Review problem 6

In this modulation problem you will be examining periodic signals and their associated discrete-time Fourier series (DTFS) coefficients. Recall that a periodic signal $x[n]$ with period N has DFTS coefficients given by

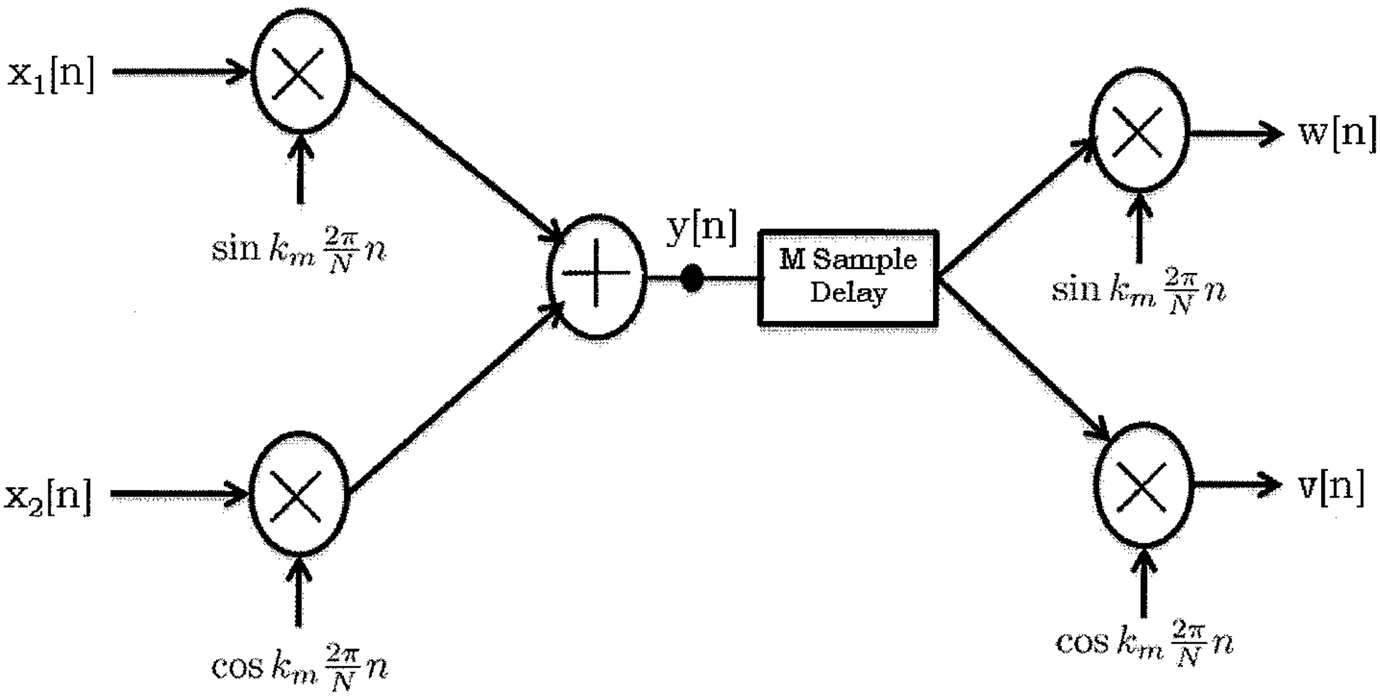
$$X[k] = \frac{1}{N}\sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi}{N}kn}$$

and that the signal $x[n]$ can be reconstructed from the DFTS coefficients using

$$x[n] = \sum_{k=-K}^{K-1} X[k]e^{j\frac{2\pi}{N}kn}$$

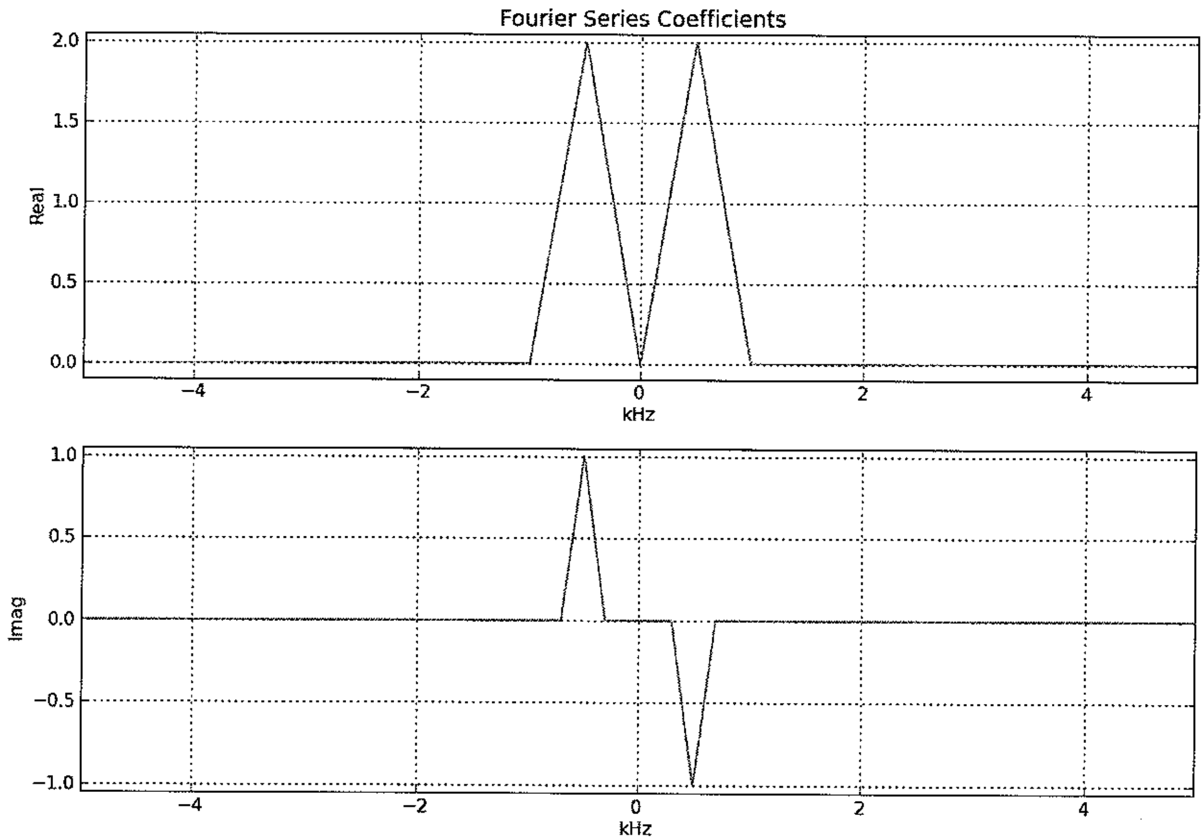
where N is the period of the signal, $-K \leq k < K$ with $K = \frac{N}{2}$.

All parts of this question pertain to the following modulation-demodulation system, where all signals are periodic with period $N = 10000$ and therefore $K = 5000$. Please also assume that the sample rate associated with this system is 10000 samples per second, so that k is both a coefficient index and a frequency. In the diagram, the modulation frequency, k_m, is 500.



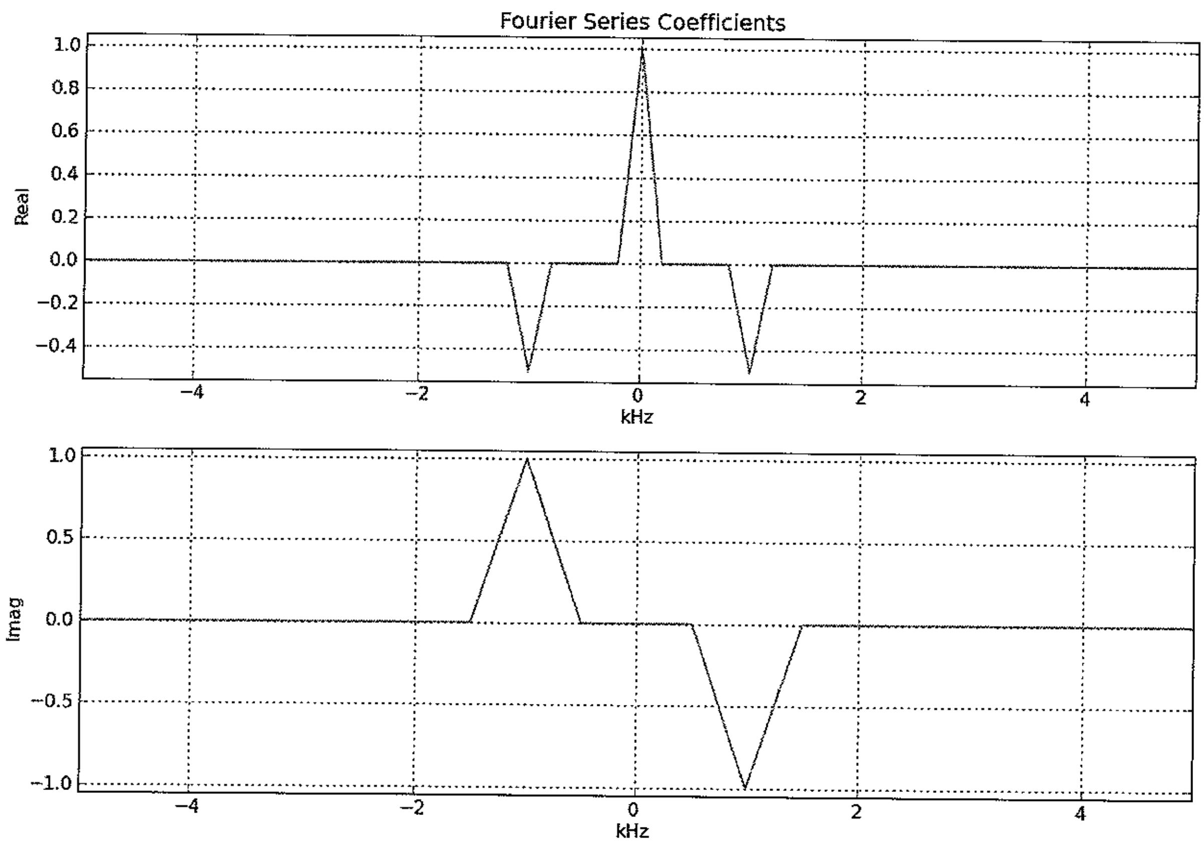
$x_1[n]$
$\sin k_m \frac{2\pi}{N} n$
$x_2[n]$
$\cos k_m \frac{2\pi}{N} n$
y[n]
M Sample Delay
w[n]
$\sin k_m \frac{2\pi}{N} n$
v[n]
$\cos k_m \frac{2\pi}{N} n$

(A) Suppose the DFTS coefficients for the signal $y[n]$ in the modulation/demodulation diagram are as plotted below.

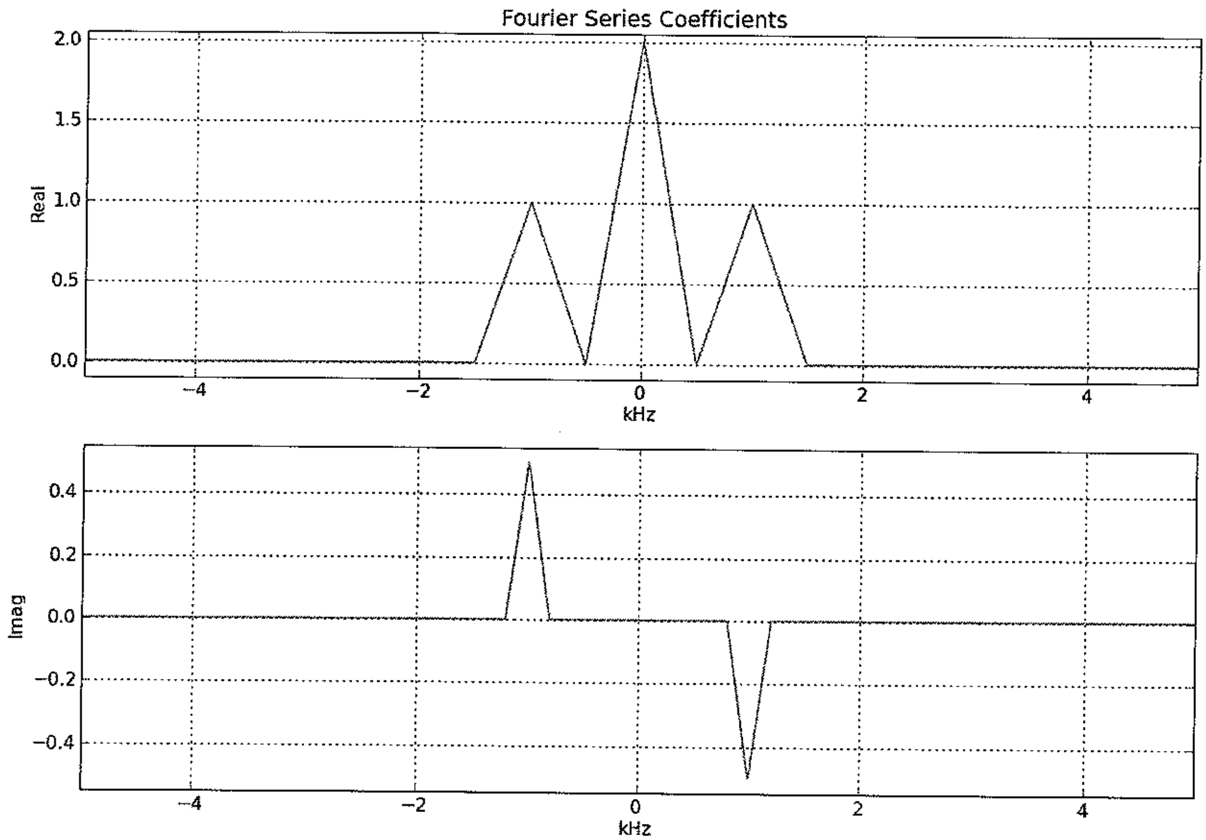


Assuming that $M = 0$ for the M-sample delay (no delay), on the two sets of axes on the next pages, please plot the DFTS coefficients for the signals **w** and **v** in the modulation/demodulation diagram. Be sure to label key features such as values and coefficient indices for peaks.

Plot of DFTS coefficients for w

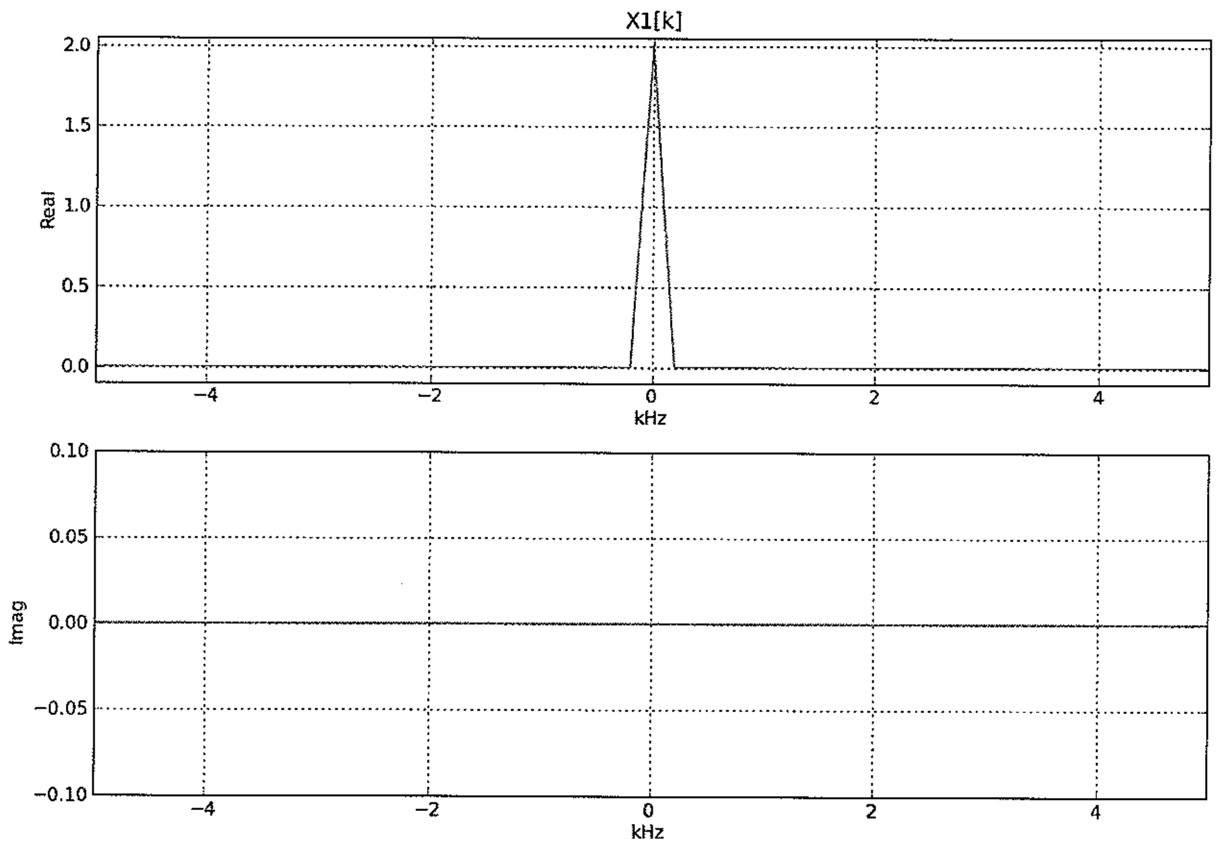


Plot of DFTS coefficients for v

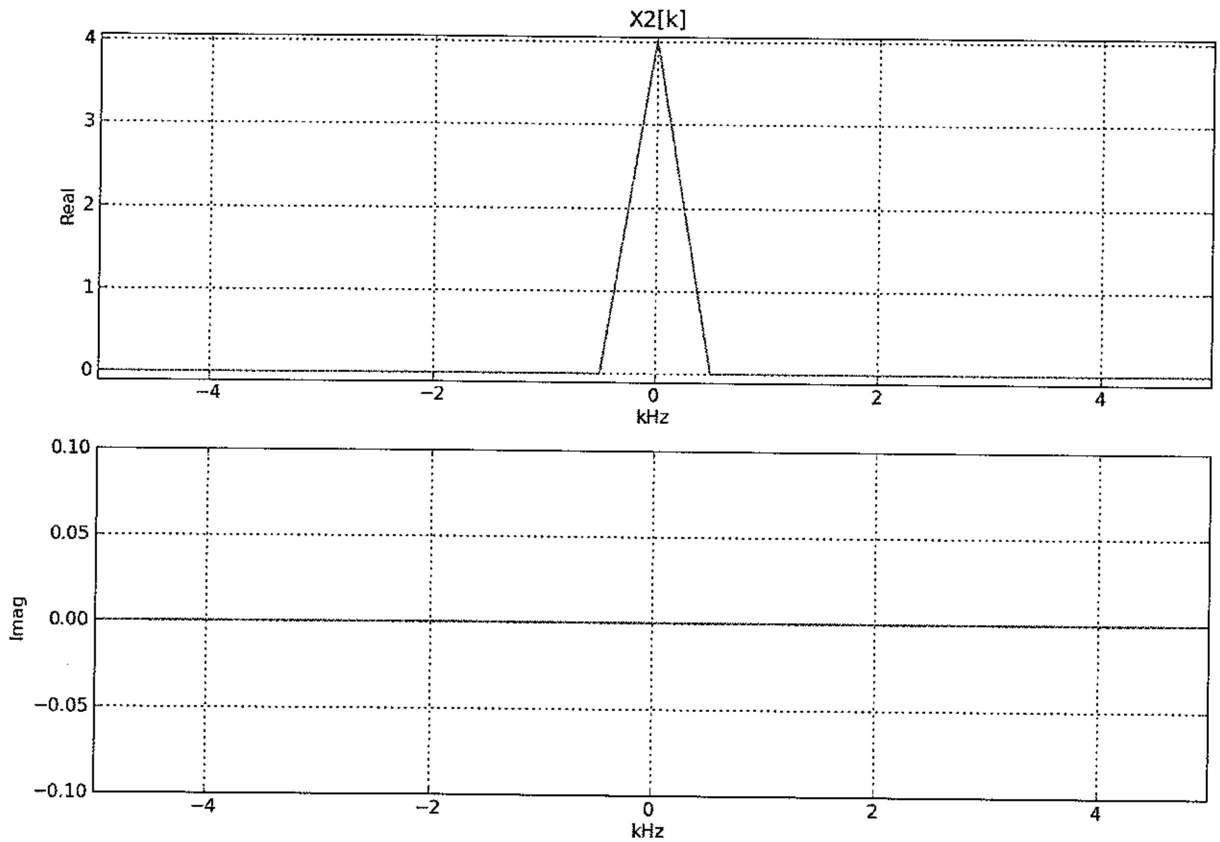


(B) Assuming the DFTS coefficients for the signal $y[n]$ are the same as in part A, on the axes below, please plot the DFT coefficients for the signal x_1 in the modulation/demodulation diagram. Be sure to label key features such as values and coefficient indices for peaks.

Plot of DFTS coefficients for x_1

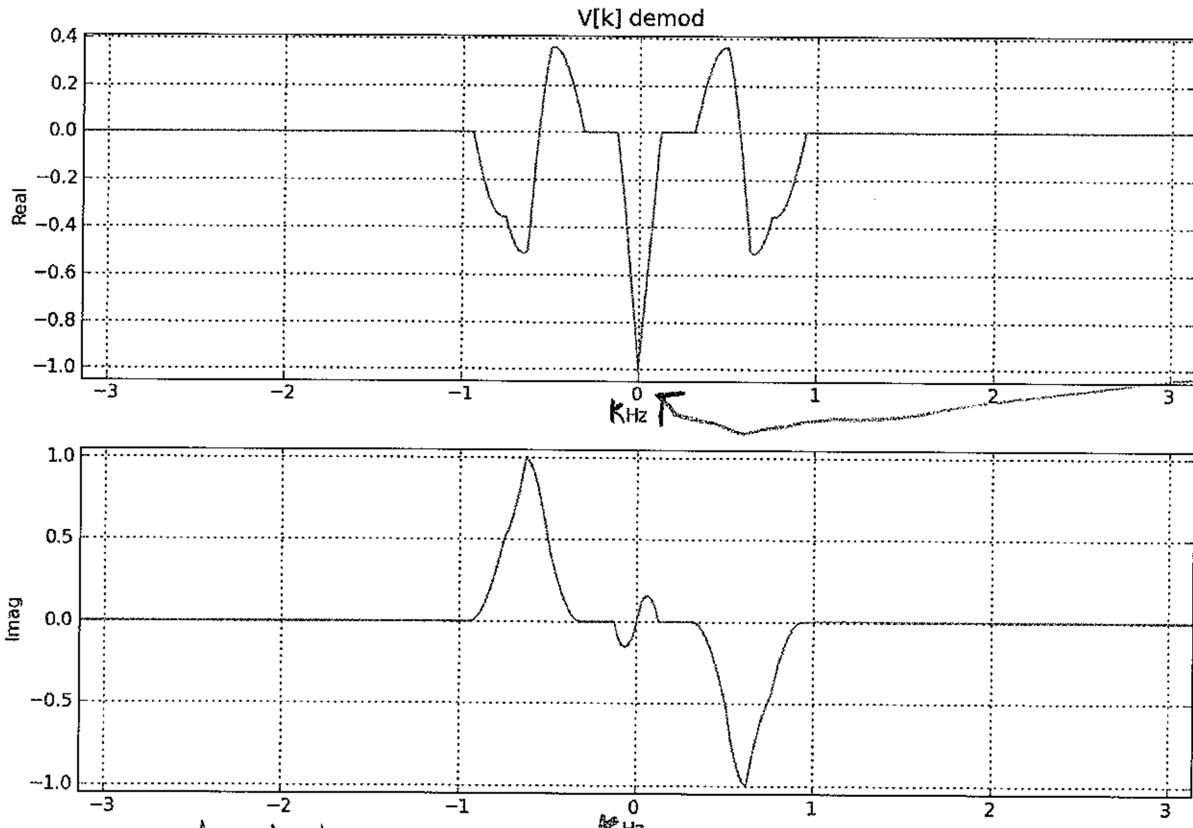


Plot of DFTS coefficients for x_2



(C) If the M-sample delay in the modulation/demodulation diagram has the right number of samples of delay, then it will be possible to nearly perfectly recover $x_I[n]$ by low-pass filtering $v[n]$. Please determine the smallest positive number of samples of delay that are needed and the cut-off frequency for the low-pass filter. Please be sure to justify your answer, using pictures if appropriate.

Plot of DFTS coefficients for v with 5 sample delay



Want M such that

$$\sin\left(K_m \frac{2\pi}{N}(n-M)\right) \simeq \cos\left(K_m \frac{2\pi}{N} n\right)$$

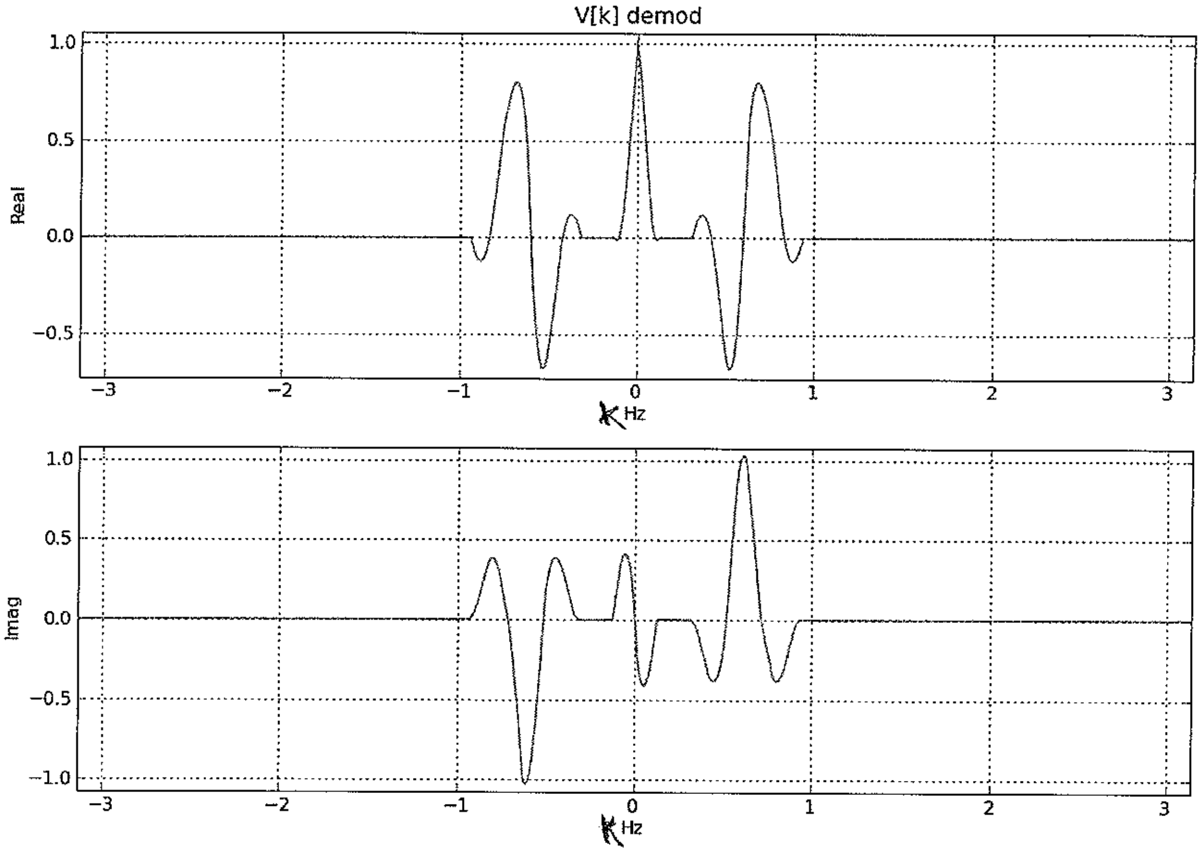
so that cosine demod (V output) will work. With $M = 5$

$$\sin\left(2\pi \frac{500}{10000}(n-5)\right) = \sin\left(\frac{2\pi\, 500}{10000}n - \frac{\pi}{2}\right)$$

$$= -\cos\left(\frac{2\pi\, 500}{10000}\right)$$

M = 5 and an LPF with gain −1 and cutoff at 250 Hz

Plot of DFTS coefficients for v with 15 sample delay



Second Soln $M = 15$

$$\sin\left(k_M \frac{2\pi}{N}(n - M)\right) = \sin\left(\frac{2\pi 500}{10000} n - \frac{3}{2}\pi\right)$$

$$= \cos\left(\frac{2\pi 500}{10000} n\right)$$

$M = 15$ and LPF with gain of 1 with cutoff at 500Hz

Solution to part C continued.

Either

M = 5 LPF with gain of -1
and cutoff at 250Hz

or

M = 15 LPF with gain of 1
and cutoff at 250Hz

Smallest M (number of samples of delay) > 0 = _______________
Cutoff Frequency of Low Pass Filter = _______________