Ex. 1: Eye diagram from bit response + bit period
(+, possibly bits reference)

Bits sent → 1 1 0 1 0 0 1 1 1 0 0 0 0 0 0 0 1

⇒ Received signal

⇒ Bit period

Ex. 2: Eye diagram from step response
(see the 14.09.2010 rec. handout)

Given: Step response

\[ \text{Given: Step response} \]

\[ \text{Bit period: } T = D \]

Possible responses over a bit period:

- "0" (after "1")
- "1" (after "0")
- "0" (after "0")
- "1" (after "1")
Ex. 2 (cont.)

Eye diagram over 3 bit periods

Ex. 3 Eye diagram for step response from Ex. 2 with \( T = D/2 \) (see 14.09.2010)

Some things do look like eye openings, but there are really none.

Unit sample response

By definition: response to \( x[n] = \delta[n] = \begin{cases} 1, & n=0 \\ 0, & n \neq 0 \end{cases} \)

Ex. 4 If unit step response is this:

\[ m[n] = \]

What is the unit sample response \( h[n] = ? \)

Math: \( h[n] = m[n] - m[n-1] \)

Answer: \( \ldots \)

\[ \begin{array}{c}
  0 \\
  -2 \\
  2 \\
  3 \\
  -D \\
  2D \\
  -D \\
  \vdots \\
  \ldots \\
  \end{array} \]
CONVOLUTION

\[ y = h * x \quad \text{means} \quad y[n] = \sum_{i} h[i]x[n-i] = \sum_{i} h[n-i]x[i] \]

This is what LTI systems do!

\[ x[n] \rightarrow [h[n]] \rightarrow y = h * x \]

"Sanity Checks":

If \( x[n] = 0 \) outside \( \{ N_x, N_x+1, \ldots, N_x+L_x-1 \} \)
\( h[n] = 0 \) outside \( \{ N_h, N_h+1, \ldots, N_h+L_h-1 \} \)

then \( y[n] = 0 \) outside \( \{ N_x+N_h, N_x+N_h+1, \ldots, N_x+N_h+L_x+L_h-2 \} \)

i.e. \( \text{BEGINNING} \ (x * h) = \text{BEGINNING} \ (x) + \text{BEGINNING} \ (h) \)
\( \text{END} \ (x * h) = \text{END} \ (x) + \text{END} \ (h) \)
\( \text{LENGTH} \ (x * h) = \text{LENGTH} \ (x) + \text{LENGTH} \ (h) \)

"Width"\

\[ [1, 2] \text{ start at 5} \quad * \quad [-1, 1] \text{ start at 7} = \]

\[ (-1)[1, 2] = \begin{bmatrix} -1 & -2 \\ \end{bmatrix} \]

\[ 1 \times [1, 2] = \begin{bmatrix} 1 & 2 \\ \end{bmatrix} \]

\[ [-1, 1] \times [1, 2] = \begin{bmatrix} -1 & -1 & 2 \\ \end{bmatrix} \]

**Answer:** \( [-1, -1, 2] \) start at 12

\[ \begin{array}{c}
5 \\
6 \\
\hline
1 \times 0 \\
\hline
20 \\
\hline
\end{array} \]
**Deconvolution:** Given \( y[n] \) and \( h[n] \), find \( x[n] \) such that \( y = h \ast x \)

If width \( h \) = \( m \), i.e., \( h[n] = 0 \) for \( n < 0 \) or \( n > m \)

\[ y[n] = h[0]x[n] + h[1]x[n-1] + \ldots + h[m]x[n-m] \]

\[ x[n] = \frac{1}{h[0]} \left( y[n] - h[1]x[n-1] - h[2]x[n-2] - \ldots - h[m]x[n-m] \right) \]

Ex. \( 6 \)

\[ h[n] = 2\delta[n] + \delta[n-1] \]

\[ y[n] = \delta[n] \]

\[ x[n] = ? \]

\[ x[0] = \frac{1}{2} (1 - 1 \cdot 0) = \frac{1}{2} \]

\[ x[1] = \frac{1}{2} (0 - 1 \cdot \frac{1}{2}) = -\frac{1}{4} \]

\[ x[2] = \frac{1}{2} (0 - 1 \cdot \frac{1}{4}) = \frac{1}{8} \]

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Ex. \( 7 \)

\[ h[n] = \delta[n] + 2\delta[n-1] \]

\[ y[n] = \delta[n] \]

\[ x[n] = \] (failure)

\[ x[0] = 1 - 2 \cdot 0 = 1 \]

\[ x[1] = 0 - 2 \cdot 1 = -2 \]

\[ x[2] = 0 - 2 \cdot (-2) = 4 \]

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Ex. \( 8 \)

\[ h[n] = \delta[n] + 2\delta[n-1] \]

\[ y[n] = \delta[n] + 3\delta[n-1] + 2\delta[n-2] \]

\[ x[n] = \] (noise amplification)

\[ x[0] = 1 - 2 \cdot 0 = 1 \]

\[ x[1] = 3 - 2 \cdot 1 = 1 \]

\[ x[2] = 2 - 2 \cdot 1 = 0 \]

\[ x[3] = 0 - 2 \cdot 0 = 0 \]

**Noise Amplification**

**Instability**