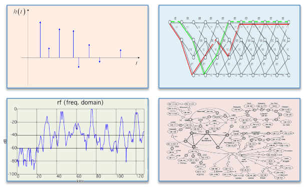


INTRODUCTION TO EECS II

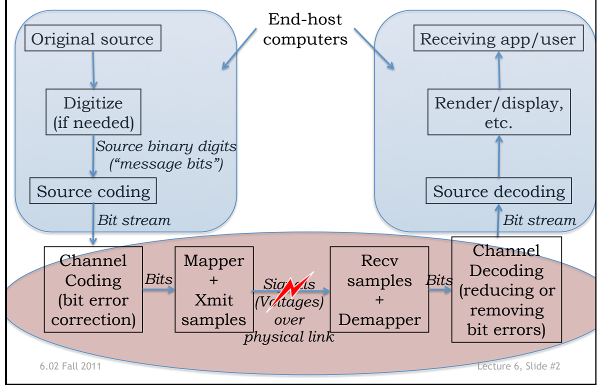
DIGITAL COMMUNICATION SYSTEMS

6.02 Fall 2011
Lecture #4

- Noise: bad things happen to good signals!
- Additive white Gaussian noise (AWGN)
- Bit error rate analysis
- Signal-to-noise ratio and decibel (dB) scale
- Binary symmetric channel (BSC) abstraction

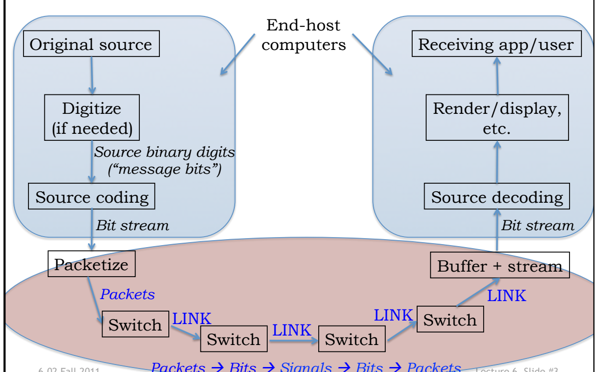
6.02 Fall 2011 Lecture 6, Slide #1

Single Link Communication Model



6.02 Fall 2011 Lecture 6, Slide #2

Network Communication Model
Three Abstraction Layers: Packets, Bits, Signals

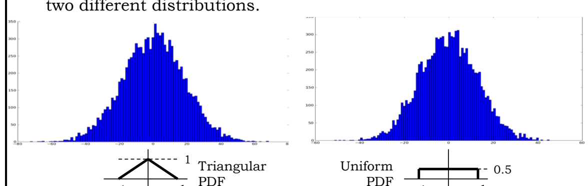


6.02 Fall 2011 Lecture 6, Slide #3

Noise on a Communication Channel

The net noise observed at the receiver is often the sum of many small, independent random contributions from many factors. If these independent random variables have finite mean and variance, the Central Limit Theorem says their sum will be a *Gaussian*.

The figure below shows the histograms of the results of 10,000 trials of summing 100 random samples draw from [-1,1] using two different distributions.

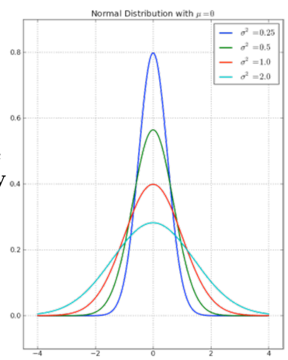


6.02 Fall 2011 Lecture 6, Slide #4

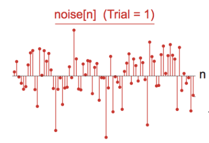
The Gaussian Distribution

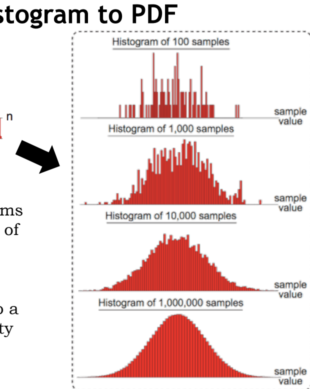
A Gaussian distribution with mean μ and variance σ^2 has a PDF described by

$$f_x(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$



From Histogram to PDF





Experiment: create histograms of sample values from trials of increasing lengths.

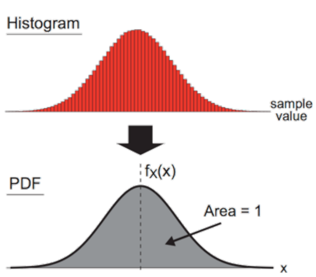
If distribution is stationary, then histogram converges to a shape known as a probability density function (PDF)

Formalizing the PDF Concept

Define x as a random variable whose PDF has the same shape as the histogram we just obtained.

Denote the PDF of x as $f_x(x)$ and scale $f_x(x)$ such that its overall area is 1:

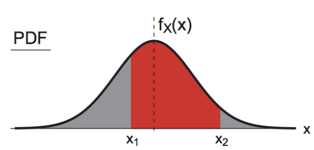
$$\int_{-\infty}^{\infty} f_x(x)dx = 1$$



Formalizing Probability

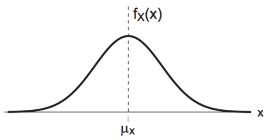
The probability that random variable *x* takes on a value in the range of x_1 to x_2 is calculated from the PDF of x as:

$$p(x_1 \le x < x_2) = \int_{x_1}^{x_2} f_x(x)dx$$



A PDF is NOT a probability – its integral is.
Note that probability values are always in the range of 0 to 1.

Mean and Variance



The *mean* of a random variable x, μ_x, corresponds to its average value and computed as:

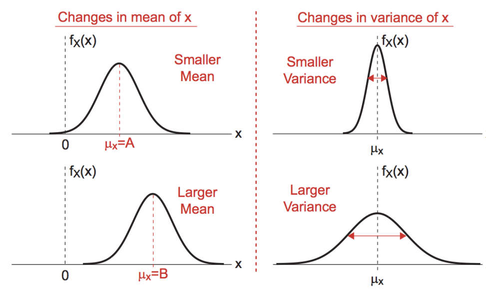
$$\mu_x = \int_{-\infty}^{\infty} x\, f_x(x)dx$$

The *variance* of a random variable x, σ_x^2, gives an indication of its variability and is computed as:

$$\sigma_x^2 = \int_{-\infty}^{\infty} (x-\mu_x)^2 f_x(x)dx$$

Compare with power calculation

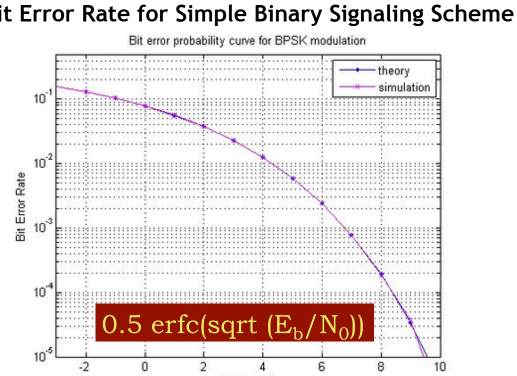
Visualizing Mean and Variance



Changes in mean shift the center of mass of PDF

Changes in variance narrow or broaden the PDF (but area is always equal to 1)

Bit Error Rate for Simple Binary Signaling Scheme



http://www.dsplog.com/2007/08/05/bit-error-probability-for-bpsk-modulation/

Signal-to-Noise Ratio (SNR)

The Signal-to-Noise ratio (SNR) is useful in judging the impact of noise on system performance:

$$\text{SNR} = \frac{\tilde{P}_{signal}}{\tilde{P}_{noise}}$$

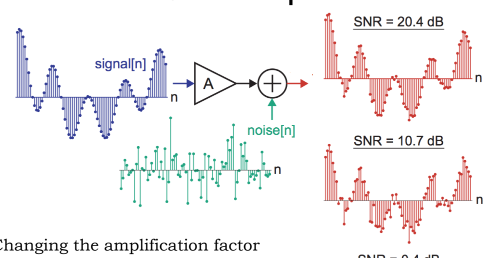
SNR is often measured in decibels (dB):

$$\text{SNR (db)} = 10\log\left(\frac{\tilde{P}_{signal}}{\tilde{P}_{noise}}\right)$$

10logX	X
100	10000000000
90	1000000000
80	100000000
70	10000000
60	1000000
50	100000
40	10000
30	1000
20	100
10	10
0	1
-10	0.1
-20	0.01
-30	0.001
-40	0.0001
-50	0.000001
-60	0.0000001
-70	0.00000001
-80	0.000000001
-90	0.0000000001
-100	0.00000000001

3db is a factor of 2

SNR Example



Changing the amplification factor (gain) A leads to different SNR values:

- Lower A → lower SNR
- Signal quality degrades with lower SNR