Problem 4. Modulation [35 Points]

In answering the four parts of this question, please refer to the modulation-demodulation system below. For this problem, please assume $N = 128$ and that the sampling frequency, $f_s$, is 128 samples per second.

\[ x_1[n] \rightarrow [\times] \rightarrow \cos f_1 \frac{2\pi}{N} n \]
\[ x_2[n] \rightarrow [\times] \rightarrow \cos f_2 \frac{2\pi}{N} n \]
\[ \cos 25 \frac{2\pi}{N} n \]
\[ \cos f_3 \frac{2\pi}{N} n \]
\[ \text{Ideal Low-Pass Filter} \rightarrow y[n] \]
(A) Below are plots of the real and imaginary parts of the Fourier coefficients versus frequency for point A of the modulation-demodulation system. On the axes below, please plot the real and imaginary parts of the Fourier coefficients versus frequency for the signal at point B, assuming $f_B = 25$. For this problem, you need only plot the Fourier Series coefficients for frequencies in the range $-25$ to $25$. Please be sure to label critical frequencies and values in your graph.
Signal at Point B Fourier Series Coefficients:
(B) Repeated below are plots of the real and imaginary parts of the Fourier coefficients versus frequency for point \( A \) of the modulation-demodulation system (that is, the same plot as in part A). If \( x_1[n] = \alpha + \beta \cos \left( \frac{2\pi}{N} n \right) \) and \( x_2[n] = \frac{1}{2} \cos \left( \frac{2\pi}{N} n \right) + \frac{1}{2} \sin \left( \frac{2\pi}{N} n \right) \), please determine the two modulation frequencies, \( f_1 \) and \( f_2 \), and the two amplitudes, \( \alpha \) and \( \beta \).

\[ f_1 = 10 \]

\[ f_2 = 20 \]

\[ \alpha = 2.0 \quad (4 \times 0.5) \]

\[ \beta = 2.0 \quad (4 \times 0.25) \]
(C) Now suppose \( x_2[n] = 0 \), \( x_1[n] = \frac{1}{2} \cos \left( \frac{2\pi}{N} n \right) + \frac{i}{2} \sin \left( \frac{2\pi}{N} n \right) \), and \( f_1 = 15 \) (not one of the answers to part B!). For what values of \( f_3 < 64 \) (there is more than one) will \( y[n] = x_1[n] \), assuming the low-pass filter has been designed correctly. In addition, what should the magnitude be for the low frequency response of the low-pass filter? Please show your reasoning, with pictures if needed.

\[
\begin{align*}
X[n] & \quad \ast \quad \ast \quad \ast \quad \ast \\
\cos \frac{15}{N} & \quad \cos \frac{25}{N} & \quad \cos \frac{f_2}{N}
\end{align*}
\]

Values for \( f_3 = 10 \) and \( 40 \)

Magnitude of the low-pass filter's low frequency response = 4

End of Quiz 2!