Rec 16
ADDENDUM: Some figures to help you remember how modulation operates in the DTFS (i.e., spectral) domain:

N even, $R_c = \frac{2\pi}{N}$, $R_1 = \frac{k_1 2\pi}{N}$

Carrier frequency

(Real) Time function $\leftrightarrow \frac{1}{N} \sum_k \mathcal{X}[k]$; $R_c = \frac{k_c 2\pi}{N}$

$\cos R_c n \leftrightarrow \begin{pmatrix} \frac{1}{2} & j\frac{1}{2} \end{pmatrix} + j \begin{pmatrix} 0 \end{pmatrix}$
$\sin R_c n \leftrightarrow \begin{pmatrix} 0 \end{pmatrix} + j \begin{pmatrix} -\frac{1}{2} & \frac{1}{2} \end{pmatrix}$

So:

$j \cos R_c n \leftrightarrow \begin{pmatrix} 0 \end{pmatrix} + j \begin{pmatrix} -\frac{1}{2} & \frac{1}{2} \end{pmatrix}$

$j \sin R_c n \leftrightarrow \begin{pmatrix} \frac{1}{2} \end{pmatrix} + j \begin{pmatrix} 0 \end{pmatrix}$

These 4 cases set the pattern for determining the DTFS of a modulated signal, as we now see...
Suppose first that
\[ x[n] \leftrightarrow \left( \begin{array}{c}
A \\
0
\end{array} \right) + j \left( \begin{array}{c}
0
\end{array} \right) \]
i.e., a purely real spectrum. Then it turns out that the DTFS's of the modulated signals \( x[n] \cos \omega_c n \) and \( x[n] \sin \omega_c n \) have a structure that parallels the upper two figures on p.1:

\[ x_R[n] \cos \omega_c n \leftrightarrow \left( \begin{array}{c}
A_2 \\
-2c
\end{array} \right) + j \left( \begin{array}{c}
0
\end{array} \right) \]
\[ x_R[n] \sin \omega_c n \leftrightarrow \left( \begin{array}{c}
0
\end{array} \right) + j \left( \begin{array}{c}
A_2 \\
-2c
\end{array} \right) \]

If on the other hand
\[ x_I[n] \leftrightarrow \left( \begin{array}{c}
0
\end{array} \right) + j \left( \begin{array}{c}
B
\end{array} \right) \]
i.e., a purely imaginary spectrum, then it's the case that the DTFS's of the modulated signals \( x_I[n] \cos \omega_c n \) and \( x_I[n] \sin \omega_c n \) have a structure that parallels the lower two figures on p.1:
\[ x[n] \cos \omega_c n \leftrightarrow \Re \{ X[k] \} + j \Im \{ X[k] \} \]
\[ x[n] \sin \omega_c n \leftrightarrow -j \Re \{ X[k] \} + \Re \{ X[k] \} \]

A general \( x[n] \) can be written as
\[ x[n] = x_R[n] + j x_I[n] \]
for some \( x_R[n] \) that has a purely real spectrum and \( x_I[n] \) that has a purely imaginary spectrum:

\[
\begin{bmatrix}
\Re \{ X[R][k] \} \\
\Im \{ X[R][k] \}
\end{bmatrix} = 
\begin{bmatrix}
\Re \{ X[I][k] \} \\
\Im \{ X[I][k] \}
\end{bmatrix} = 
\begin{bmatrix}
\Re \{ X[R][k] \} \\
\Im \{ X[R][k] \}
\end{bmatrix}
\]

So
\[ x[n] \cos \omega_c n = x_R[n] \cos \omega_c n + x_I[n] \cos \omega_c n \]
and
\[ x[n] \sin \omega_c n = x_R[n] \sin \omega_c n + x_I[n] \sin \omega_c n \]

and we know how to do all four of these!