Recitation 12

10-25-2011

Today:

① Review of LTI Preliminaries ~ causality, stability.

② Examples & Graphical evaluation of convolution

Recall:

① $h[n] \triangleq \mathcal{R}\{\delta[n]\}$ – Unit sample response (aka impulse response)

$\delta[n] \longrightarrow \boxed{LTI} \longrightarrow h[n]$

$\delta[n-k] \longrightarrow \qquad \longrightarrow h[n-k]$ for LTI system.

Arbitrary input

$x[n] \longrightarrow \boxed{LTI} \longrightarrow y[n] = x[n] * h[n]$

② Causal.

if $h[n] = 0$ for $n < 0$, system is causal.

(ie. output depends only on present and past inputs, not future).

③ Stable (in BIBO sense)

if $\sum_{k=-\infty}^{\infty} |h[k]|$ is finite (ie output is bounded for bounded input).

Example 1: Given the LTI system with unit sample (impulse) response $h[n] = \alpha^n u[n]$.

(a) Is the system causal?

Soln: Since $h[n] = 0 \ \forall n < 0$, system is causal.

Recall: $\sum_{n=0}^{\infty} \alpha^n = \frac{1}{1-\alpha}, |\alpha| < 1$

(b) Is the system BIBO stable?

Soln: $\sum_{k=-\infty}^{\infty} |h[k]| = \sum_{k=-\infty}^{\infty} |\alpha^k u[k]| = \sum_{k=0}^{\infty} |\alpha^k| = \frac{1}{1-|\alpha|}$, for $|\alpha| < 1$

Thus system is BIBO stable if $|\alpha| < 1$, and unstable if $|\alpha| \geq 1$.

Example 2: The unit sample response $h[n]$ of a discrete-time LTI-system is given by

$$h[n] = u[n] - u[n-3]$$

If the input to the system is

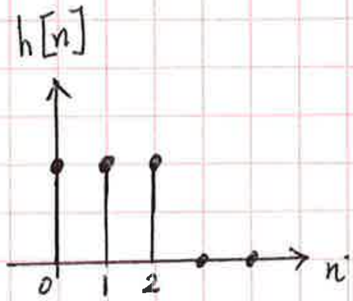
$$x[n] = \delta[n-1] - \delta[n-3]$$

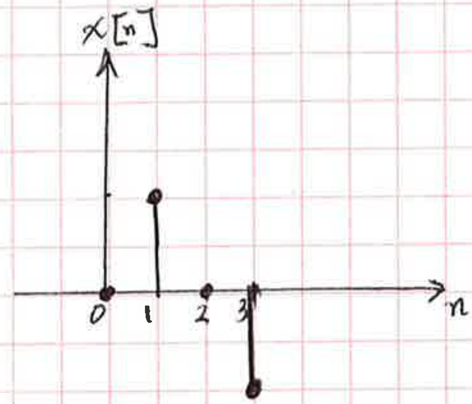
(a) Sketch $h[n]$ and $x[n]$

(b) Obtain the output $y[n]$ and sketch it. (3 diff. ways :-))

Soln:

(a)





Method 1:

(b) Since system is linear and time-invariant, and by the defn of the unit sample (impulse) response,

$$y[n] = h[n-1] - h[n-3] \qquad \left(\begin{array}{l} \delta[n-1] \rightarrow h[n-1] \\ \delta[n-3] \rightarrow h[n-3] \end{array}\right)$$

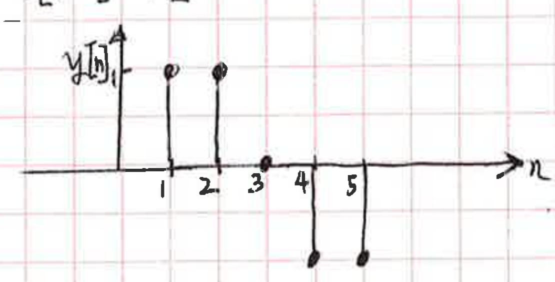
Since $h[n] = u[n] - u[n-3]$,

$$h[n-1] = u[n-1] - u[n-1-3] = u[n-1] - u[n-4]$$

and $h[n-3] = u[n-3] - u[n-3-3] = u[n-3] - u[n-6]$

And thus,

$$y[n] = u[n-1] - u[n-3] - u[n-4] + u[n-6]$$, which is sketched below



Method 2: (Analytical Approach)

$$x[k] = \delta[k-1] - \delta[k-3] = \begin{cases} 1, & k=1 \\ -1 & k=3 \\ 0 & \text{elsewhere} \end{cases}$$

So that

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \underbrace{x[1]\,h[n-1]}_{k=1} + \underbrace{x[3]\,h[n-3]}_{k=3}$$

$$= 1 \cdot h[n-1] + (-1) \cdot h[n-3]$$

$$= h[n-1] - h[n-3] \quad \rightarrow \text{as before.}$$

Using values for $h[n] = u[n] - u[n-3]$ and LTI properties, we obtain

$$y[n] = u[n-1] - u[n-3] - u[n-4] + u[n-6] \quad \text{as before.}$$

Method 3: Graphical Mthd:

⊛ Review of the Convolution Sum Operation:

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]\,h[n-k]$$ — convolution sum.

Evaluation Steps:

① Obtain $h[-k]$ from time-reversal of $h[k]$ about origin.

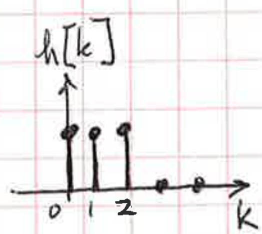
② Shift $h[-k]$ (function of k) by n (fixed parameter) to the right to obtain $h[n-k]$ ie. $h[-(k-n)]$.

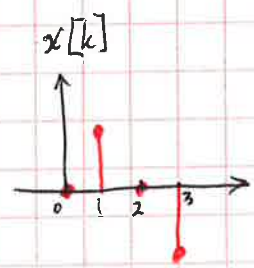
③ Obtain product $x[k]h[n-k]$ over all k, at fixed n; sum the product $\forall k$ to produce single output sample at n, ie. $y[n]$.

④ Repeat steps ② and ③ for different values of n from $-\infty$ to ∞.

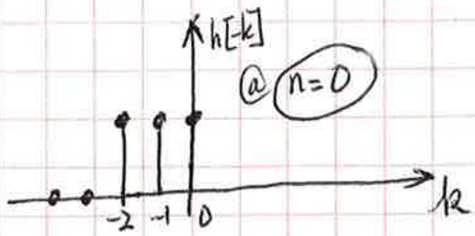
Returning to our example...

$$y[n] = (x*h)[n] = \sum_{k=-\infty}^{\infty} x[k]\,h[n-k]$$ where





Step 1: Obtain $h[-k]$:



ie $y[0]=0$

So $y[n=0]=0$, since no overlap!

Step 2: Obtain $h[n-k]$ @ $n=1$ shift 1 to the right

$y[n=1]$, $y[1] = (1)(1) = 1$

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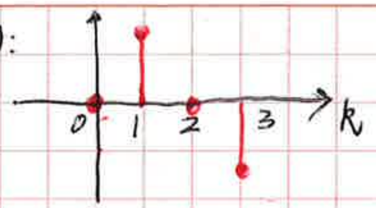
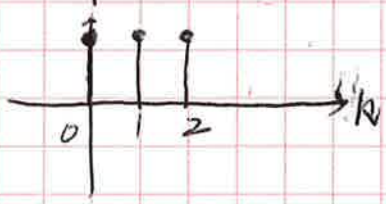
$x[k]$:



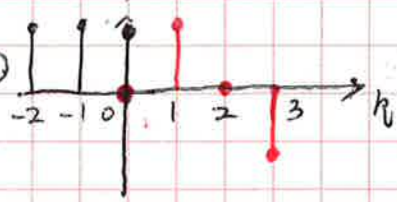
$x[k] = \delta[k-1] - \delta[k-3]$

$h[k]$



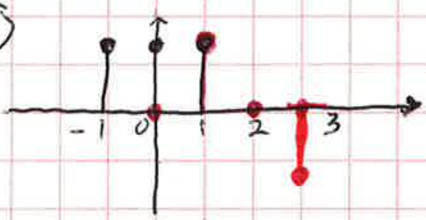
$h[k] = u[k] - u[k-3]$

(A) $n=0$



$y[0] = 0$

(B) $n=1$

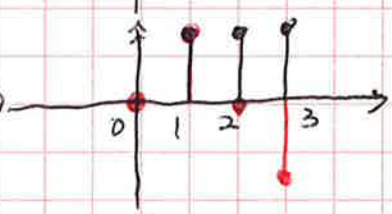


$y[1] = 1$

(C) $n=2$

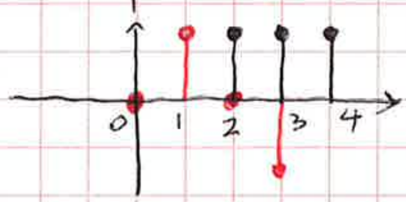
$y[2] = 1$

(D) $n=3$



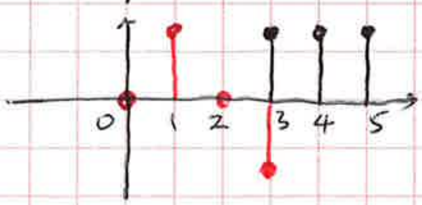
$y[3] = 0$

(E) $n=4$



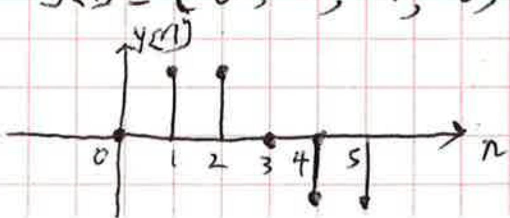
$y[4] = -1$

(F) $n=5$



$y[5] = -1$

So: $y[n] = (0, 1, 1, 0, -1, -1)$



which is identical to what we obtained before.