MAC PROTOCOLS: UTILIZATION, THROUGHPUT

we do analytical calculations for the setups without stabilization algorithms (i.e. \( p = \text{const} \)) with stabilization, we resort to simulation

"UN" SLOTTED ALOHA

\[ \text{slots} \]

\[ 1 \text{ packet} = T \text{ slots} \]

\[ p = 1P \left( \text{start sending a packet now} \right) \]

\[ \text{at a given node} \]

\[ 1P \left( \text{a packet is sent out at time } n \right) \text{ with no collision} \]

\[ = \sum_{i \in \text{backlogged nodes}} \sum_{k=0}^{T-1} 1P \left( \text{packet started at node } i \text{ at time } n-k, \text{ no packets started at other nodes at times } n-k-T+1, n-k-T+2, \ldots, n-k+T-1 \right) \]

\[ \geq N T P \left( 1-P \right) \]

\[ \text{"=} \text{ for "compulsive-obsessive" nodes, which keep starting new packets even while transmitting older ones} \]

MAXIMIZATION w.r.t. "\( p \)"

\[ \log (\ldots) = \text{const} + \log (p) + [(2T-1)N-1] \log (1-p) \]

DERIVATIVE:

\[ \frac{1}{p} - \frac{(2T-1)N-1}{1-p} \quad (= 0 \text{ at } p = \frac{1}{(2T-1)N}) \]
"un"slotted Aloha (continued)

Maximal IP (doing something useful at a given slot)

\[ IP = NT \frac{1}{(2T-1)N} \left( 1 - \frac{1}{(2T-1)N} \right)^{(2T-1)N-1} \rightarrow \frac{1}{2e} \quad \text{as } T \to \infty \]

(Reasonable: twice the number of collision if compared to slotted Aloha)

Ex. 1 (#5 in lecture 10/11 notes)

Calculating throughput when 2-way collisions are OK (but 3-way gets everything lost)

\( N \) backlogged nodes

\[ IP(\text{send}) = p \]

IP (getting exactly 1 packet in a given slot)

\[ IP(\text{get exactly 1 packet in some slot}) = Np(1-p)^{N-1} \]

IP (get exactly 1 packet in some slot)

\[ IP(\text{get exactly 2 packets from two given slots}) = p^2 (1-p)^{N-2} \]

IP (get exactly 2 packets from some slots)

\[ = \binom{N}{2} p^2 (1-p)^{N-2} = \frac{N(N-1)}{2} p^2 (1-p)^{N-2} \]

Average throughput

\[ = 1 \cdot Np(1-p)^{N-1} + 2 \cdot \frac{N(N-1)}{2} p^2 \]

Max. throughput = ? (either 1 or 2)
\[
\text{UTILIZATION} = \frac{\text{AVG. THROUGHPUT}}{\text{MAX. THROUGHPUT}}
\]

Ex. 2

100 nodes
20 backlogged
Token-passing TDMA MAC protocol

\[
\frac{\text{DATA PACKET TIME}}{\text{TOKEN TIME}} = 3
\]

"Trivial":

Time around: \(100 \cdot \text{tt} + 20 \cdot 3 \cdot \text{tt}\)

Packets sent: 20

\[
\frac{20}{160 \cdot \text{tt}}
\]

Throughput

"Token case:

Time around: \(100 \cdot 3 \cdot \text{tt}\)

Packets sent: 20

\[
\frac{20}{300 \cdot \text{tt}}
\]

Throughput

\(~ \frac{300}{160} \) times better.