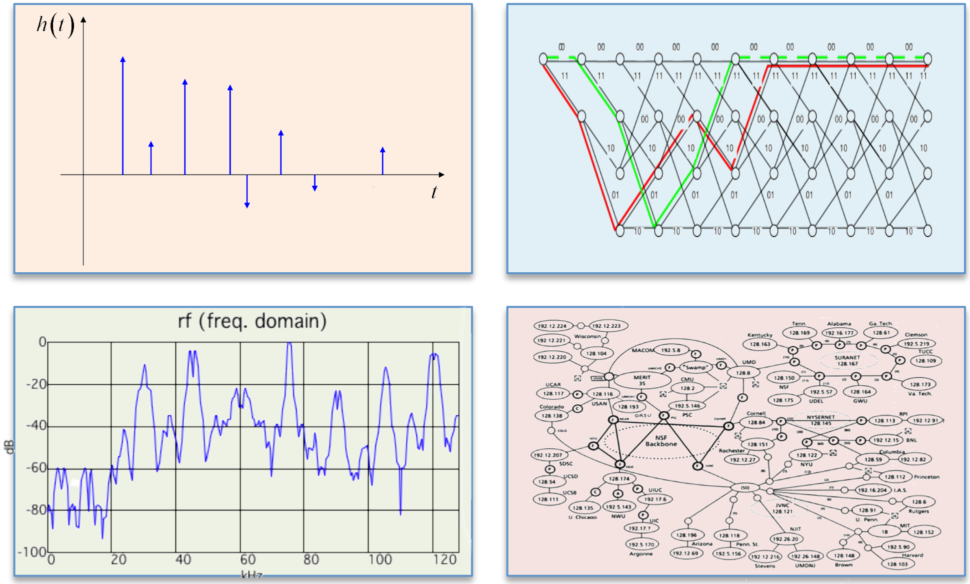


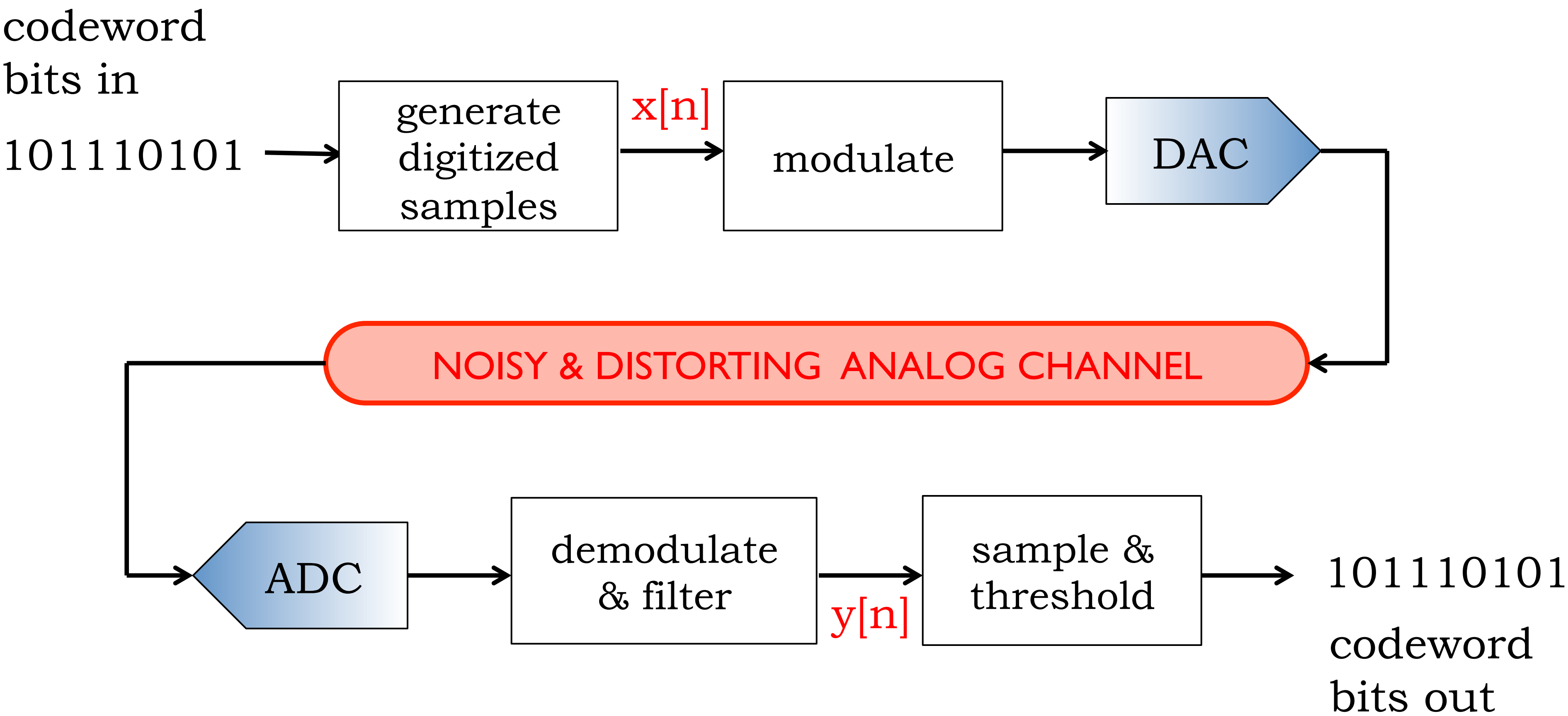
INTRODUCTION TO EECS II

DIGITAL COMMUNICATION SYSTEMS

6.02 Fall 2011
Lecture #16

- More on modulation and demodulation, FDM
- Effects of phase errors and channel delays
- Quadrature components
- Brief comments on: pulse shaping; spectral content of noise; optimal filtering at receiver

From Baseband to Modulated Signal, and Back

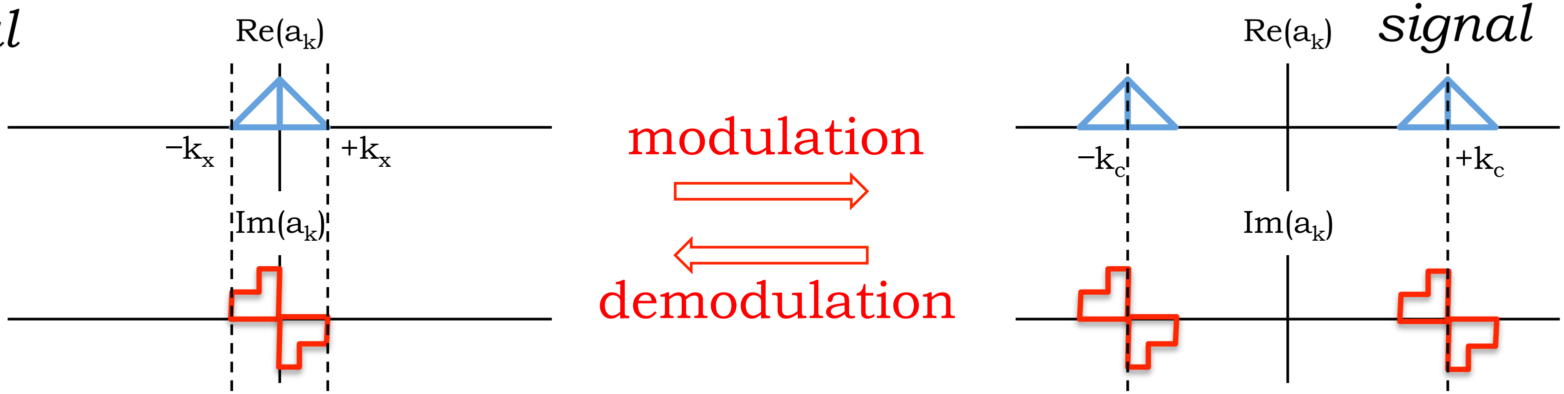


Using Some Piece of the Spectrum

- You have: a band-limited signal x[n] at *baseband* (i.e., centered around 0 frequency).
- You want: the same signal, but centered around some specific frequency $k_c\Omega_1$.
- Modulation: convert from baseband up to $k_c\Omega_1$.
- Demodulation: convert from $k_c\Omega_1$ down to baseband

Spectrum of ***baseband*** *signal*

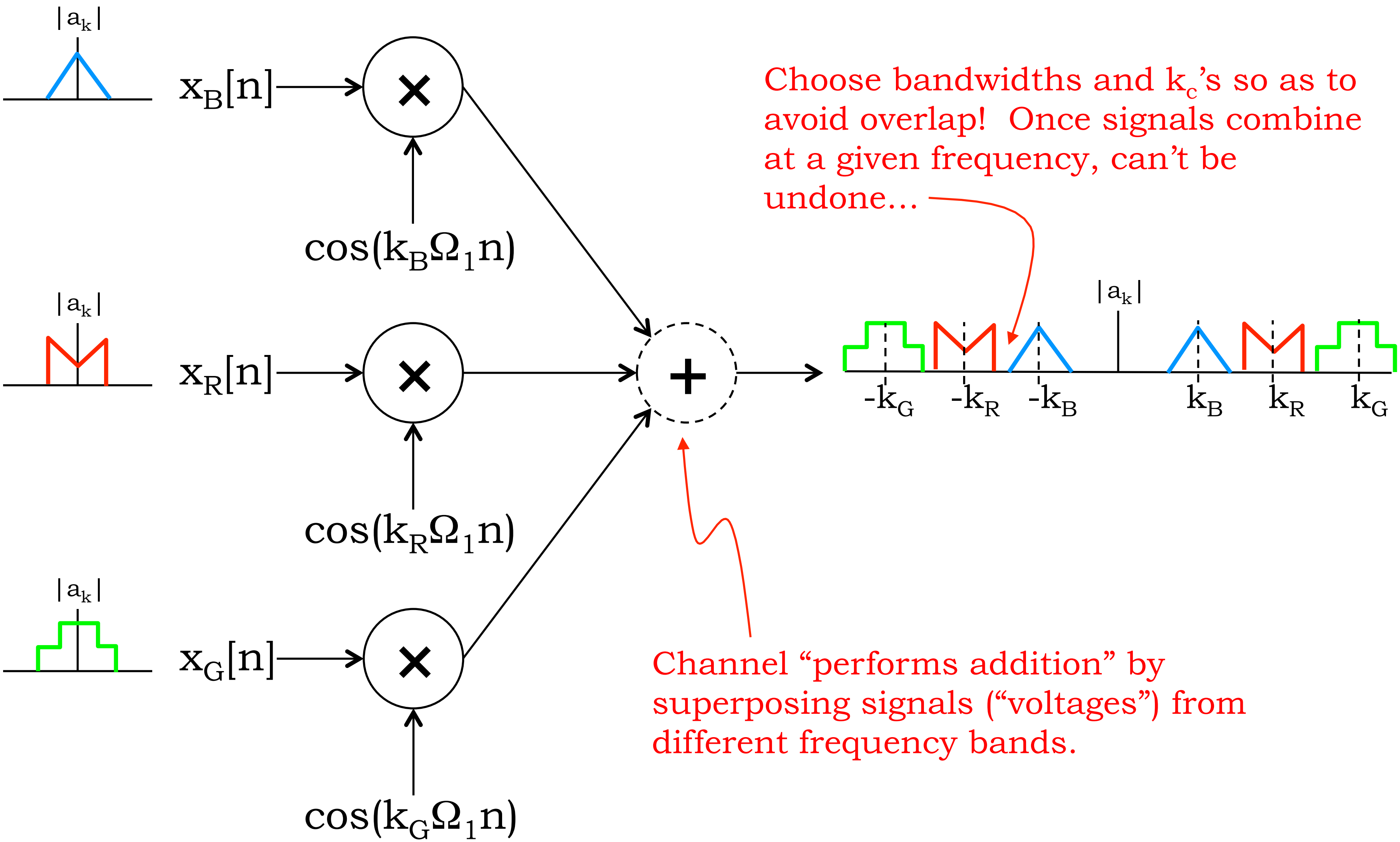
Spectrum of ***transmitted*** *signal*



Signal centered at 0

Signal centered at k_c

Multiple Transmitters: *Frequency Division Multiplexing (FDM)*



Choose bandwidths and k_c's so as to avoid overlap! Once signals combine at a given frequency, can't be undone…

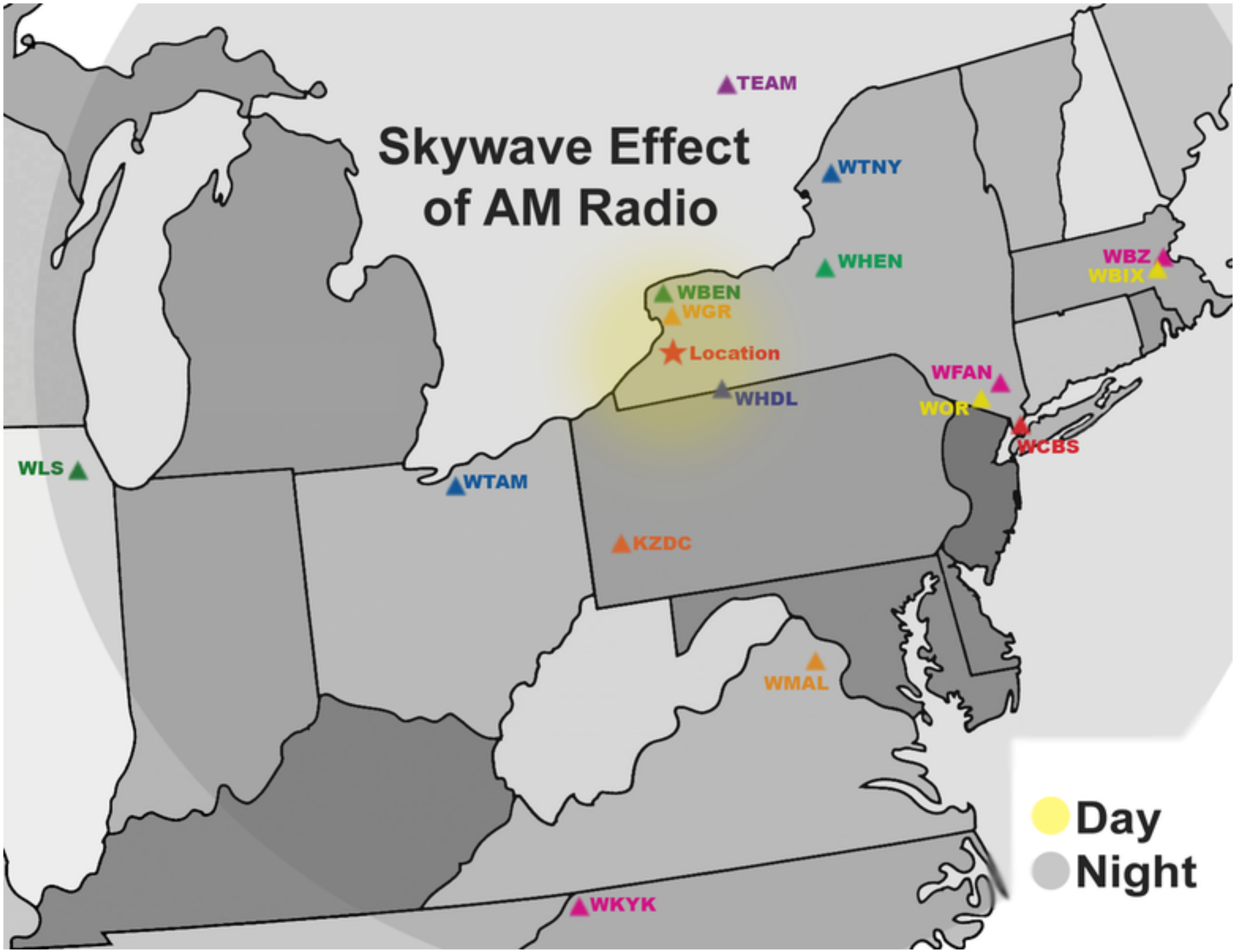
Channel "performs addition" by superposing signals ("voltages") from different frequency bands.

AM Radio

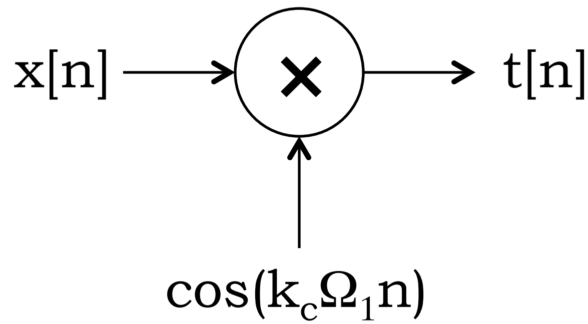
AM radio stations are on 520 – 1610 kHz ('medium wave") in the US, with carrier frequencies of different stations spaced 10 kHz apart.

Physical effects very much affect operation. e.g., EM signals at these frequencies propagate much further at night (by "skywave" through the ionosphere) than during the day (100's of miles by "groundwave" diffracting around the earth's surface), so transmit power may have to be lowered at night!

AM Radio


Skywave Effect
of AM Radio
TEAM
WTNY
WHEN
WBEN
WGR
Location
WHDL
WFAN
WOR
WCBS
WBZ
WBIX
WLS
WTAM
KZDC
WMAL
WKYK
Day
Night

Modulation

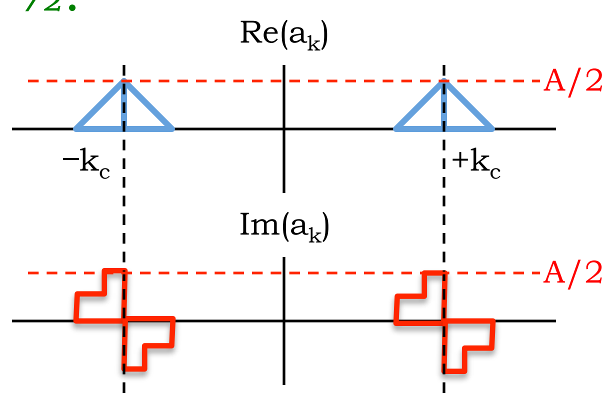


For band-limited signal A_k are nonzero only for small range of $\pm k$

$$t[n] = \left[\sum_{k=-k_x}^{k_x} A_k e^{jk\Omega_1 n}\right]\left[\frac{1}{2}e^{jk_c\Omega_1 n} + \frac{1}{2}e^{-jk_c\Omega_1 n}\right]$$

$$= \frac{1}{2}\sum_{k=-k_x}^{k_x} A_k e^{j(k+k_c)\Omega_1 n} + \frac{1}{2}\sum_{k=-k_x}^{k_x} A_k e^{j(k-k_c)\Omega_1 n}$$

i.e., just replicate baseband signal at $\pm k_c$, and scale by ½.



"Heterodyne principle" (Fessenden)

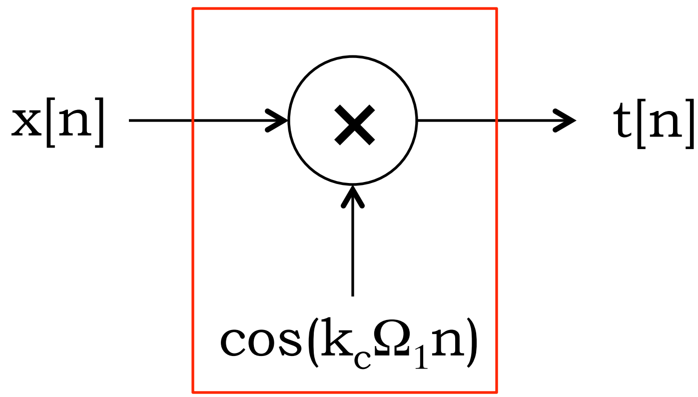
$$\cos(\Omega_{\text{mod}} n).\cos(\Omega_{car} n)$$

$$=$$

$$0.5\{\cos[(\Omega_{car} + \Omega_{\text{mod}})n] + \cos[(\Omega_{car} - \Omega_{\text{mod}})n]\}$$

Multiplying two sinusoids causes the
sum and difference frequencies to appear.

Is Modulation Linear? Time-Invariant? …



… as a system that takes input x[n] and produces output t[n]?

Yes, linear!

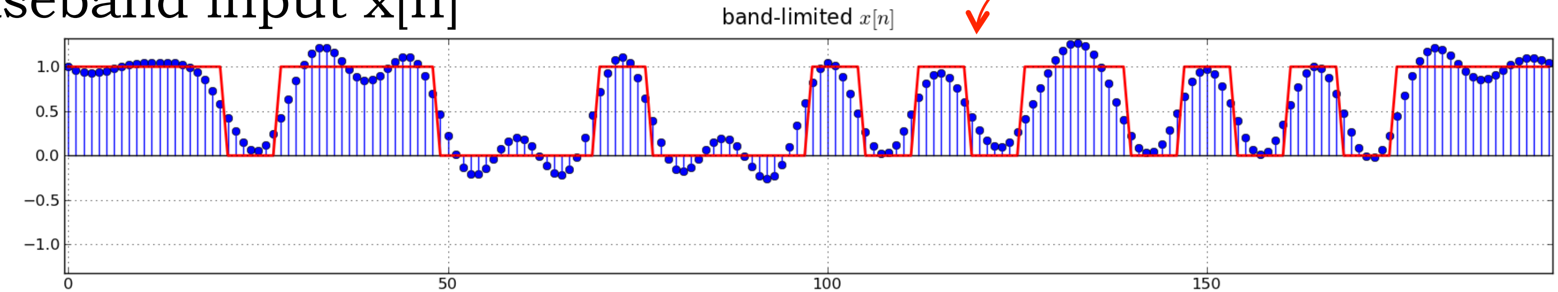
No, not time-invariant!

Apply definitions from Ch. 10 notes

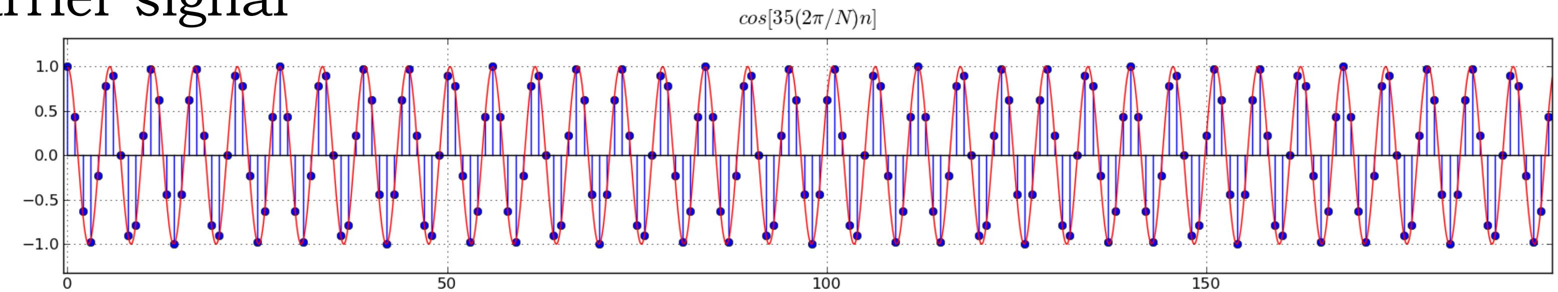
Example: Modulation (time)

Shaped pulses! Chosen because we know the channel is bandlimited

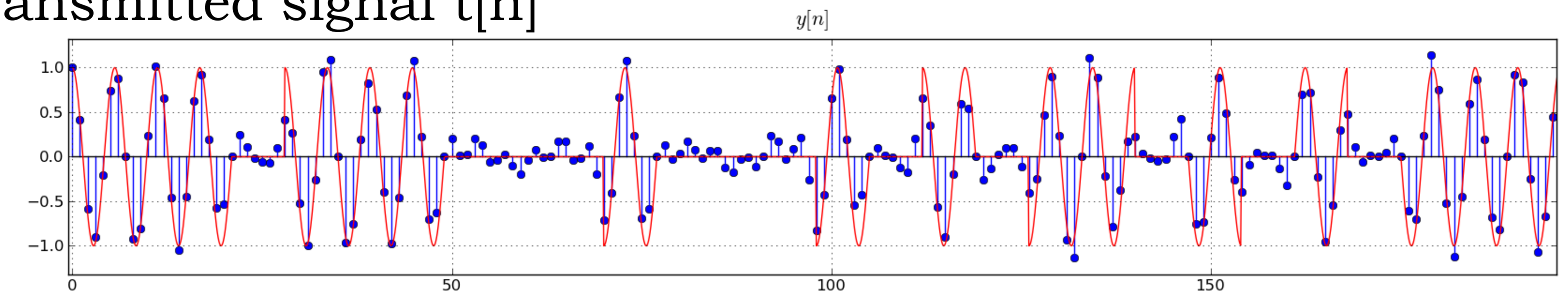
Baseband input x[n]



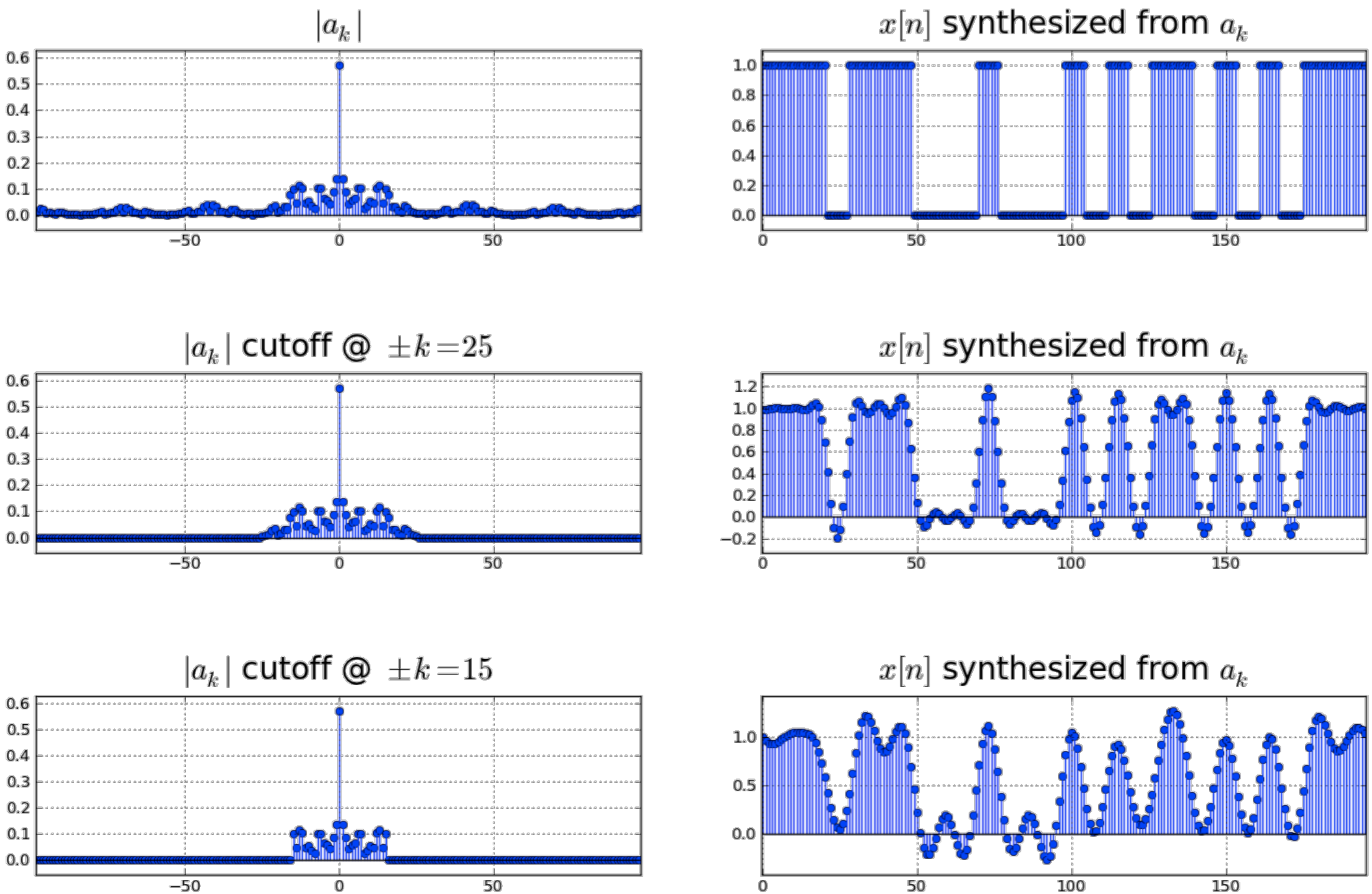
Carrier signal



Transmitted signal t[n]

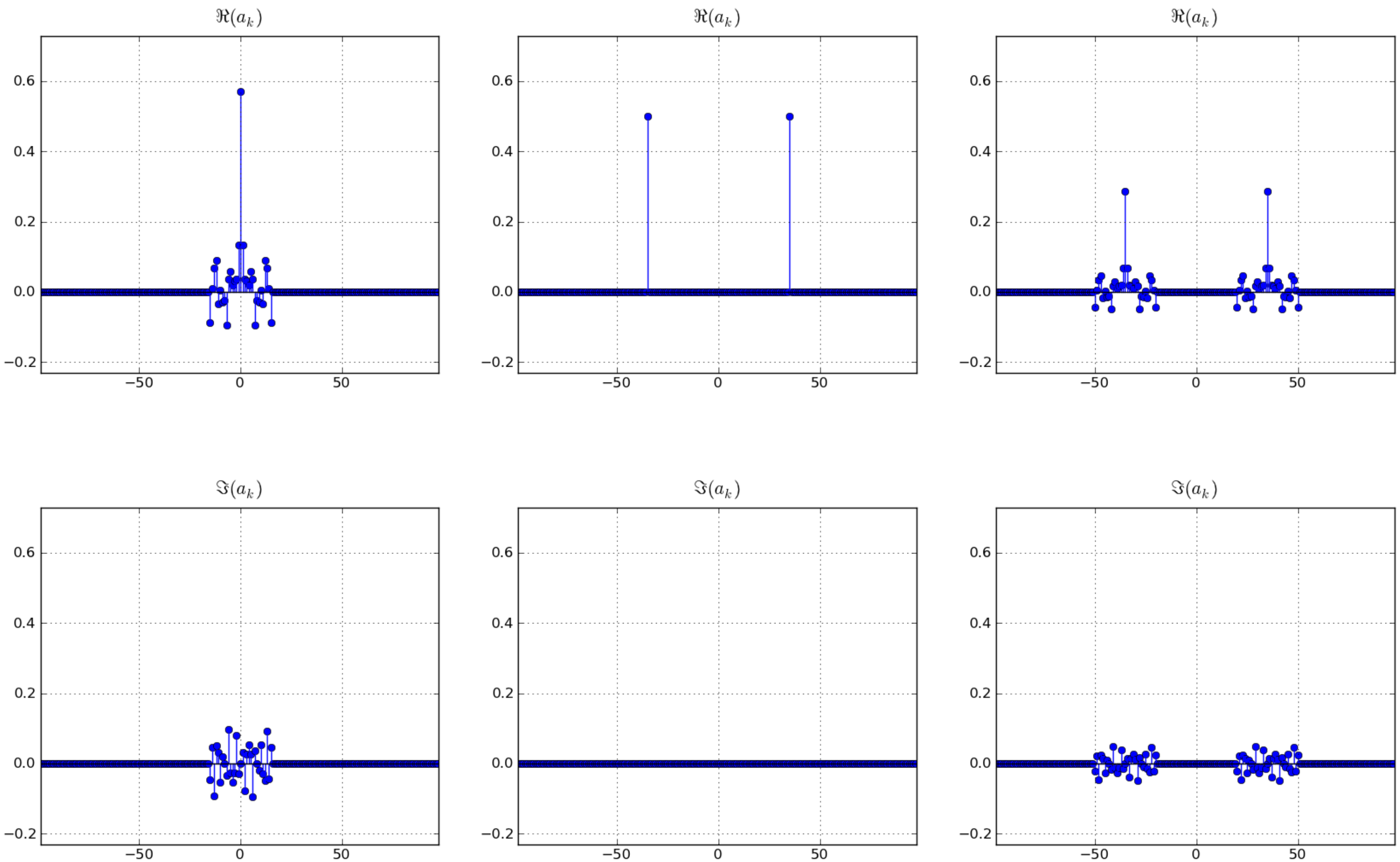


Effect of Band-limiting a Transmission



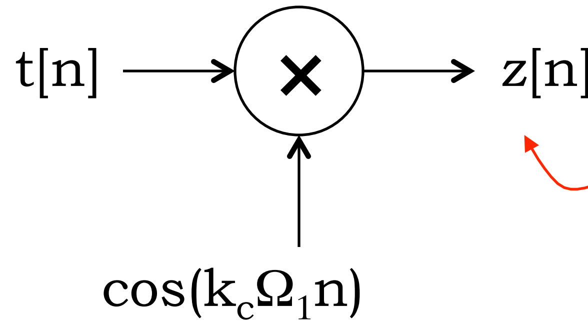
Example: Modulation (freq domain picture)

Band-limited x[n] $\cos(35\Omega_1 n)$ t[n]



Demodulation

Assuming no distortion or noise on channel, so what was transmitted is received



Hmm. So z[n] has what we want at baseband, but has signal we don't want at $\pm 2k_c\Omega_1$

$$z[n] = t[n]\left[\frac{1}{2}e^{jk_c\Omega_1 n} + \frac{1}{2}e^{-jk_c\Omega_1 n}\right]$$

$$= \left[\frac{1}{2}\sum_{k=-k_x}^{k_x} A_k e^{j(k+k_c)\Omega_1 n} + \frac{1}{2}\sum_{k=-k_x}^{k_x} A_k e^{j(k-k_c)\Omega_1 n}\right]\left[\frac{1}{2}e^{jk_c\Omega_1 n} + \frac{1}{2}e^{-jk_c\Omega_1 n}\right]$$

$$= \frac{1}{4}\sum_{k=-k_x}^{k_x} A_k e^{j(k+2k_c)\Omega_1 n} + \underbrace{\frac{1}{2}\sum_{k=-k_x}^{k_x} A_k e^{jk\Omega_1 n}}_{\text{What we want}} + \frac{1}{4}\sum_{k=-k_x}^{k_x} A_k e^{j(k-2k_c)\Omega_1 n}$$

What we want

That is just Fessenden's heterodyne principle at work again:

Taking the sum and difference frequencies of

$$\Omega_c$$

with the sum and difference frequencies of

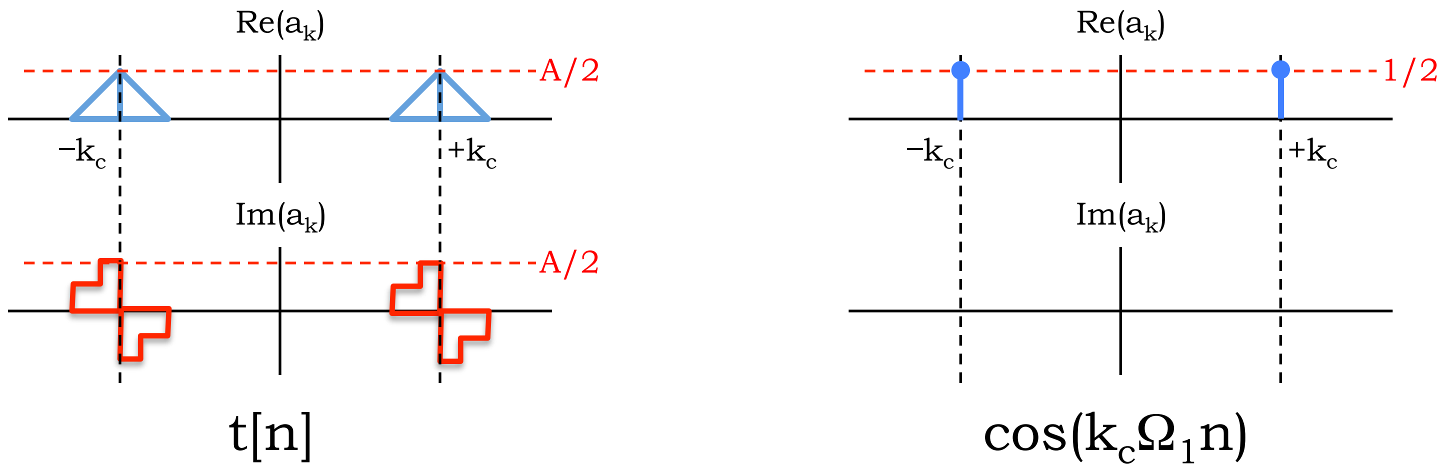
$$\Omega_c \text{ and } \Omega_m ,$$

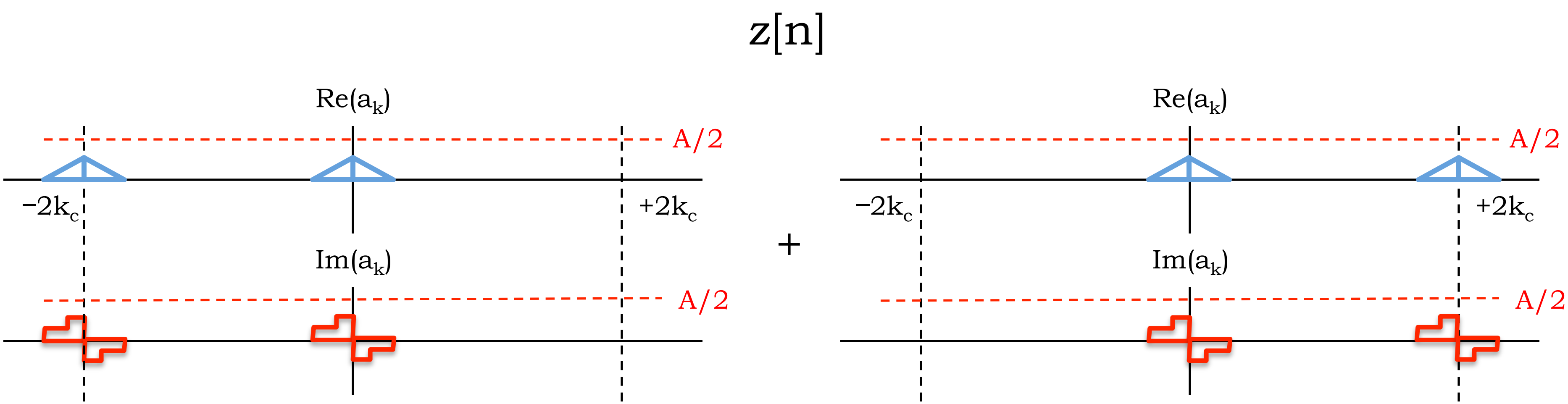
i.e., $\pm\Omega_c \pm \Omega_m$

results in components at frequencies that are

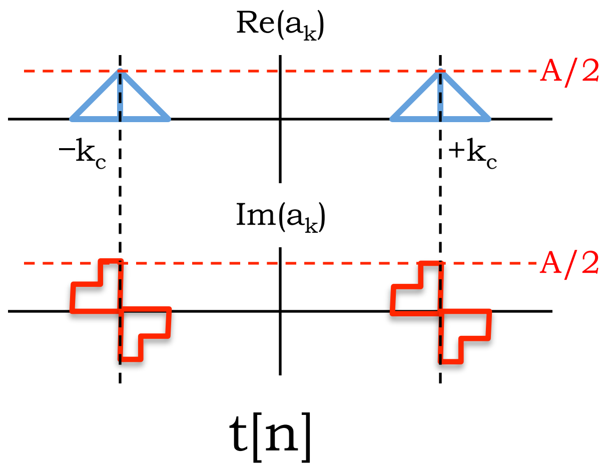
$$\pm\Omega_m \text{ away from } 0, -2\Omega_c , +2\Omega_c$$

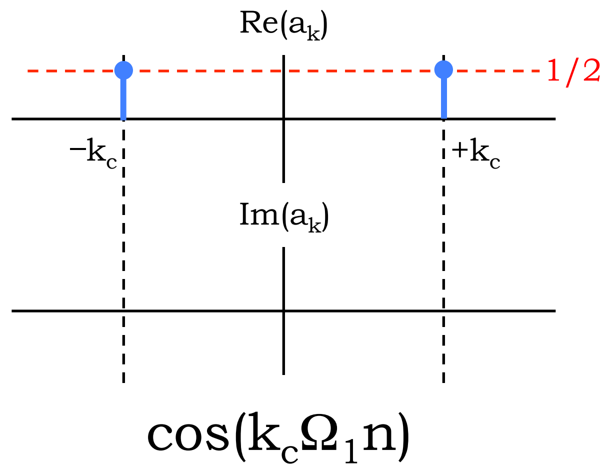
Demodulation Frequency Diagram



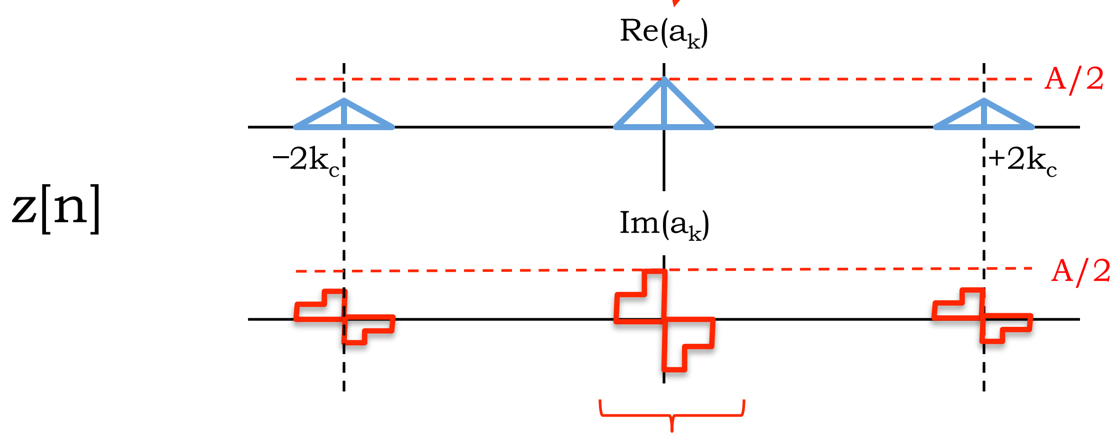


Demodulation Frequency Diagram





Note combining of signals around 0 results in doubling of amplitude



What we want

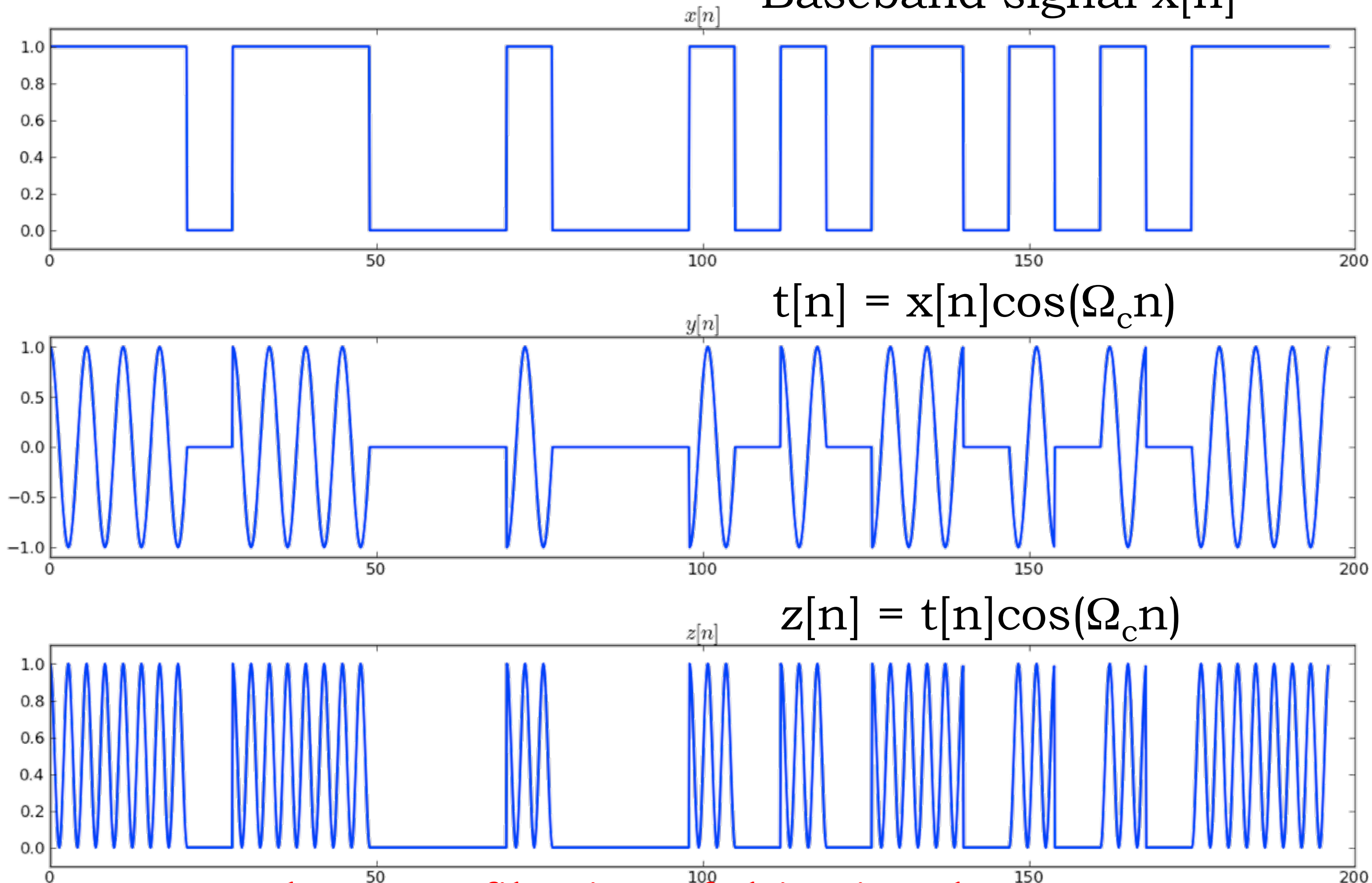
Example: Demodulation (time)

Showing idealized signals ---
no bandwidth limit on channel

Baseband signal x[n]

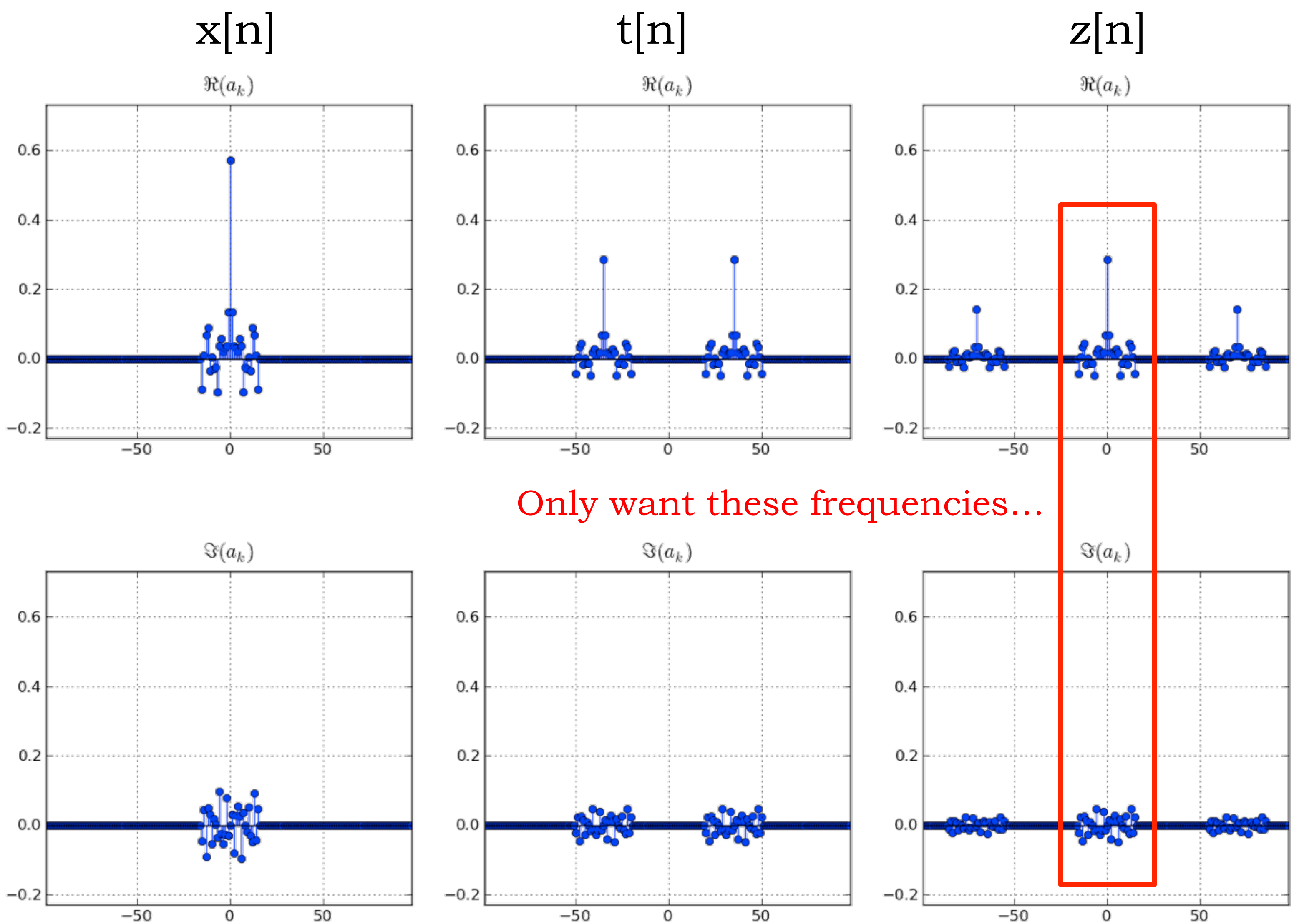
$t[n] = x[n]\cos(\Omega_c n)$

$z[n] = t[n]\cos(\Omega_c n)$

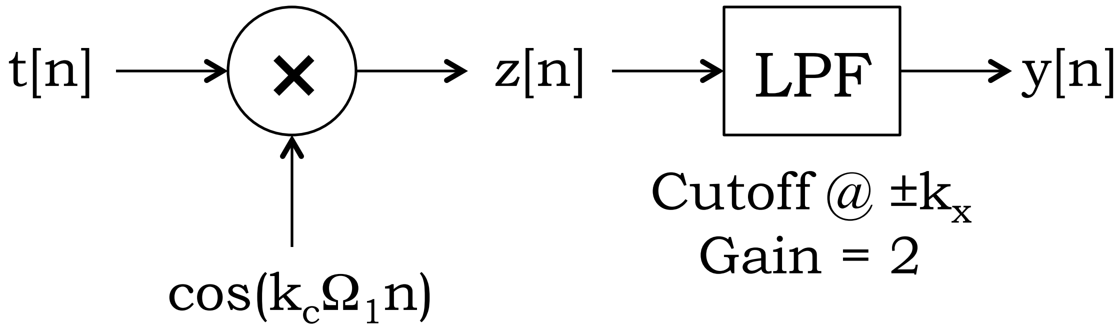


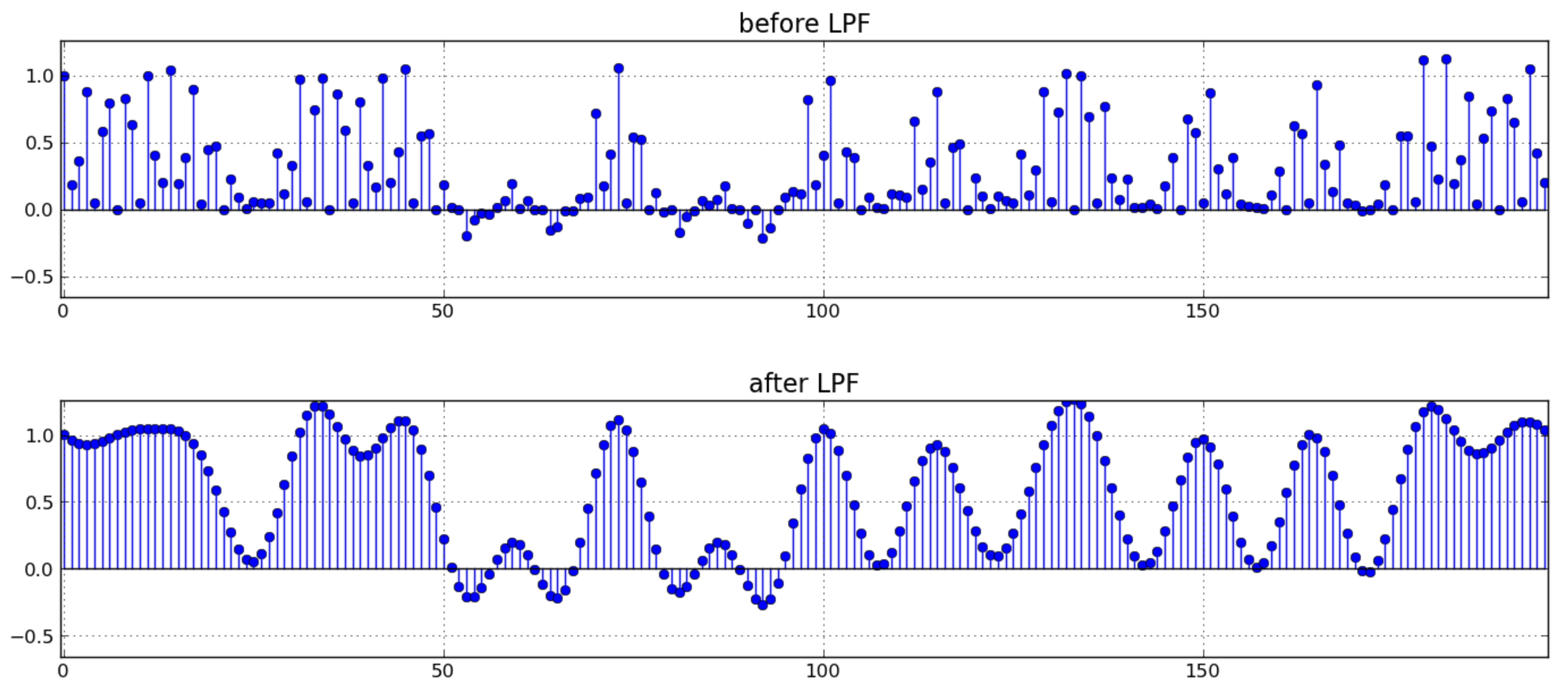
Note: lowpass filtering of this signal will yield x[n]/2 !

Example: Demodulation (freq)

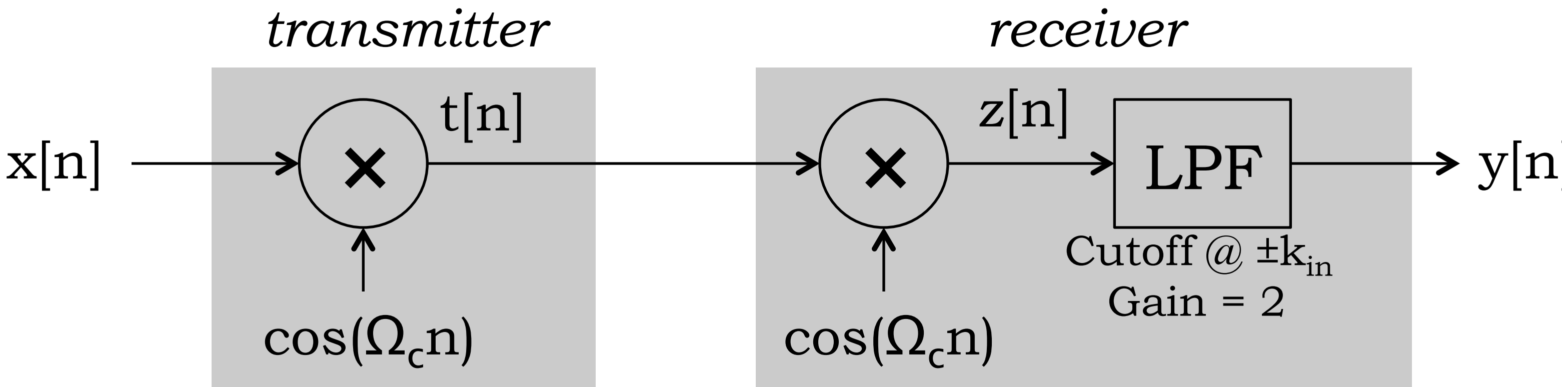


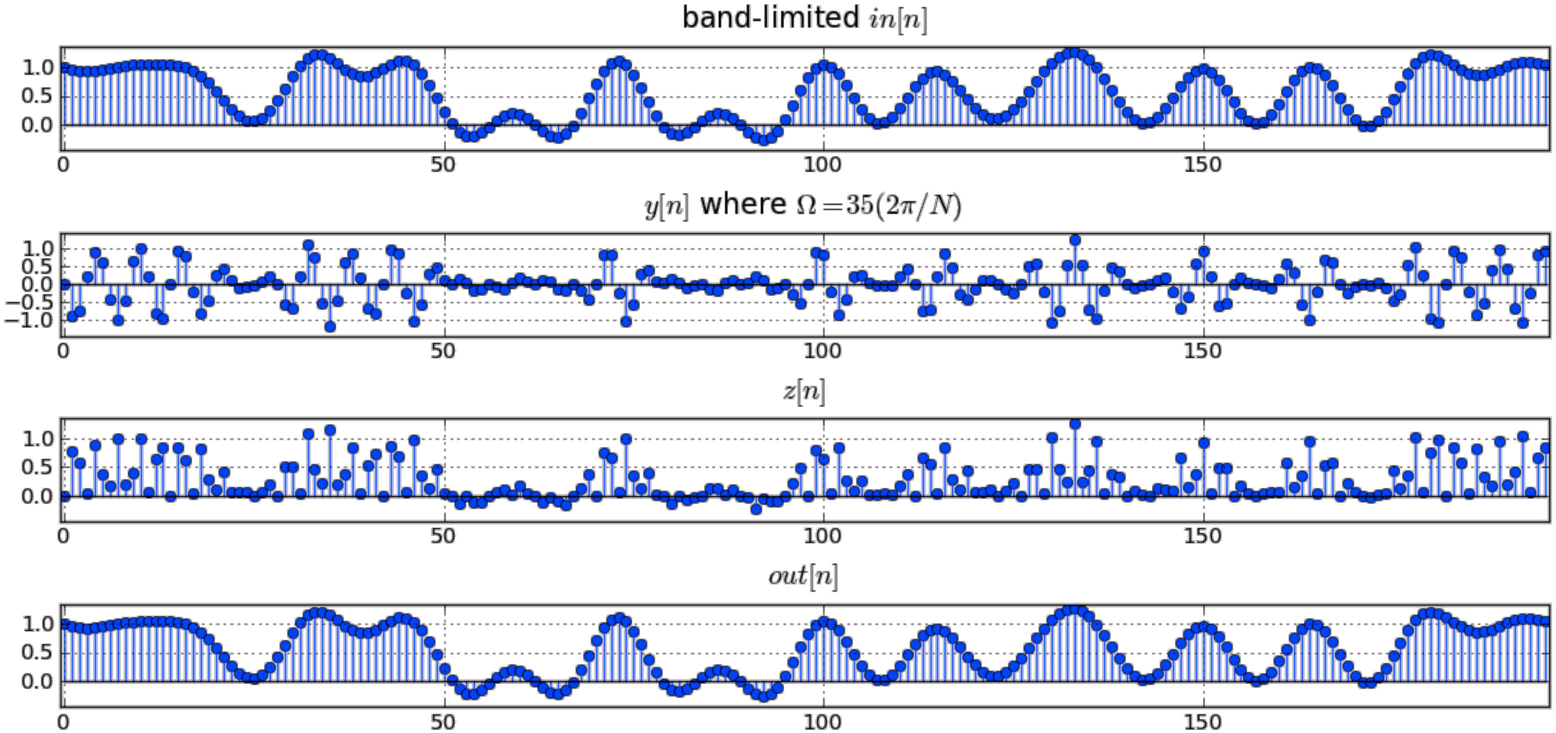
Demodulation + LPF



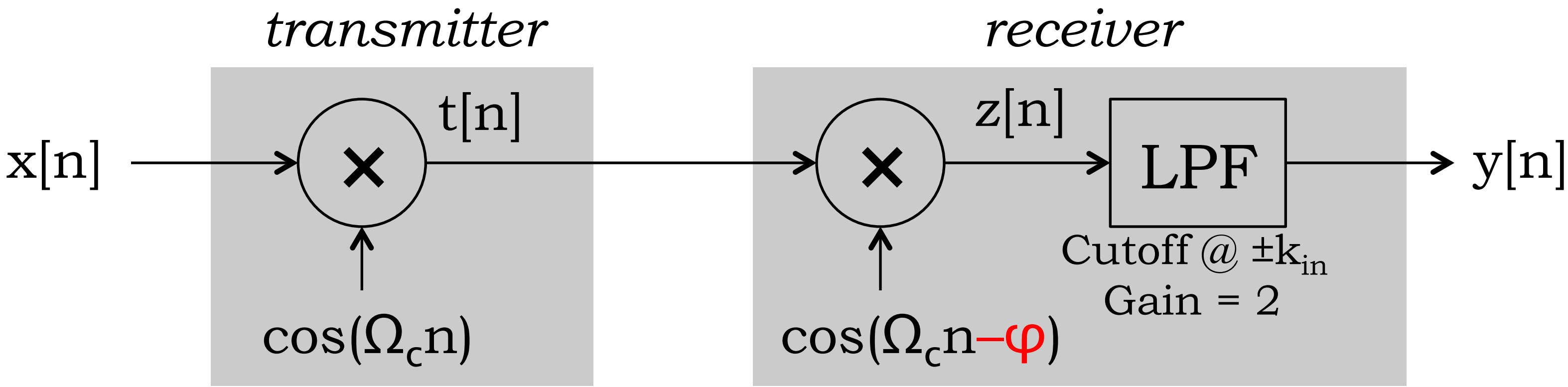


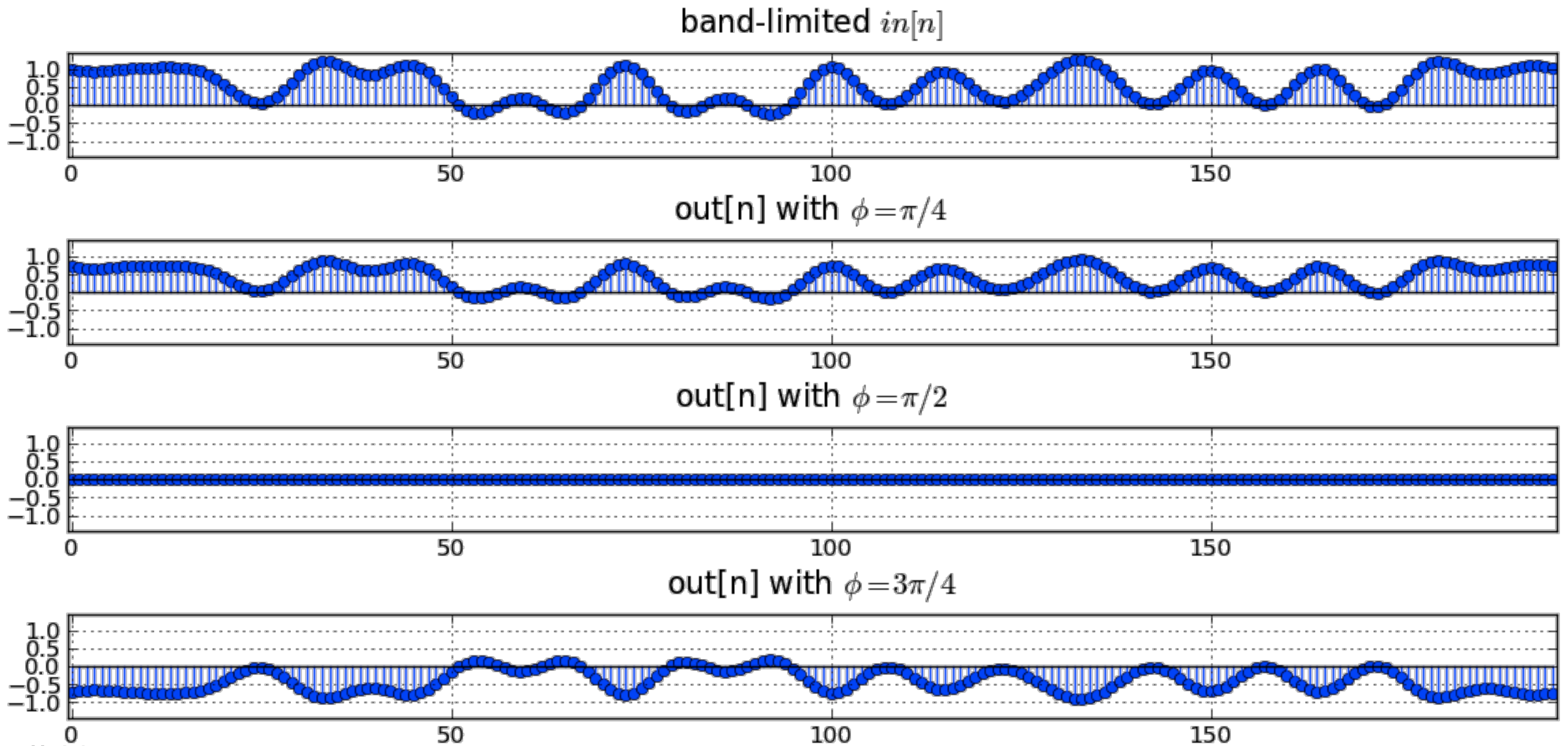
Ideal Modulation/Demodulation





Phase Error in Demodulator





Phase Error Math

Let's derive an equation for z[n]:

$$z[n] = t[n] \cdot \cos(\Omega_c n - \varphi) = x[n] \cdot \cos(\Omega_c n) \cdot \cos(\Omega_c n - \varphi)$$

But

$$\cos(\Omega_c n).\cos(\Omega_c n - \varphi) = 0.5\{\cos(2\Omega_c n - \varphi) + \cos(\varphi)\}$$

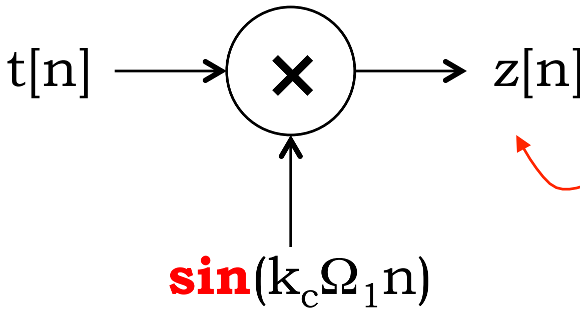
It follows that the demodulated output, after the LPF of gain 2, is

$$y[n] = x[n].\cos(\varphi)$$

So a phase error of φ results in amplitude scaling by cos(φ).

Note: in the extreme case where φ=π/2, we are demodulating by a sine rather than a cosine, and we get y[n]=0 .

Demodulation with sin($k_c\Omega_1 n$)



Hmm. So z[n] no longer has the signal we want at baseband!

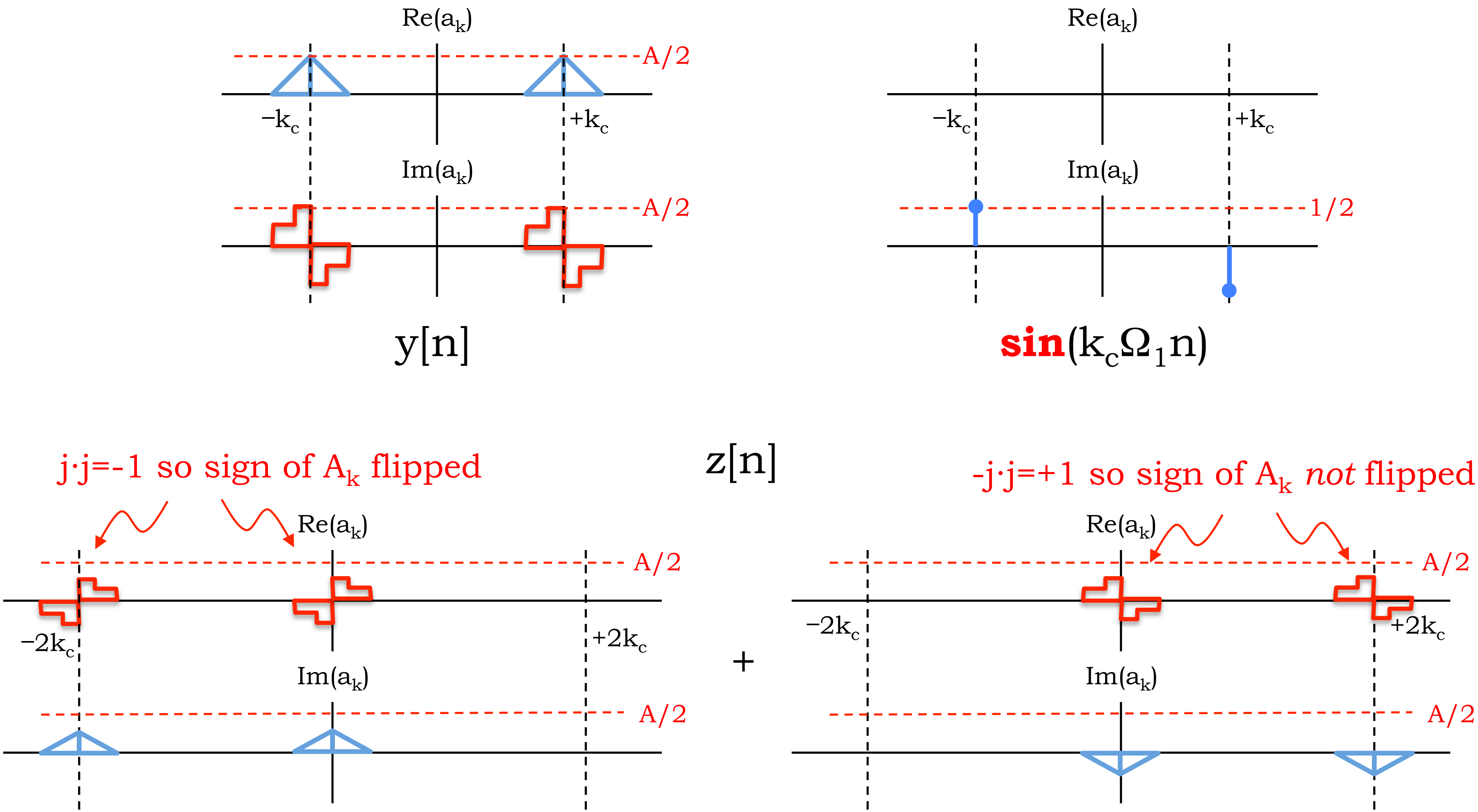
$$z[n] = t[n]\left[-\frac{j}{2}e^{jk_c\Omega_1 n} + \frac{j}{2}e^{-jk_c\Omega_1 n}\right]$$

$$= \left[\frac{1}{2}\sum_{k=-k_x}^{k_x} A_k e^{j(k+k_c)\Omega_1 n} + \frac{1}{2}\sum_{k=-k_x}^{k_x} A_k e^{j(k-k_c)\Omega_1 n}\right]\left[-\frac{j}{2}e^{jk_c\Omega_1 n} + \frac{j}{2}e^{-jk_c\Omega_1 n}\right]$$

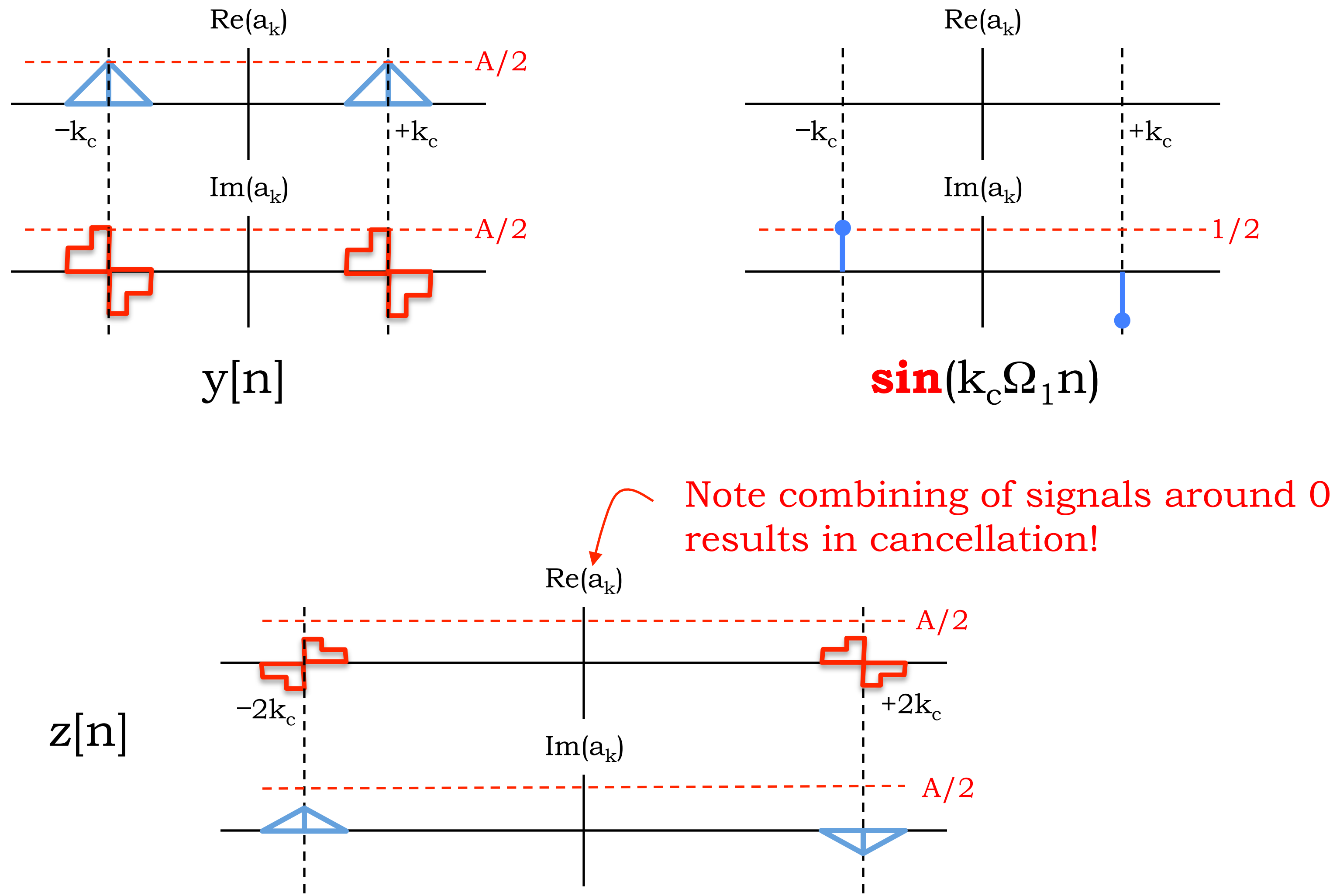
$$= -\frac{j}{4}\sum_{k=-k_x}^{k_x} A_k e^{j(k+2k_c)\Omega_1 n} + \frac{j}{4}\sum_{k=-k_x}^{k_x} A_k e^{j(k-2k_c)\Omega_1 n}$$

Oops, no baseband signal!

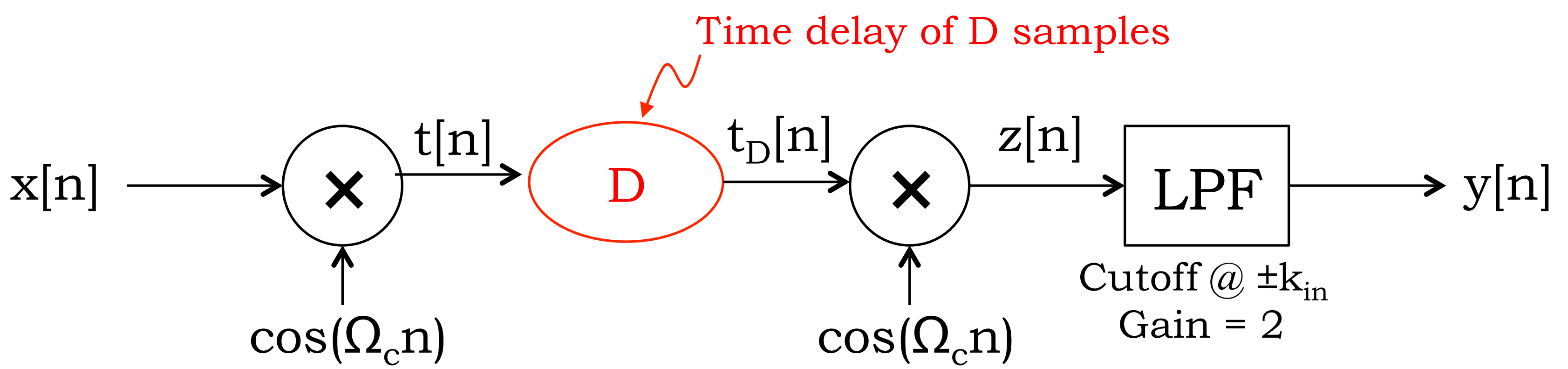
Demodulation (sin) Frequency Diagram



Demodulation (sin) Frequency Diagram



Channel Delay



Very similar math to the previous "phase error" case:

$$z[n] = t_D[n] \cdot \cos(\Omega_c n) = t[n-D] \cdot \cos(\Omega_c n)$$
$$= x[n-D] \cdot \cos[\Omega_c (n-D)] \cdot \cos(\Omega_c n)$$

Passing this through the LPF:

$$y[n] = x[n-D] \cdot \cos(\Omega_c D)$$

Looks like a phase error of $\Omega_c D$

Channel Delay Math

$$z[n] = t_D[n] \cdot \cos(\Omega_c n) = t[n-D] \cdot \cos(\Omega_c n)$$
$$= x[n-D] \cdot \cos[\Omega_c (n-D)] \cdot \cos(\Omega_c n)$$

But

$$\cos[\Omega_c (n-D)] \cdot \cos(\Omega_c n) = 0.5\{\cos(2\Omega_c n - \Omega_c D) + \cos(\Omega_c D)\}$$

It follows that the demodulated output (after the LPF of gain 2) is

$$y[n] = x[n-D].\cos(\Omega_c D)$$

So a channel delay of D results in amplitude scaling by $\cos(\Omega_c D)$.

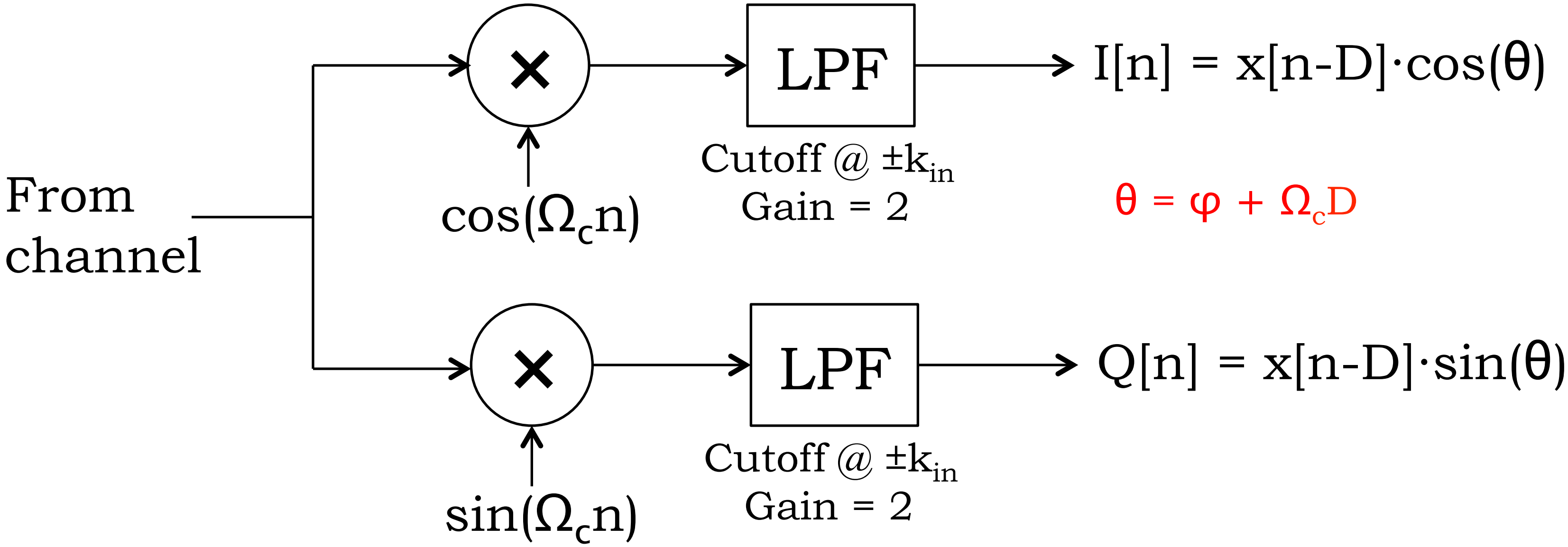
If $\Omega_c D$ is an odd multiple of $\pi/2$, then y[n]=0 !!

Fixing Phase Problems in the Receiver

So phase errors and channel delay both result in a scaling of the output amplitude, where the magnitude of the scaling can't necessarily be determined at system design time:

- channel delay varies on mobile devices
- phase difference between transmitter and receiver is arbitrary

One solution: *quadrature demodulation*



Quadrature Demodulation

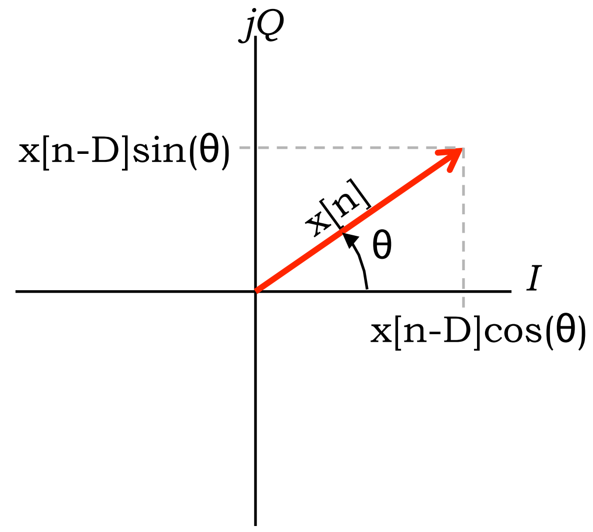
If we let

$$w[n] = I[n] + jQ[n]$$

then

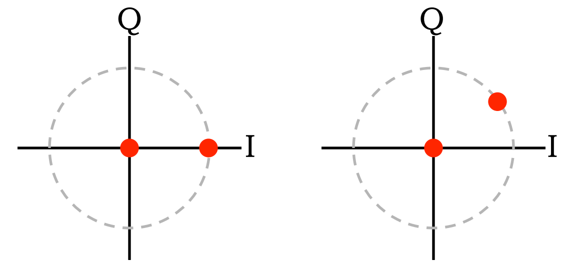
$$\left|w[n]\right| = \sqrt{I[n]^2 + Q[n]^2}$$
$$= |x[n-D]|\sqrt{\cos^2\theta + \sin^2\theta}$$
$$= |x[n-D]|$$

OK for recovering x[n] if it never goes negative, as in on-off keying



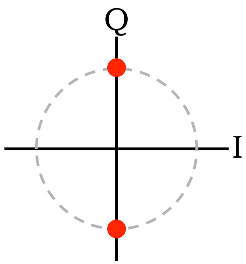
Constellation diagrams:

x[n] = { 0, 1 }

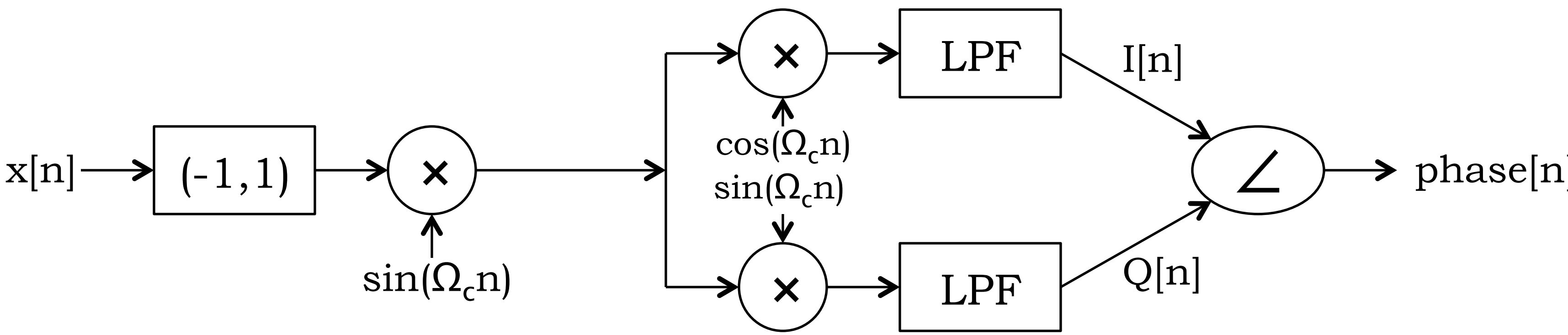


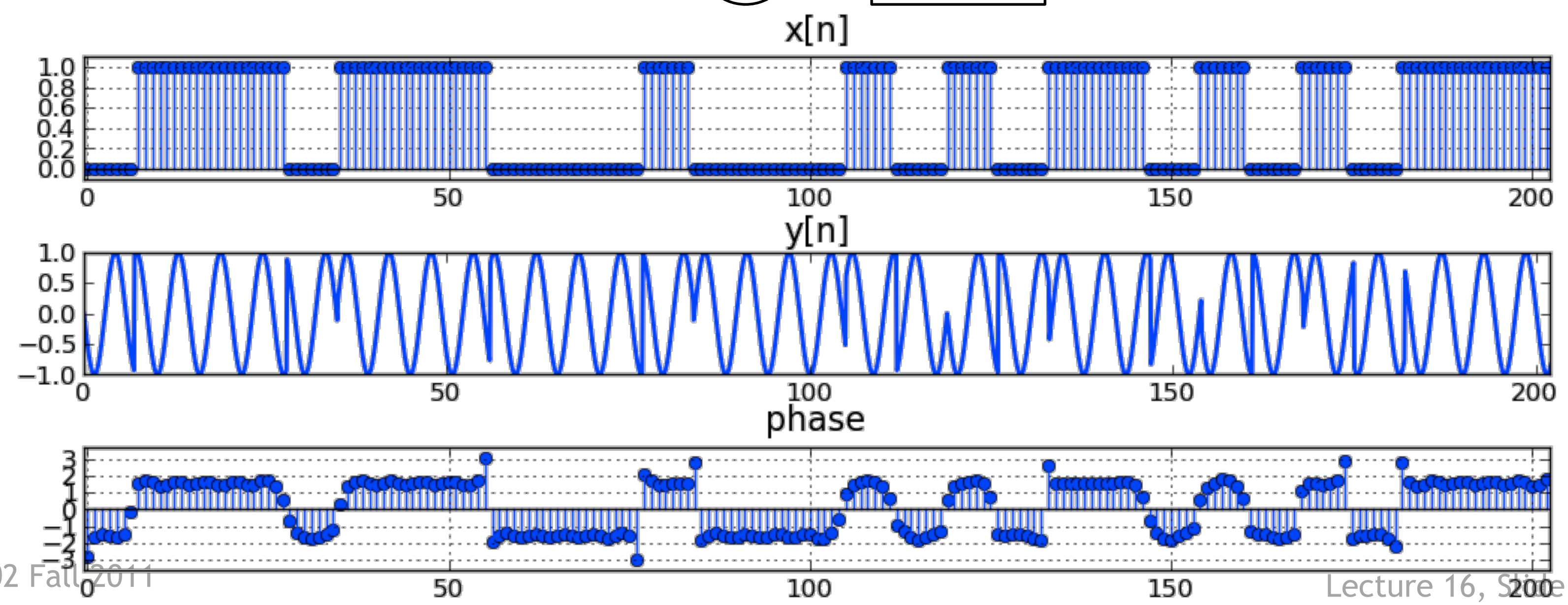
transmitter receiver

BPSK

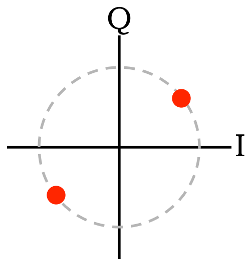


In binary phase-shift keying (BPSK), the message bit selects one of two phases for the carrier, e.g., π/2 for 0 and –π/2 for 1.



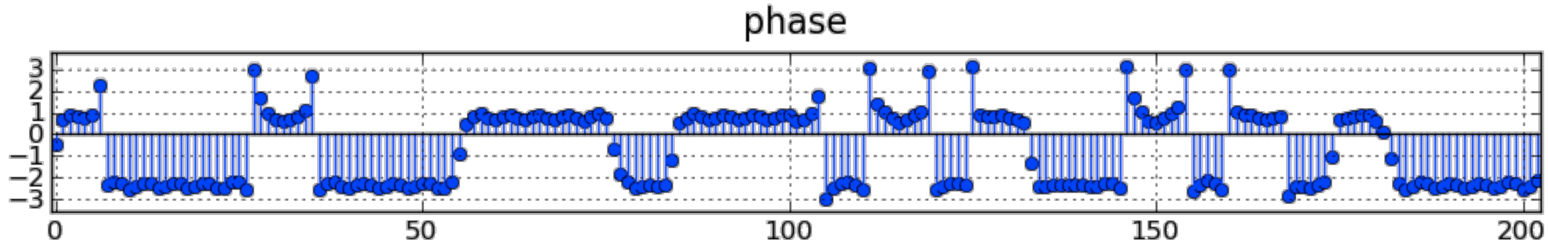


Dealing With Phase Ambiguity



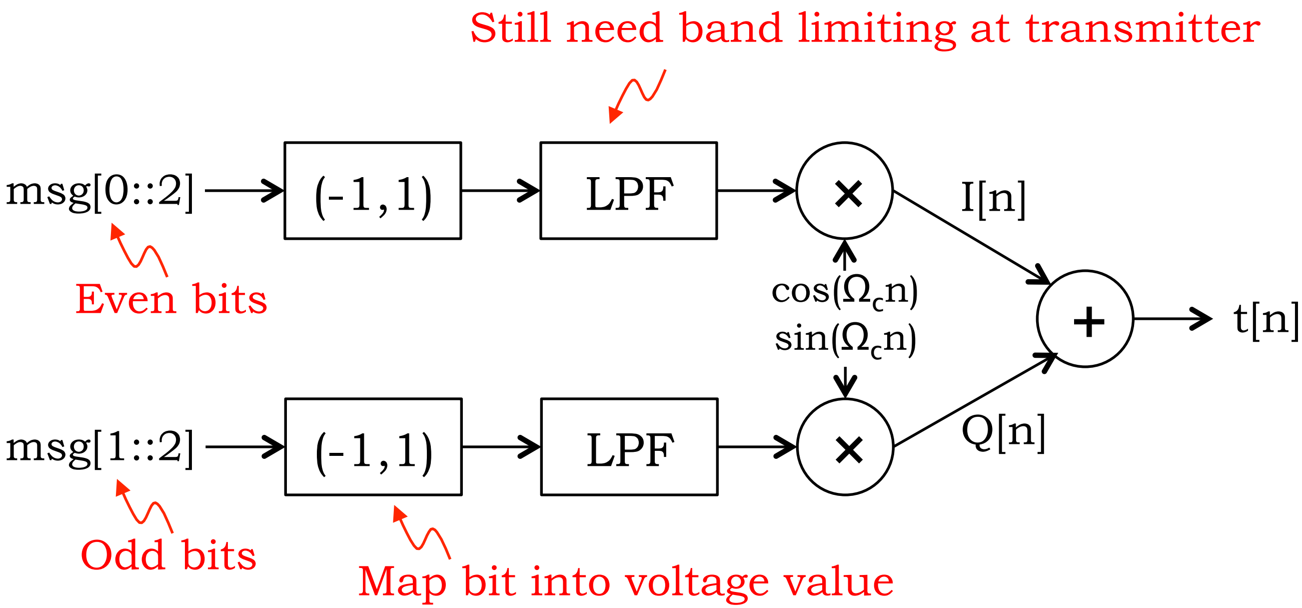
BPSK is also subject phase changes introduced by channel delays or phase difference between xmit and rcv: the received constellation will be rotated with respect to the transmitter's constellation. Which phase corresponds to which bit?

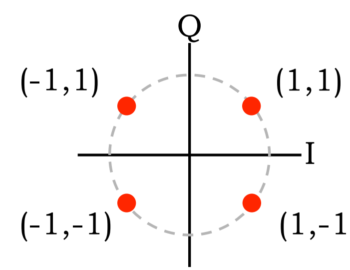
The fix? Think of the phase encoding as *differential*, not absolute: a change in phase corresponds to a change in bit value. Assume that, by convention, messages start with a single 0 bit, i.e., prepend a 0 to each to message. Then the first phase change represents a 0→1 transition, the second phase change a 1→0 transition, and so on.



QPSK Modulation

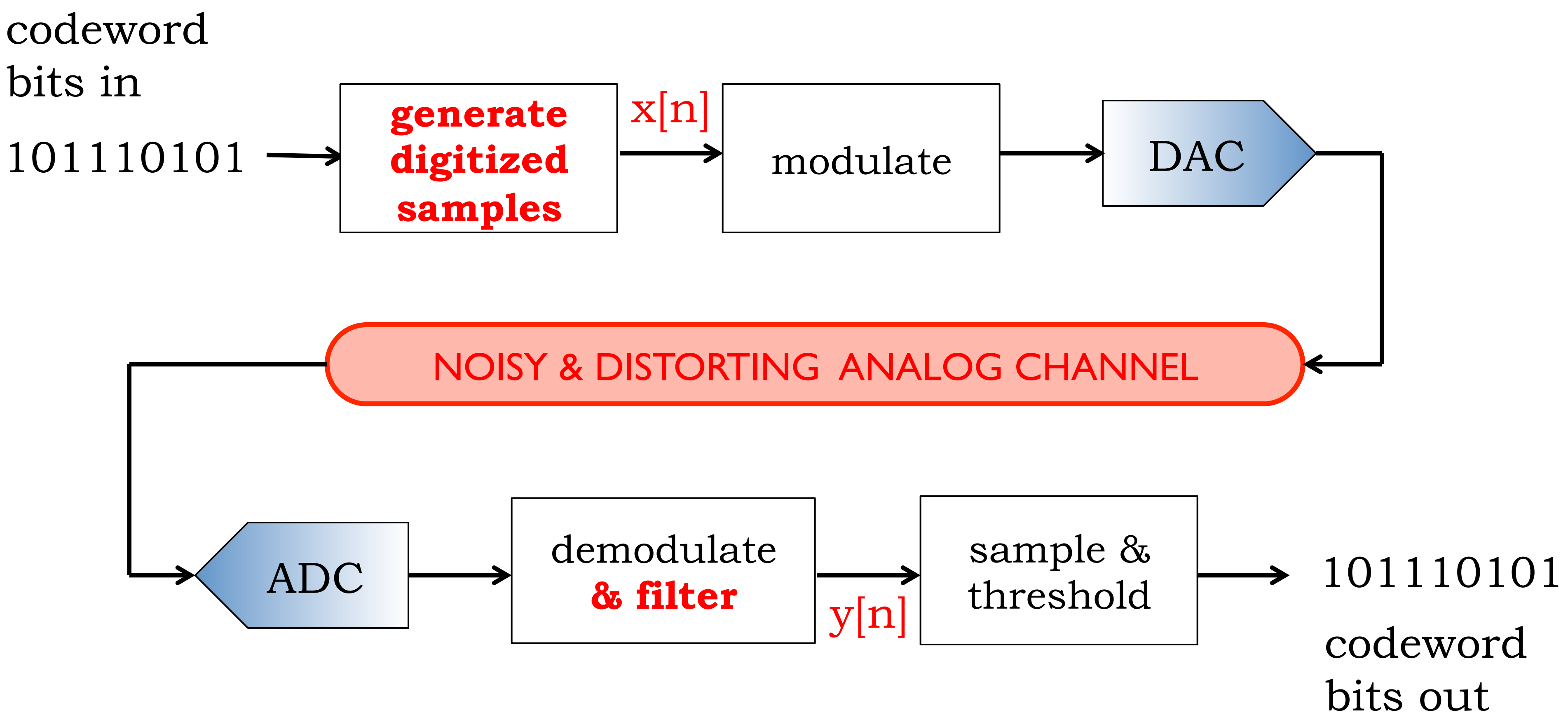
We can use the quadrature scheme at the transmitter too:



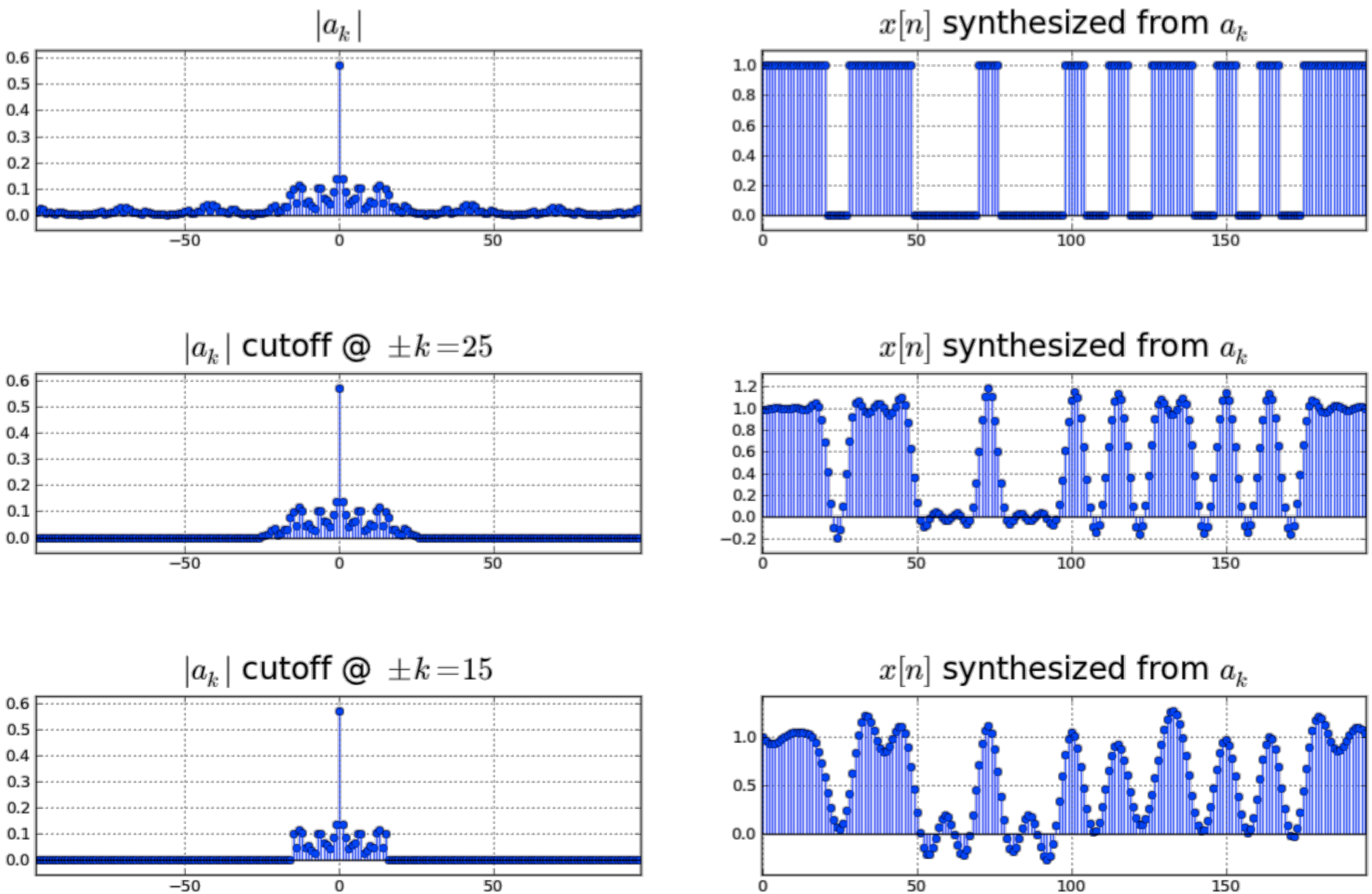


When mapping bits to voltage values, we should choose the values so that the maximum amplitude of t[n] is 1. For QPSK (also referred to as QAM-4) that would mean $\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right) = (.707,.707)$

From Baseband to Modulated Signal, and Back



Effect of Band-limiting a Transmission



Brief note 1: Shaping the Baseband Pulse

Rather than generating a rectangular-wave x[n] at the source and then lowpass filtering to get bandlimited signal for transmission, use pulse-amplitude modulation (PAM):

$$x[n] = \sum_k b[k]p[n-kT]$$

where b[k] is the voltage level for the k-th bit slot (e.g., 0 and 1, or –1 and 1), T is the bit-slot duration, and p[n] is a basic pulse shape.

Our rectangular-wave x[n] is of this type, but with a rectangular pulse for p[n], and the subsequent bandlimiting can make this a poor signal for the receiver to deal with.

Other choices for p[n] can fare much better on a bandlimited channel, and allow higher transmission rates.

Brief note 2: Spectral Content of Noise

What about the spectral content of noise? We need to know that in order to do optimal processing at the receiver.

It turns out that the expected power of white noise (i.e., uncorrelated or independent from sample to sample) is uniformly distributed (flat) over all frequencies, $-\pi$ to π. (That's actually why it's called white noise!)

Other kinds of noise will have spectral variation or coloring (hence called colored noise).

Brief note 3: Optimal Filtering at the Receiver

Can we do better than basing our 0,1 decision on just one well-chosen sample in each bit slot?

Yes, take a weighted average over all the relevant samples! Averaging increases the signal-to-noise ratio (because the variance of N uncorrelated measurements is 1/N of the variance of a single measurement).

This leads to the idea of "matched filtering".

Courses to consider downstream for elaboration on such topics: 6.011 and 16.36