Recitation 10 – Quiz 1 Review

10-13-2011

Today

Quiz-1 Quick Review:

- ~ Huffman, LZW, Information
- ~ ECC
  - Properties of Linear Codes
  - Rectangular Codes
  - CC's & Viterbi
- ~ BER Calculation (if time permits)

Announcements:

1. Office Hrs F 10/14, 1-4pm, 38-246
2. Quiz-1 T 10/18, check website for time & room
3. No recitation T 10/18, but will hold Office Hrs @ 10AM and 1PM.
4. TA Review N 10/16

LZW Compression & Decompression

Q: Suppose receive 97, 98, 257, 256, 258

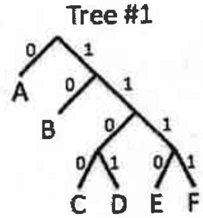
Decode received string.

(cf next page)

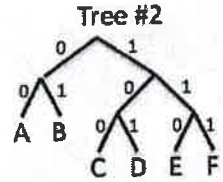
Huffman

Q: ~~For~~ For the trees below of messages composed of six symbols A, B, C, D, E, F with corresponding probabilities p(A), ... p(F)

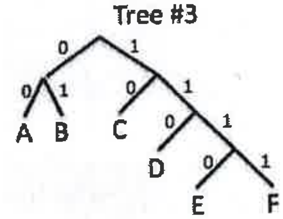
Tree #1



Tree #2



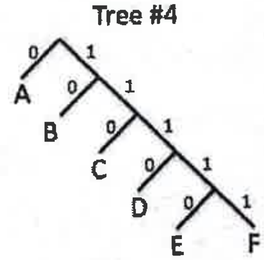
Tree #3



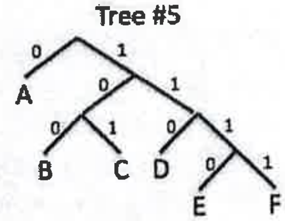
(A) Which Trees are possible H.Cs with $p(A) > 0.5$?

Soln: 1, 4, and 5

Tree #4



Tree #5



(B) Which trees are possible H.Cs with equiprobable symbols?

Soln: 2 only.

LZW Decoding:

1. Initialize table as encoder
2. //decode first symbol.
   Output Table [index]
   Previous_string = Table [index]

3. while indexes still exist
   //next index
   Lookup Table [index]
   if Table [index] NOT Valid
      current_string = Previous_string + Previous_string [0]
   else //Table [index] contains valid data
      current_string = Table [index]

   Output current_string
   Update table with Previous_string + Current_string [0]
   Previous_string = current_string

Ex: Codes: 97 98 257 256 258

Received index	Valid/ NOT Valid	Current string	Output	Update Table P_S + C_S [0]	Previous string
97			a		a
98	V	b	b	table[256] = ab	b
257	NV	bb	bb	table [257] = bb	bb
256	V	ab	ab	table [258] = bba	ab
258	V	bba	bba	table [259] = abb	bba

Result: a b bb ab bba

$\boxed{\text{Huffman}}$ cont'd.

Q: Consider Huffman decoding tree with 10 symbols chosen at random. Let l = length of decoding tree for symbol least likely to occur.

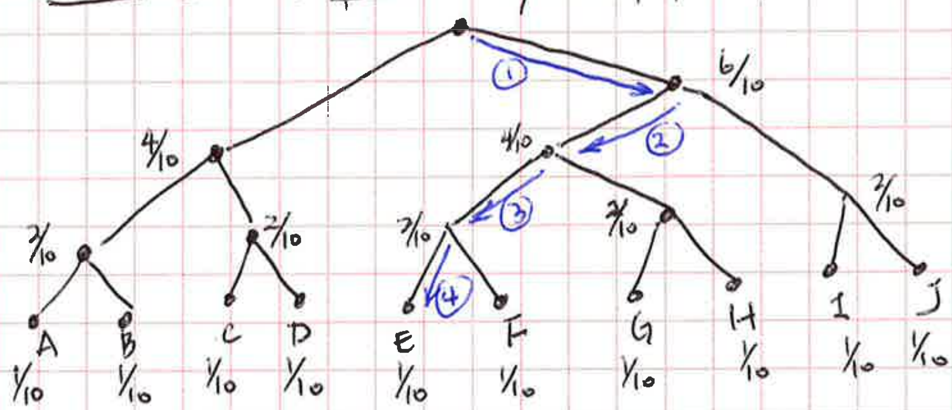
(A): What's minimum possible value of l?

(B): What's maximum possible value of l?

Soln:

(A) Min l @ equiprobable symbol probability case.

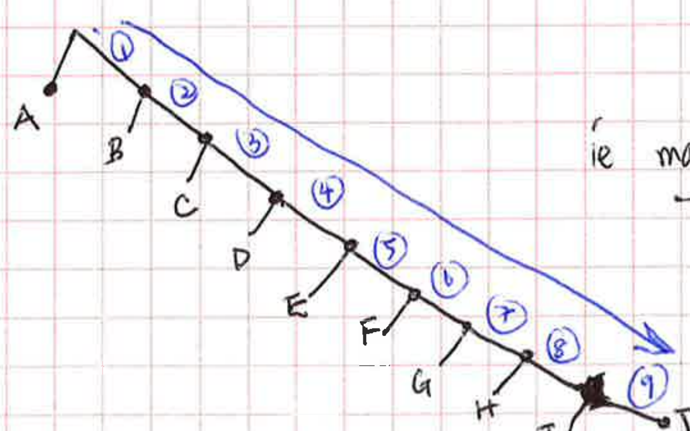
(eg)



ie. $\min l = \lceil \log_2 N \rceil$ for equiprobable case!

$= \lceil \log_2 10 \rceil = 4$

(B) Max l



ie $\max l = N-1 = 9$

Linear Block Codes

Q: Consider $\mathcal{G} = \{(00100), (10010), (01001), (11111)\}$

(A) Is $\mathcal{G}$ linear?

Soln: NO, must contain (00000).

Recall $\vec{c}_i + \vec{c}_j \in \mathcal{G} \ \forall \ i, j$ for linearity!

(B) $d_{min} = ?$

Soln: 3   Note: Not linear, so cannot infer d_{min} from min weight!

(C) $t_{detect}, t_{correct} = ?$

Soln: $t_{detect} = d_{min} - 1 = 3 - 1 = 2$

$$t_{correct} = \left\lfloor \frac{d_{min} - 1}{2} \right\rfloor = 1$$

(D) Find ML codeword for received $\vec{r}_1 = (00111)$ and $\vec{r}_2 = (11110)$

Soln: From $\mathcal{G}$ above, $\hat{\vec{r}}_1 = (00100)$ OR (11111) – pick between them!
$d_H = 2$

$\hat{\vec{r}}_2 = (11111)$
$d_H = 1$

Q: What's the minimum required redundancy for a single-error correcting code of length-7?

Soln:

Recall: $2^{n-k} \geq 1 + \binom{n}{1} + \binom{n}{2} + \dots + \binom{n}{t}$ for t-error correcting.

For $n=7$ and $t=1$, need

$$2^{7-k} \geq 1 + \binom{n}{1} = 1 + \binom{7}{1}$$

ie $$2^{7-k} \geq 1 + 7 = 8 = 2^3$$

ie $7-k \geq 3$ OR $k \leq 4$

Thus min redundancy is 3

Rectangular Codes

Consider (A)

$$\begin{array}{cccc|c} 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ \hline 1 & 0 & 0 & 0 & \end{array}$$

If (no parity-check error) great!

(B)

$$\begin{array}{cccc|c} 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ \hline 1 & 0 & 0 & 0 & \end{array}$$

If (single-row OR single-column only) ~ corresponding parity in error!

(C)

$$\begin{array}{cccc|c} 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ \hline 1 & 0 & 0 & 0 & \end{array}$$

If (single-row AND single-column only) ~ Data bit in crosshairs error! ~ Flip to correct.

(D)

$$\begin{array}{cccc|c} 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ \hline 1 & 0 & 0 & 0 & \end{array}$$

Else, multiple errors! No proper action.

VD of CC

Given:

$\vec{g}^{(0)} = (1\ 1)$

$\vec{g}^{(1)} = (1\ 0)$

$\vec{g}^{(2)} = (0\ 1)$

$m = 1$

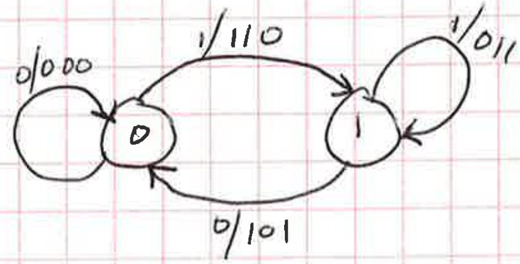
Q1: $K = ?$ $r = ?$ Num. States $= ?$

2    $1/3$ 2

Q2: Given $\vec{x} = (0110)$ what is $\vec{y}$?

Assume start state is 0-state.

Soln:



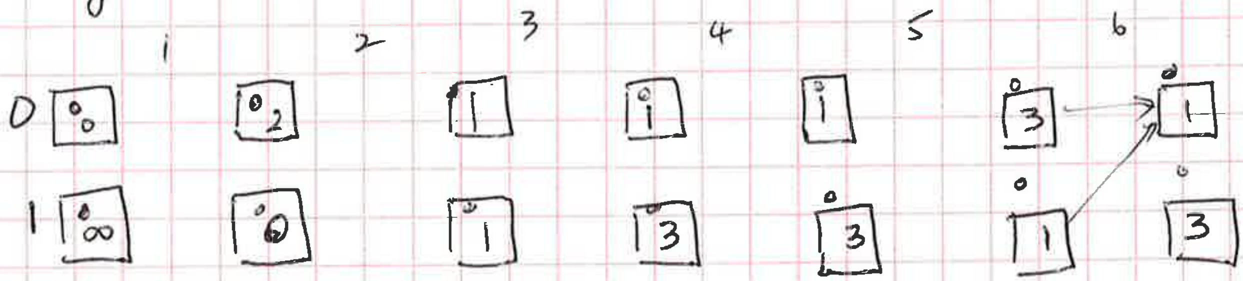
Soln:

$\vec{x} = (0110)$

$\vec{y} = (000, 110, 011, 101)$

Q3: Suppose $\vec{r} = (110, 001, 101, 000, 110, 101)$

Decode $\hat{y}$ and $\hat{x}$.



Info

Q1: Bob is told that a 5-bit number contains exactly two-1's. How much information did he receive?

Soln: Recall $\boxed{I = \log_2\left(\frac{1}{P}\right)}$ bits.

$${}^5C_2 = \frac{5!}{3!\,2!} = 10 \text{ possibilities out of a total of } 2^5 = 32$$

i.e. $P\{\text{5-bit num contains exactly two-1's}\} = \frac{10}{32} \sim P_B$

$$I_B = \log_2\left(\frac{1}{P_B}\right) = \log_2\left(\frac{32}{10}\right) = 1.68 \text{ bits}$$

Entropy & Expected length

Recall $\boxed{H(X) = E[I(X)] = \sum p(x)\cdot\log_2\frac{1}{p(x)}}$

Q2: Consider an unfair die with $P(H) = 0.98$ and $P(T) = 0.02$. In 1000 throws of the die, what is the number bits are needed to represent the sequence? (How many)

Soln:

Expected length of representation for single throw $= P_H\cdot\log_2\left(\frac{1}{P_H}\right) + P_T\cdot\log_2\left(\frac{1}{P_T}\right)$

$$= (0.98)\log_2\left(\frac{1}{0.98}\right) + (0.02)\log_2\left(\frac{1}{0.02}\right)$$

$$= 0.14 \text{ bits.}$$

Thus for 1000-throws, need $(1000)(0.14) = 140$-bits