

# Time ↔ "Relationships" ↔ Frequency

<p style="text-align: center;"><i>"Decomposition"</i></p> $x[n] = \sum_{k=\langle N \rangle} a_k e^{j(\frac{2\pi k}{N})n}$ <p>Must know</p> <p><math>x[n]</math> has <math>N</math> samples /periodic in <math>N</math></p> <p><math>x_1[n] + x_2[n]</math></p> <p><math>\alpha \cdot x[n]</math></p>	<p style="text-align: center;"><i>"Get Freq. coefs."</i></p> $a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-j(\frac{2\pi k}{N})n}$ <p><math>a_k = a_{k+N}</math> "periodic in <math>N</math>"</p> <p><math>a_k = b_k + c_k</math> "Linear"</p> <p><math>x_1[n] \leftrightarrow b_k, x_2[n] \leftrightarrow c_k</math></p> <p><math>\alpha \cdot a_k</math> "Linear"</p>
<p style="text-align: center;">Properties</p> <p><math>x[n]</math> is real</p> <p><math>x[n]</math> real and <math>x[n] = x[-n]</math></p> <p><math>x[n - \alpha]</math></p> <p><math>x[n] \cdot e^{j(\frac{2\pi \alpha}{N})n}</math></p>	<p><math>a_k = a_{-k}^*</math> "Conjugate Sym."</p> <p><math>a_k</math>'s are real</p> <p><math>a_k \cdot e^{-j(\frac{2\pi k}{N})\alpha}</math> "Time Shift"</p> <p><math>a_{k-\alpha}</math> "Freq. Shift"</p>
<p style="text-align: center;">Examples</p> <p><math>x[n] = 1</math></p> <p><math>x[n] = \delta[n]</math></p> <p><math>x[n] = \cos\left(\left(\frac{2\pi \ell}{N}\right)n\right)</math></p> <p><math>x[n] = \sin\left(\left(\frac{2\pi \ell}{N}\right)n\right)</math></p>	<p><math>a_k = \delta[k]</math> "DC-constant"</p> <p><math>a_k = \frac{1}{N}</math> "unit sample"</p> <p><math>a_k = \frac{1}{2}\delta[\ell] + \frac{1}{2}\delta[-\ell]</math></p> <p><math>a_k = \frac{1}{2j}\delta[\ell] - \frac{1}{2j}\delta[-\ell]</math></p>

$$e^{j\left(\frac{2\pi k}{N}\right)n} = \cos\left(\left(\frac{2\pi k}{N}\right)n\right) + j \sin\left(\left(\frac{2\pi k}{N}\right)n\right)$$

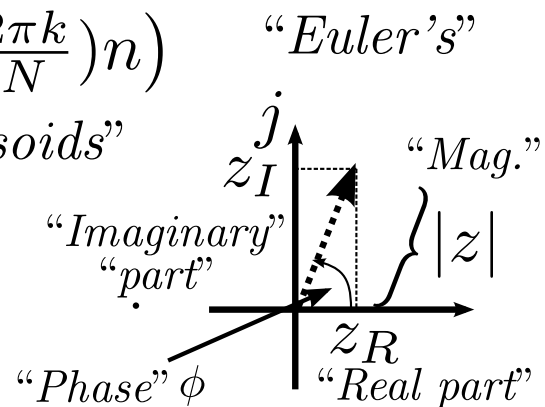
*"Freq."*  
(fraction of  $2\pi$ )

*"discrete-time sinusoids"*

*"A complex number has 4 parts"*

$$z = z_R + jz_I = |z|e^{j\phi}$$

$$|z| \cos(\phi) \quad |z| \sin(\phi)$$



*"Conjugation"*

$$z^* = z_R - jz_I = |z|e^{-j\phi}$$

$ z ^2 = z_R^2 + z_I^2$	$\phi = \tan^{-1}\left(\frac{z_I}{z_R}\right)$
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