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Recitation 18

Today: ① MAC Protocols
② Filters (if time permits)

Examples of Shared Media: ① satellite ntwk ② Wireless (radio freq.) ntwk ③ Wired Bus (eg. Ethernet) ④ FDM radio & TV broadcast

MAC Protocol: Defines rules governing how a common (shared) communication channel (medium) is shared among different nodes.

1| Performance Metrics:

(1.) Throughput (bps) ~ Num packets delivered successfully per unit time (s).

*(A) Utilization ~ (independent of channel rate): ratio of total throughput to max data rate of channel.
$(0 \le U \le 1)$

(eg1) Given: _ channel rate ~ 10 Mbps.
~ 4 nodes share channel with throughputs 2Mbps, 2Mbps, 2Mbps, 1 Mbps

Reqrd: Utilization?

Soln: $U = \dfrac{\text{Total Throughput}}{\text{Max Channel Data Rate}} = \dfrac{(2+2+2+1)}{10} = \dfrac{7}{10} = 0.7$

*(B) Fairness ~ notions of fairness?
"Allocation with smaller s.d. fairer than one with larger standard deviation

Fairness Index $F_i = \dfrac{\left(\sum_N x_i\right)^2}{N \sum_N x_i^2}$, x_i is throughput of node i and N backlogged nodes.

(eg2) For eg1 above, what is F Index? $F_i = \dfrac{(2+2+2+1)^2}{4 \cdot (2^2+2^2+2^2+1^2)} = \dfrac{49}{52} = 0.94$ Fair? (Yes $\approx$ 1 - perfect fairness) $\frac{1}{N} \le F_i \le 1$

2. TDMA

A simple TDMA protocol:

If the current time is t, and N nodes share the channel, then the node with ID i transmits if and only if

it is backlogged and $(t \bmod N) == i$ $\quad i \in \{0, N-1\}$

Pros:

- It is fair
- Simple to implement if there is a central coordinator.

Cons:

- Under-utilize the medium in skewed traffic pattern
- become complex if there is not a central coordinator (i.e. distributed)

3. Aloha

Protocol:

If a node is backlogged, it sends a packet with probability p.

Utilization:

count slots without collisions! (Exactly one node transmits)

$U(p) = P(\text{Exactly one node sends})$

$= N\,p(1-p)^{N-1}$ where N is the number of backlogged nodes, p: each node transmits with probability p.

The maximum value of U occurs

when $p = \frac{1}{N}$

and the maximum value of U is

$$U_{max} = \left(1 - \frac{1}{N}\right)^{N-1}$$

As $N \to \infty$, $U_{max} \to \frac{1}{e} \approx 37\%$

Example 1. Given: A and B share a channel.

The probability that A sends a packet is: P_a

The probability that B sends a packet is P_b.

A is 2/3 as likely to send a packet as B,

i.e. $P_a = \frac{2}{3} P_b$

Assume both A and B are backlogged and each packet is one slot long.

Question: ① what is the utilization?

② what is P_a and P_b such that the utilization is maximized?

③ what is maximized utilization?

Solution:

① utilization is the probability that exactly one user sends.

So: $U = P(A \text{ xmits}) * P(B \text{ doesn't xmits})$

$+ P(B \text{ xmits}) * P(A \text{ doesn't xmits})$

$= P_a * (1 - P_b) + P_b * (1 - P_a)$ ⟆ plug $P_a = \frac{2}{3} P_b$

$= \frac{2}{3} P_b (1 - P_b) + P_b \left(1 - \frac{2}{3} P_b\right)$

$= \frac{5}{3} P_b - \frac{4}{3} P_b^2$

② the utilization U is maximized when $\frac{dU}{dP_b} = 0$.

So: $0 = \frac{dU}{dP_b} = \frac{d\left(\frac{5}{3}P_b - \frac{4}{3}P_b^2\right)}{dP_b}$

$= \frac{5}{3} - \frac{8}{3}P_b$

$\Rightarrow P_b = \frac{5}{8}$, $P_a = \frac{2}{3}P_b = \frac{2}{3} * \frac{5}{8} = \frac{5}{12}$.

③ Plug in $P_b = \frac{5}{8}$, we got maximized U

$U_{max} = \frac{5}{3}P_b - \frac{4}{3}P_b^2$

$= \frac{5}{3} * \frac{5}{8} - \frac{4}{3} * \left(\frac{5}{8}\right)^2$

≈ 0.52

Example 2: Given: A and B shares a channel.

The probability that A sends packet is: $P_a = 0.3$.

we want B get three time the throughput as A whenever both nodes are backlogged

Question? what is the transmission probability of B, P_b in order to achieve our throughput goal?

Solution:

A's throughput = P (A xmits) * P (B doesn't)

$= 0.3 \quad * (1 - P_b)$

B's throughput = P (B xmits) * P (A doesn't)

$= P_b \quad * (1 - P_a)$

$= P_b \quad * (1 - 0.3)$

$= 0.7 P_b$

B's throughput = 3 * (A's throughput)

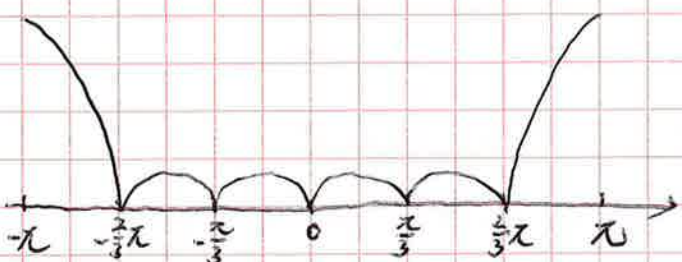
$\Rightarrow 0.7 P_b = 3 * [0.3 * (1 - P_b)] \Rightarrow 1.6 P_b = 0.9$

$\Rightarrow P_b \approx 0.56$

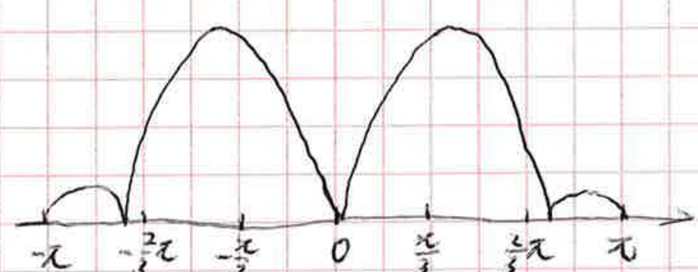
Filter:

Given: consider three LTI filters, denoted by A, B, C.
The magnitudes of their frequency response $|H_A(e^{j\Omega})|$, $|H_B(e^{j\Omega})|$, $|H_C(e^{j\Omega})|$ are given below.

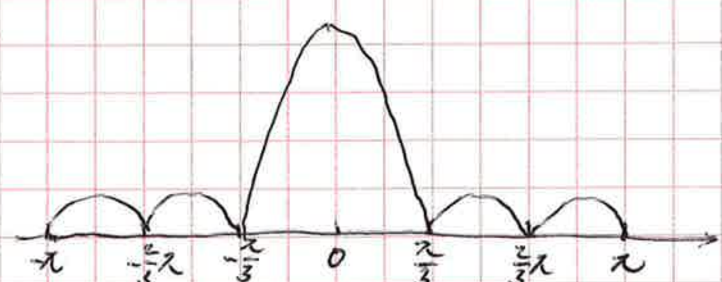
① $|H_A(e^{j\Omega})|$



② $|H_B(e^{j\Omega})|$

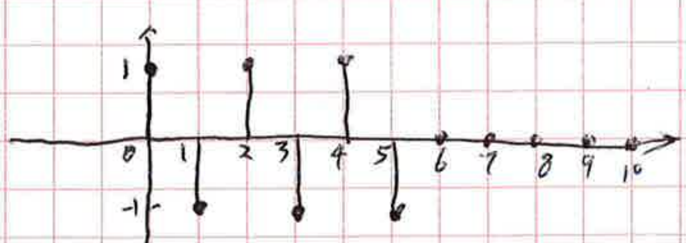


③ $|H_C(e^{j\Omega})|$



Question:

which frequency response of filters (A, B, or C) corresponding to the following unit sample response?



Solution:

(Evaluate $|H(e^{j\Omega})|$ at several frequencies and see what we found)

Noticed: $h[0]=1,\ h[1]=-1,\ h[2]=1$

$h[3]=-1,\ h[4]=1,\ h[5]=-1,\quad h[n]=0$ otherwise.

Then, $H(\Omega)=\sum h[m]\,e^{-j\Omega m}$

At $\Omega=0$, $|H(e^{j\Omega})|=|H(e^{j0})|=\sum h[m]\,e^{-j0\cdot m}$ $\quad\rfloor\ e^{-j\cdot 0\cdot m}=e^{0}=1$

$= \sum h[m]$

$=h[0]+h[1]+h[2]+h[3]+h[4]+h[5]$

$=0$

At $\Omega=\pi$, $|H(e^{j\Omega})|=|H(e^{j\pi})|=\sum h[m]\,e^{-j\pi\cdot m}$ $\quad\rfloor\ e^{-j\pi m}=(-1)^m$

$=\sum h[m]\cdot(-1)^m$

$=h[0]-h[1]+h[2]-h[3]+h[4]-h[5]$

$=6$

The only plot with a frequency response of 0 at frequency 0, and non-zero value at frequency π is the one for filter A.

So: The filter corresponding is A.