1. Overview of Comm. System:

- Source
- Data Representation, Compression (source coding)
- Channel Coding
- Modulation
- Demodulation
- Channel Decoder
- Source Decoder
- (Destination)

2. Information Representation and Measure:

- More likely event → less info
- Less likely event → more info

\[ I \propto \frac{1}{P} \]

Better more quantitative measure: \[ I = \log_2 \left( \frac{1}{P_i} \right) \text{ bits} \] - How much info?

- If 2 events are equally likely, i.e., \( P = \frac{1}{2} \), then \( I = 1 \text{ bit} \).
  - e.g., represent events as "0" and "1".

- If 2 independent events both happen, \( P = P_i P_j \).
  - So that \( I = I_i + I_j \) - information additive.

- For \( N \) equiprobable events, if told that \( M \) has occurred, how much info?
  \[ I = \log_2 \left( \frac{1}{\frac{M}{N}} \right) = \log_2 \left( \frac{N}{M} \right) \text{ bits} \]

- If told drawn card is "spade" (13 possible cards), info you get
  \[ I = \log_2 \left( \frac{1}{13/52} \right) = \log_2 \left( \frac{52}{13} \right) = 2 \text{ bits} \]
2-2. Entropy ~ expected value of information.

\[ H(X) = E[I(X)] = \sum p(x) \log\left(\frac{1}{p(x)}\right) \]

~ average amt. of information needed to resolve all uncertainty.
~ lower bound on number of bits needed to encode message.

Some examples:

* Recall: fair die.
  \[ p(H) = p(T) = \frac{1}{2} \]
  one flip of a fair die: \[ p_i = \frac{1}{2} \]
  \[ \text{Amount of information } I = \log_2\left(\frac{1}{\frac{1}{2}}\right) = 1 \text{ bit}. \]
  \[ \text{(ideal)} \]
  \[ \text{Expected length of representation } = \sum p_i \log_2\left(\frac{1}{p_i}\right) = \left(\frac{1}{2}\right)(1) + \left(\frac{1}{2}\right)(1) = 1 \text{ bit} \]

Now: unfair die.

\[ p(H) = 0.98 \quad p(T) = 0.02 \]
\[ \text{Expected length of representation } = (0.98)\log_2\left(\frac{1}{0.98}\right) + (0.02)\log_2\left(\frac{1}{0.02}\right) \]
\[ = 0.14 \text{ bits} \ll 1 \text{ bit}. \]

What does this mean? "0.14" bits ???

* Coin almost always comes up "Heads" -> average uncertainty small

* Thus, for 1000 throws, need only 140 bits to represent sequence!

Why — Most likely is "H", so only need to indicate where "T" occurs — need only 140 bits to do that.
Another Example:

1. Alice, Bob, Charlie, Deb trying to guess a 3-bit binary number.
   - Alice is told "number is odd"
   - Bob: "not a multiple of 3" (i.e., not 0, 3, 6)
   - Charlie: "contains exactly two 1's"
   - Deb: given all the above clues.

2. How much information did each player receive about the number?

Solu:

- \( A : \{001, 011, 101, 111\} \sim 4 \) possibilities out of 8.
  \[ P_A = \frac{4}{8} = \frac{1}{2} \]
  \[ I_A = \log_2 \left( \frac{1}{\frac{1}{2}} \right) = 1 \text{-bit} \]

- \( B : \) could be \( \{1, 2, 4, 5, 7\} \sim 5 \) possibilities out of 8.
  \[ P_B = \frac{5}{8} \]
  \[ I_B = \log_2 \left( \frac{1}{\frac{5}{8}} \right) = 0.678 \text{-bit} \]

- \( C : \{011, 101, 110\} \) or \( \binom{3}{2} = 3 \) possibilities out of 8
  \[ P_C = \frac{3}{8} \]
  \[ I_C = \log_2 \left( \frac{1}{\frac{3}{8}} \right) = 1.42 \text{-bit} \]

- \( D : \{000, 001, 010, 011, 100, 101, 110, 111\} \)
  \[ P_D = \frac{1}{8} \]
  \[ I_D = \log_2 \left( \frac{1}{\frac{1}{8}} \right) = 3 \text{-bit} \]
Chapter 2 (All the questions come from exercises of chapter 2 note)

Q1: Alice: Possibility = $\frac{4}{8}$  
Bob: Possibility = $\frac{5}{8}$  
Charlie: Possibility = $\frac{3}{8}$  
Deb: Possibility = $\frac{1}{8}$

\[ I = \log_2 \left( \frac{1}{\frac{4}{8}} \right) = 1 \]
\[ I = \log_2 \left( \frac{1}{\frac{5}{8}} \right) = 0.678 \]
\[ I = \log_2 \left( \frac{1}{\frac{3}{8}} \right) = 1.415 \]
\[ I = \log_2 \left( \frac{1}{\frac{1}{8}} \right) = 3 \]

Q2: (a) \[ I = \log_2 \left( \frac{1}{p} \right) = \log_2 \left( \frac{1}{0.001} \right) = \log_2 1000 \]

(b) \[ \sum_{i} p_i \log_2 \left( \frac{1}{p_i} \right) = 0.25 \log_2 \left( \frac{1}{0.25} \right) + 0.75 \log_2 \left( \frac{1}{0.75} \right) \]

Q3: \[ C_2^4 = \frac{4 \times 3}{2 \times 1} = 6 \]
\[ I = \log_2 \left( \frac{1}{p} \right) = \log_2 \left( \frac{1}{6 \times \frac{16}{16}} \right) = \log_2 2.667 \]

Q4: \[ p = \frac{49}{52} \]
\[ I = \log_2 \frac{1}{p} = \log_2 \frac{52}{49} = 0.0857 \]