Recitation 6

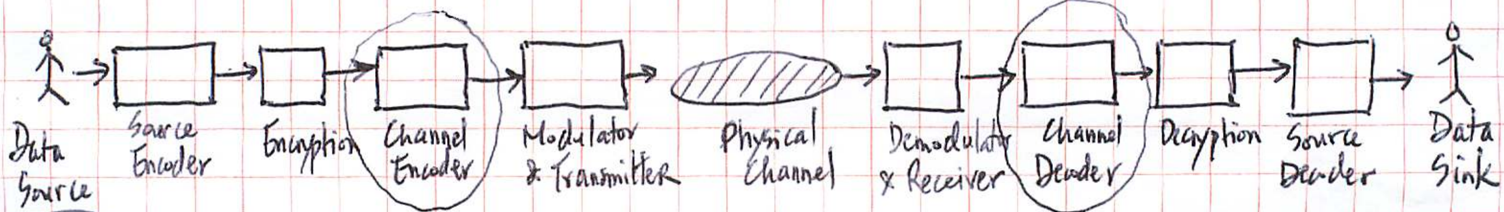
9-27-2011

Today:

- channel coding (detection & correction) ✓
- detection ~ parity-check codes
- correction ~ replication (repetition) codes
- linear block codes and properties

- Block codes
- Defns. & linear block codes

Recall: Model of Digital Communication System:



Claims

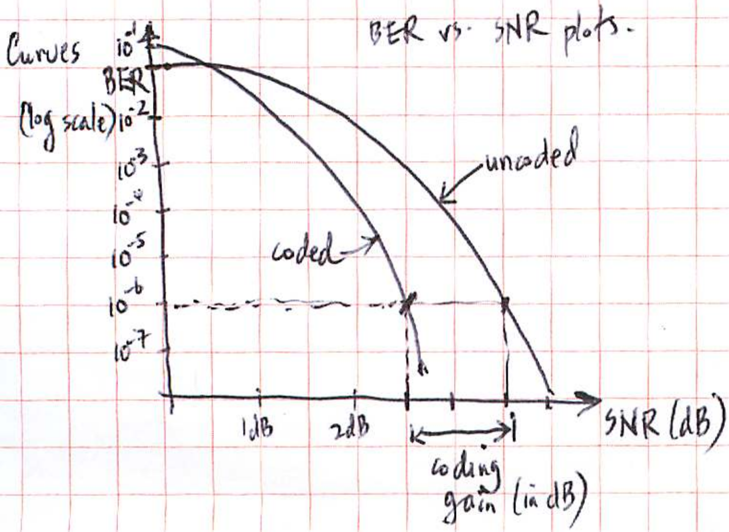
(1) **Thm:** Shannon's Noisy Channel Theorem (Error-free transmission possible!)

For every channel, we can associate a "channel capacity" C.

There exists error control codes such that information can be transmitted across the channel at rates less than C with arbitrarily low bit error rate. ✓

(2) Coding Gain: ECC can be energy-efficient!

Waterfall Curves   BER vs. SNR plots.



Embeddings (and ~~repetition~~ replication codes):

(*) Consider, want to send {Apple (A), Banana (B)}

Case I:

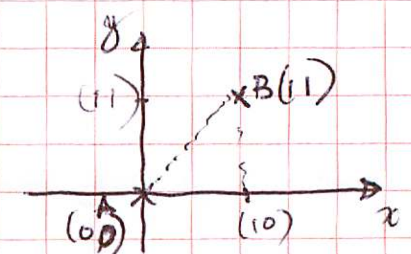
$$\left.\begin{array}{l} A - 0 \\ B - 1 \end{array}\right\} d_H = 1$$

1-D:

cannot detect anything!

Case II:

$$\left.\begin{array}{l} A - 00 \\ B - 11 \end{array}\right\} d_H = 2$$

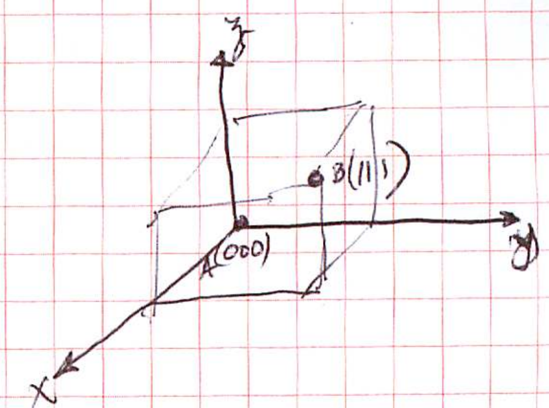


CAN detect single-error (10, or 01)

CANNOT correct!

Case III:

$$\left.\begin{array}{l} A - 000 \\ B - 111 \end{array}\right\} d_H = 3$$



CAN detect up to 2 errors!

CAN correct single-error!

⋮

Can continue like this to see that

$$\boxed{t_{detect} \leq d_{min} - 1} \qquad \boxed{t_{correct} \leq \left\lfloor \frac{d_{min} - 1}{2} \right\rfloor}$$

<u>Minimum Distance</u>: of a block code is the minimum Hamming distance btw all distinct pairs of codewords in $\mathcal{C}$.

$$d_{min} = \min\left(d_H(\vec{v}_i, \vec{v}_j)\right) \; \forall \, i \neq j$$

(*) For an error to go undetected, it must change the symbol values in at least d_{min} coordinates for one codeword to look like another in $\mathcal{C}$.

(eg) codebook codewords: 00, 11

For error to be undetectable, must change by 2 coordinates (2-bits)

ie. $d_{min} = 2$.

Thm: A code with d_{min} can detect all error patterns of weight less than or equal to $d_{min} - 1$

ie $\boxed{t_{detect} \leq d_{min} - 1}$

(eg) The length-4 ~~binary~~ repetition code has two codewords (0000) and (1111).

How many errors can it detect?

Soln: $t_{detect} = d_{min} - 1 = 4 - 1 = 3$ errors at most.

ie can detect all error patterns of weight 3!

ECC: Once error detected:

1. ARQ — request retransmission (feedback exists, reliability premium over BW)
2. Muting — tag and pass unto sink (delay critical, but can live with some errors eg. audio)
3. FEC — attempt correction (no feedback mechanism, BW essential)

P2

Example: Given: bit error rate p.
Question: For following coding schemes, the probability of undetected error $P(\text{undetected error}) = ?$

① No code, word length = 4

Answer: No code ⇒ can't detect any error.

So: $P(\text{undetected error}) = P(1\text{ error}) + P(2\text{ errors}) + P(3\text{ errors}) + P(4\text{ errors})$

$$= C_1^4 p(1-p)^3 + C_2^4 p^2(1-p)^2 + C_3^4 p^3(1-p) + C_4^4 p^4$$

$$= \sum_{i=1}^{4} C_i^4 p^i (1-p)^{4-i}$$

② Even parity, word length = 4

Answer: Even parity ⇒ can detect 1 error and 3 errors
can NOT detect 2 errors and 4 errors

So: $P(\text{undetected error}) = P(2\text{ errors}) + P(4\text{ errors})$

$$= C_2^4 p^2(1-p)^2 + C_4^4 p^4$$

For replication coding, with word length of c, bit error rate p.

$$P(\text{decoding error}) = \begin{cases} \sum_{i=\lceil \frac{c}{2} \rceil}^{c} \binom{c}{i} p^i (1-p)^{c-i} & \text{if } c \text{ odd} \\ \sum_{i=\frac{c}{2}+1}^{c} \binom{c}{i} p^i (1-p)^{c-i} + \frac{1}{2}\binom{c}{c/2} p^{c/2}(1-p)^{c/2} & \text{if } c \text{ even} \end{cases}$$

Proof: if c is odd, replication coding can NOT correct $\lceil \frac{c}{2} \rceil$ ~~bit errors~~ or more bit errors

So, $P(\text{decoding error}) = \sum_{i=\lceil \frac{c}{2} \rceil}^{c} \binom{c}{i} p^i (1-p)^{c-i}$

if c is even, replication coding can NOT correct $\frac{c}{2}+1$ ~~bit error~~ or more bit error.
and can NOT correct half of $\frac{c}{2}$ bit error.

So. $$P(\text{decoding error}) = \underbrace{\sum_{i=\frac{c}{2}+1}^{c} \binom{c}{i} p^i (1-p)^{c-i}}_{\frac{c}{2}+1 \text{ or more errors}} + \frac{1}{2} \underbrace{\binom{c}{c/2} p^{c/2} (1-p)^{c/2}}_{\frac{c}{2} \text{ errors}}$$