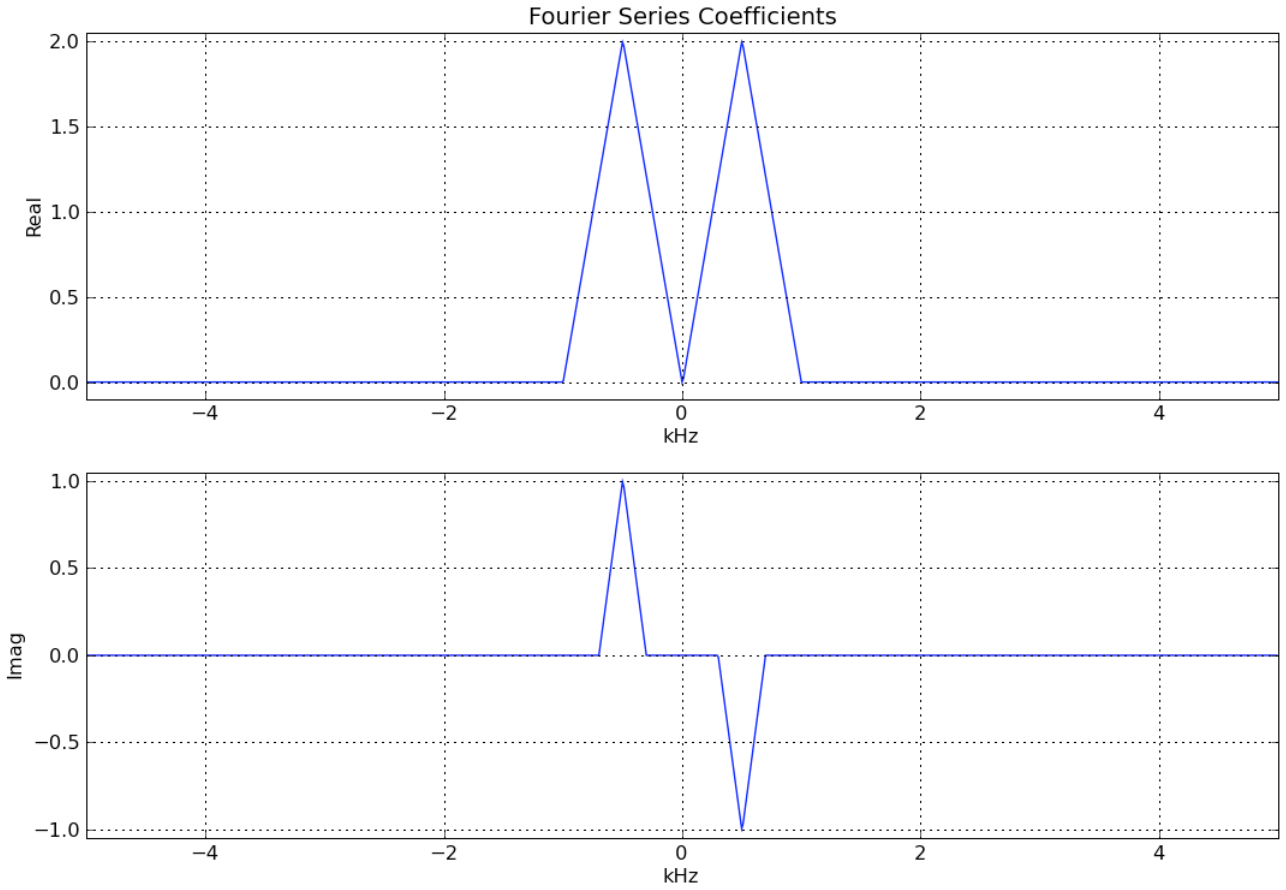
$$x[n] = \sum_{k=-K}^{K-1} X[k] e^{j\frac{2\pi}{N}kn}$$

where N is the period of the signal, $-K \leq k < K$ with $K = \frac{N}{2}$.

All parts of this question pertain to the following modulation-demodulation system, where all signals are periodic with period $N = 10000$ and therefore $K = 5000$. Please also assume that the sample rate associated with this system is 10000 samples per second, so that k is both a coefficient index and a frequency. In the diagram, the modulation frequency,k_m, is 500.

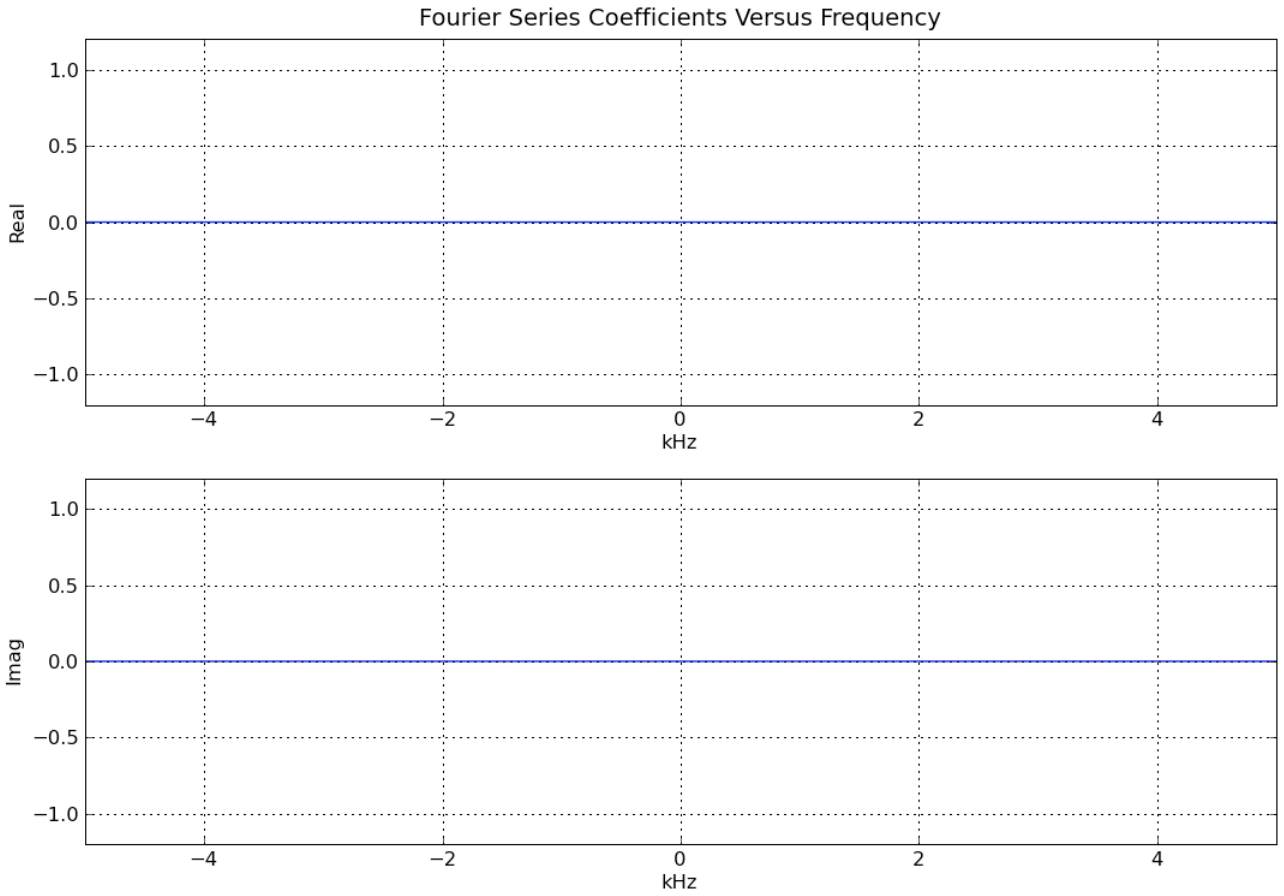

x1[n]
x2[n]
sin $k_m \frac{2\pi}{N} n$
cos $k_m \frac{2\pi}{N} n$
y[n]
M Sample Delay
w[n]
v[n]
sin $k_m \frac{2\pi}{N} n$
cos $k_m \frac{2\pi}{N} n$

(A) Suppose the DFTS coefficients for the signal $y[n]$ in the modulation/demodulation diagram are as plotted below.

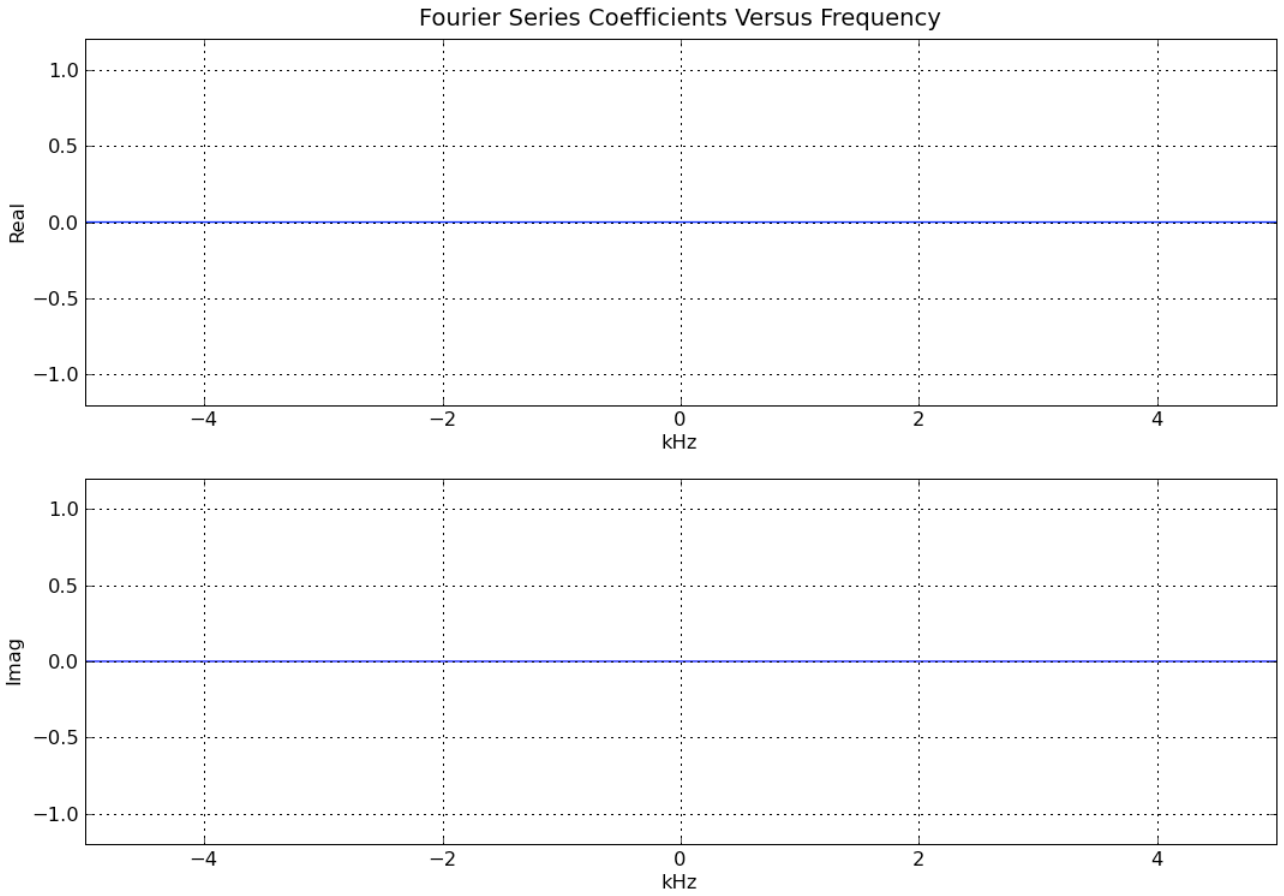


Assuming that $M = 0$ for the M-sample delay (no delay), on the two sets of axes on the next pages, please plot the DFTS coefficients for the signals **w** and **v** in the modulation/demodulation diagram. Be sure to label key features such as values and coefficient indices for peaks.

Plot of DFTS coefficients for w

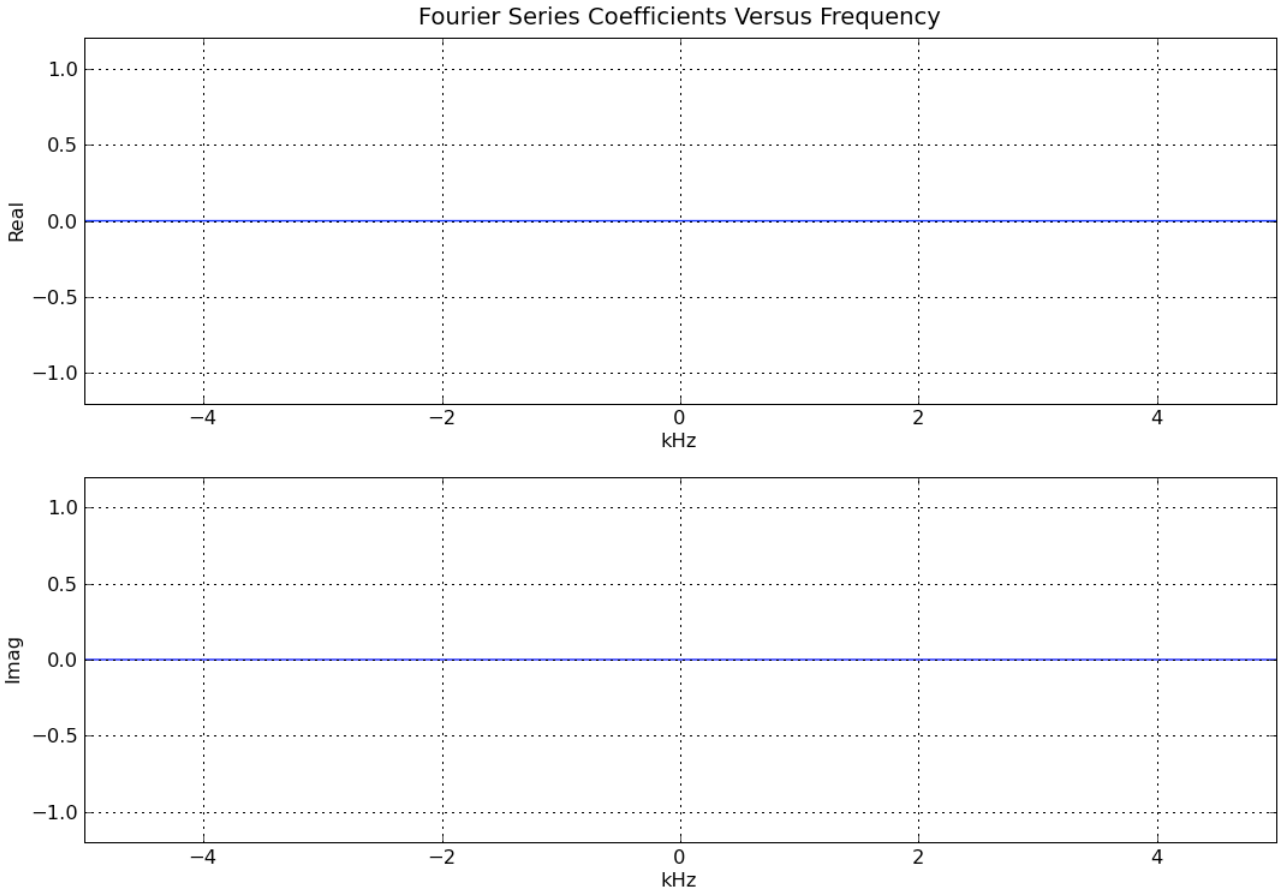


Plot of DFTS coefficients for v



(B) Assuming the DFTS coefficients for the signal $y[n]$ are the same as in part A, on the axes below, please plot the DFT coefficients for the signal x_1 in the modulation/demodulation diagram. Be sure to label key features such as values and coefficient indices for peaks.

Plot of DFTS coefficients for x_1



(C) If the M-sample delay in the modulation/demodulation diagram has the right number of samples of delay, then it will be possible to nearly perfectly recover $x_1[n]$ by low-pass filtering $v[n]$. Please determine the smallest positive number of samples of delay that are needed and the cut-off frequency for the low-pass filter. Please be sure to justify your answer, using pictures if appropriate.

Smallest M (number of samples of delay) > 0 = _____

Cutoff Frequency of Low Pass Filter = _____