Recitation 4

9-20-2011

Review of Probability Theory

Randomness – the essence of communication is randomness.

<If A knew what B would say in advance, there'd be no need to say it!>

- Communication Assumes that Transmitter is connected to a random source whose output the receiver cannot predict with certainty.

Random Processes – provide good models for
- info source (some probabilities associated $\bar{c}$ information event!)
- noise ~ imperfections in communication that cause received signal to be different from transmitted.

1. deterministic linear distortion (eg. $x(t) * h(t)$)
2. non-deterministic (random) Noise – modeled as random process!

Probability Measure P – a set function that assigns non-negative values to events in Ω samplespace.

① $0 \leq p(E) \leq 1 \quad \forall E \in \Omega$

② $p(E_1 \cup E_2) = p(E_1) + p(E_2) - p(E_1 \cap E_2)$ generally.

Certainly, if A, B are disjoint (mutually exclusive), ie. $A \cap B = \phi$,

then $p(E_1 \cup E_2) = p(E_1) + p(E_2)$.

In general, if $E_1, E_2, E_3, \ldots$ are disjoint (ie. $E_i \cap E_j = \phi \ \forall i \neq j$)

then $P\left(\bigcup_i E_i\right) = \sum_i P(E_i)$

③ $P(\Omega) = 1, \quad P(\phi) = 0$

④ If $E_1 \subset E_2$, then $P(E_1) \leq P(E_2)$

Conditional Probability:

Consider events E_1 and E_2 with corresponding probabilities $p(E_1)$ and $p(E_2)$.

If we receive info that event E_2 has in fact happened, our probability about event E_1 will no longer be $P(E_1)$.

We define the conditional probability of event E_1, given that event E_2 has occurred as

$$\boxed{P(E_1|E_2) = \frac{P(E_1 \cap E_2)}{P(E_2)}, \quad P(E_2) \neq 0.}$$

eg E_1 : ~~It's~~ Snowing in Cambridge $P(E_1) \approx 1/4$
E_2 : It's winter $P(E_2) \approx 1/3$

If $P(E_1|E_2) = P(E_1)$, then knowledge of E_2 does not change $P(E_1)$.

In this case, $P(E_1 \cap E_2) = P(E_1) \cdot p(E_2)$

and the events are statistically independent!

Total Probability Thm:

If $\bigcup_{i=1}^{n} E_i = \Omega$ and $E_i \cap E_j = \emptyset$ $\forall i \neq j, \ 1 \leq i,j \leq n$

(collectively exclusive) (mutually exclusive or disjoint)

ie. events E_i, E_j partition the sample space ...

If we have the conditional probabilities of event A (ie. $P(A|E_i)$ ~ the ith conditional probability)

Then $$\boxed{P(A) = \sum_{i=1}^{n} P(A|E_i) \cdot P(E_i)}$$

Using Bayes' rule, we can obtain

$$\boxed{P(E_i|A) = \frac{P(A|E_i) \cdot P(E_i)}{P(A)} = \frac{P(A|E_i) \cdot P(E_i)}{\sum_{i=1}^{n} P(A|E_i) \cdot P(E_i)}}$$

Example: The Binary Symmetric Channel (BSC)

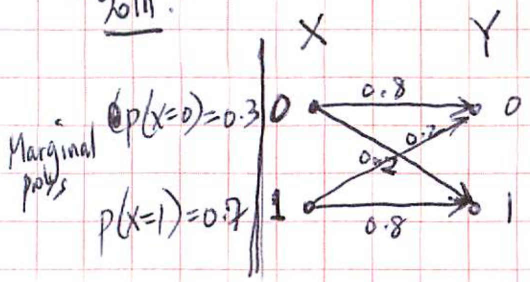
An information source produces 0 and 1 with probabilities 0.3 and 0.7 respectively. The output of the source is transmitted via a channel with crossover probability (probability of error) equal to 0.2.

What is the probability that

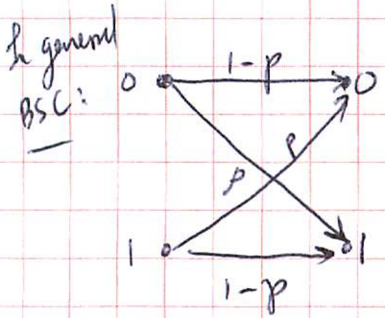
(a) A 1 is observed?

(b) A 1 was transmitted given that a 1 was observed at the channel output?

Soln:



In general BSC:



(a) $p(y=1) = ?$

By total prob thm,

$$p(y=1) = p(y=1|x=0)\cdot p(x=0) + p(y=1|x=1)\cdot p(x=1)$$

$$= (0.2)(0.3) + (0.8)(0.7)$$

$$= 0.06 + 0.56 = 0.62$$

(b) $p(x=1|y=1) = ?$

By Bayes' thm, $$P(x=1|y=1) = \frac{p(y=1|x=1)\cdot p(x=1)}{p(y=1)}$$

$$= \frac{(0.8)(0.7)}{(0.62)}$$ ~ from above

$$= 0.56/0.62$$

$$= 0.903$$ – which is reasonable for this channel!