Recitation 24

Review:
MAC
Switching
Routing
Transport Layer

Example 1: Given: Three nodes - A, B, C - share access to a shared channel using slotted Aloha.
Assume the nodes are always backlogged, and the node A has half the probability as the other two, i.e.

\[ P_B = P_C = 2P_A \]

Question:
1. If \( P_A = 0.3 \), What is average utilization?
2. What value of \( P_A \) maximizes the utilization, and what is corresponding maximum utilization?

Solution:
Utilization is the probability that exactly one user sends

\[ u = P(A \text{ xmits}) \times P(B \text{ doesn't}) \times P(C \text{ doesn't}) \]
\[ + P(B \text{ xmits}) \times P(A \text{ doesn't}) \times P(C \text{ doesn't}) \]
\[ + P(C \text{ xmits}) \times P(A \text{ doesn't}) \times P(B \text{ doesn't}) \]
\[ = P_A (1-P_B) (1-P_C) + P_B (1-P_A) (1-P_C) + P_C (1-P_A) (1-P_B) \]
\[ = P_A (1-2P_A) (1-2P_A) + 2P_B (1-P_A) (1-2P_A) + 2P_C (1-P_B) (1-2P_B) \]
\[ = 5P_A - 16P_A^2 + 12P_A^3 \]
\[ = 5 \times 0.3 + 16 \times 0.3^2 + 12 \times 0.3^3 = 0.384 \]

\[ u = 5P_A - 16P_A^2 + 12P_A^3 \]

\( u \) is maximized when \( \frac{du}{dP_A} = 0 \)
So: \( \frac{du}{d\rho_A} = \frac{d}{d\rho_A} \left( 5\rho_A - 16\rho_A^2 + 12\rho_A^3 \right) = 5 - 32\rho_A + 36\rho_A^2 \)

\[ \implies \rho_A = 0.687 \text{ or } 0.202 \]

However, \( \rho_B = 2\rho_A \quad 2\rho_A = \rho_B \leq 1 \implies \rho_A \leq 0.5 \)

So, when \( \rho_A = 0.202 \)

The value of \( \rho_A \) maximized utilization is \( \rho_A = 0.202 \)

\[ U_{\text{max}} = 5\rho_A - 16\rho_A^2 + 12\rho_A^3 \]

\[ = 5 \times (0.202) - 16 \times (0.202)^2 + 12 \times (0.202)^3 \]

\[ = 0.456 \]
You send a stream of packets of size 1000 bytes each across a network path from Cambridge to Berkeley. You find that the one-way delay varies between 50 ms (in the absence of any queueing) and 125 ms (full queue), with an average of 75 ms. The transmission rate at the sender is 1 Mbps; the receiver gets the packets at the same rate without any packet loss.

(a) What is the mean number of packets in the queue at the bottleneck link along the path (assume that any queueing happens at just one switch)?

(b) You now increase the transmission rate to 2 Mbps. You find that the receiver gets packets at a rate of 1.6 Mbps. The average queue length does not change from before.

(i) What is the packet loss rate at the switch?

(ii) What is the average one-way delay now?

Solution:

(a) Average delay one-way delay = 75 ms
    one-way delay without queueing = 50 ms

    So: Average queueing delay
    \[ D = \text{average one-way delay} - \text{one-way delay without queueing} \]
    \[ = 75 \text{ ms} - 50 \text{ ms} \]
    \[ = 25 \text{ ms} \]

    Arrival rate \( \lambda \)
    \[ \lambda = \frac{1 \text{ Mbps}}{8000 \text{ bits/packet}} \]
    \[ = \frac{1 \text{ M bits/s}}{8000 \text{ bits/packet}} \]
    \[ = 125 \text{ packets/s} \]
So: the mean number of packets in the queue

\[ \lambda \times D \]

\[ = 125 \text{ packets} \times 25 \text{ ms} \]

\[ = 3.125 \text{ packets} \]

b) i) Packet loss rate = \frac{\text{Sender transmission rate} - \text{Receiver arrival rate}}{\text{Sender transmission rate}}

\[ = \frac{2M - 1.6M}{2M} \]

\[ = \frac{2M}{2M} - \frac{1.6M}{2M} \]

\[ = 0.20 \]

\[ = 20\% \]

ii) Average one-way delay = \text{one-way delay in the absence of queueing} + \text{average queuing delay} \ D

Use Little's Law to calculate \ D:

\[ N \text{ doesn't change} \quad \text{so: } N = 3.125 \text{ packets} \]

Arrival rate \( \lambda = 1.6 \text{ Mbps} \)

\[ = \frac{1.6 \text{ Mbits/s}}{8000 \text{ bits/packet}} \]

\[ = 0.2 \text{ packets/s} \]

\[ D = \frac{N}{\lambda} = \frac{3.125 \text{ packets}}{200 \text{ packets/s}} = 15.625 \text{ ms} \]

So: average queuing delay \( D = \frac{N}{\lambda} = \frac{3.125 \text{ packets}}{200 \text{ packets/s}} = 15.625 \text{ ms} \)

So: average one-way delay = \text{one-way delay in the absence of queueing} + \text{average queuing delay}

\[ = 50 \text{ ms} + 15.625 \text{ ms} \]

\[ = 65.625 \text{ ms} \]
Ex 2. Determine whether the following statements are True or False:

1. There exists some offered load pattern for which TDMA has lower throughput than slotted Aloha.
2. Suppose there are 3 nodes which use a fixed probability of \( p = \frac{1}{3} \) when transmitting on a slotted Aloha network. If all nodes are backlogged, the utilization is \( U \approx 33\% \).
3. Using contention windows with a CSMA implementation guarantees that a packet will be transmitted successfully (i.e., without collisions) with some bounded time.

Solution:

1. **TRUE**

   Proof: (TDMA doesn't work well for unbalanced load)

   For example: There are \( N \) nodes. Only 1 node is backlogged.

   Then, throughput of TDMA = \( \frac{1}{N} \) (utilization)

   Utilization of slotted Aloha \( \approx 1 \) when we set \( p = \frac{1}{3} \).

   In this case, throughput of TDMA < throughput of slotted Aloha.

2. **FALSE**

   Proof:

   \[
   u = P(\text{node 0 xmits}) \times P(\text{node 1 doesn't}) \times P(\text{node 2 doesn't}) \\
   + P(\text{node 1 xmits}) \times P(\text{node 0 doesn't}) \times P(\text{node 2 doesn't}) \\
   + P(\text{node 2 xmits}) \times P(\text{node 0 doesn't}) \times P(\text{node 1 doesn't}) \\
   = \frac{4}{9} \neq \frac{1}{3}.
   \]

   Actually, only \( N \) (the number of all nodes) \( \rightarrow \infty \),

   \[
   u \rightarrow \frac{1}{3} \approx 33\%.
   \]
FALSE

Proof:

Recall contention window scheme with CSMA:

- **CW**: current value of the window, CW can vary between CW_{min} and CW_{max}

when a node transmits,

- picks a number randomly \( r \) in \([1, CW]\)
- sends packet after \( r \) time slots.

If a collision occurs

- doubles CW (if \( CW < CW_{\text{max}} \))
- else
  - halves CW (if \( CW > CW_{\text{min}} \))

So, contention window only guarantees a transmission attempt within bounded time, but there's no guarantee of success.

( Because all attempts can be collided )
Ex 4:

Three users X, Y, Z share a link with capacity 1 Mbps. There are two possible strategies to access the link: (1) TDMA and (2) Taking turns. In "Taking turns" we add a latency of 0.05 s before taking the turn. The user can then use the link for as long as it has data to send.

In each of the four cases, which strategy would you use and why?

1. X, Y, Z send a 320 kbps file every second.
2. X sends 640 kbps file every second, while Y, Z send 80 kbps file every second.

Solution:

1. **TDMA wins**.

   Proof: for TDMA, each user gets \( \frac{\text{link capacity}}{3} = 333 \text{ kbps} \) bandwidth.

   The required bandwidth for each user is

   \[
   320 \text{ kbps/s} < 333 \text{ kbps/s}
   \]

   So: TDMA can work well in this case for "taking turn".

   In 1 second, we should do following jobs:

   - Start
   - \( x \) sends
   - \( 0.05 \) s latency
   - \( 320 \text{ kbps} \)
   - \( 0.05 \) s latency
   - \( 320 \text{ kbps} \)
   - \( 0.05 \) s latency
   - \( 320 \text{ kbps} \)
   - \( 0.05 \) s latency
   - \( 320 \text{ kbps} \)

   \[
   \text{time} = \frac{\text{data size}}{\text{link capacity}}
   \]

   \[
   \begin{array}{c|c|c|c|c}
   \hline
   \text{time} & 0.325 & 0.325 & 0.325 & 0.325 \\
   \text{latency} & 0.05s & 0.05s & 0.05s & 0.05s \\
   \text{data size} & 320 \text{ kbps} & 320 \text{ kbps} & 320 \text{ kbps} & 320 \text{ kbps} \\
   \text{rate} & 320 \text{ kbps/s} & 320 \text{ kbps/s} & 320 \text{ kbps/s} & 320 \text{ kbps/s} \\
   \hline
   \end{array}
   \]
The total time = 0.05 + 0.32 + 0.05 + 0.32 + 0.05 + 0.32 = 1.11 s > 1 s
so: "taking turn" can't finish its job.

"Taking turn" wins

Proof: for Toma, each user still gets 33.3 kbits/s bandwidth. However, user X requires 690 kbits/s bandwidth. So X's requirement can't be met.

In 1 second, we should do following job:

- Start
- 0.05 s latency
- X sends 690 kbits
- 0.645
- Time = data size / link capacity
- 1 Mbit/s
- 1 Mbit/s
- 0.08 s
- Y sends 80 kbits
- 0.08 s
- Z sends 30 kbits
- 0.05 s

The total time = 0.05 + 0.64 + 0.05 + 0.08 + 0.05 + 0.08 = 0.85 s < 1 s
so: "taking turn" can finish its job in this case.
Example 5: Consider a network with topology shown below.

```
   A  B  C  D  E
  /   /   /   /
 C /   /   /   /
  /   /   /   /
 B /   /   /   /
  /   /   /   /
   A  B  C  D  E
```

Suppose the final routing table for node A using Dijkstra's algorithm is as below.

<table>
<thead>
<tr>
<th>Node</th>
<th>Link</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td>B</td>
<td>LD</td>
<td>5</td>
</tr>
<tr>
<td>C</td>
<td>L1</td>
<td>6</td>
</tr>
<tr>
<td>D</td>
<td>LD</td>
<td>7</td>
</tr>
<tr>
<td>E</td>
<td>L1</td>
<td>01</td>
</tr>
</tbody>
</table>

Assume: the costs obey two constraints.
1. All costs are integer ≥ 1
2. Each shortest path is unique, i.e., there aren't two possible shortest paths between any two nodes.

Question: Please make the most specific deduction about each link's cost. The cost of link (A-B, B-C, A-C, B-D, C-D, D-E) i.e., give the exact cost if possible that can be determined, or indicate ≥k if you determine that k is a lower bound on the possible link cost.
Solution:

1. Look at entry (B, L0, 5).
   
   $A - B$ is the shortest path between $A$ and $B$. $\Rightarrow \text{linkcost}(A, B) = 5$.

2. Look at entry (C, L1, 6).
   
   $A - C$ is the shortest path between $A$ and $C$. $\Rightarrow \text{linkcost}(A, C) = 6$.

   Besides, the following path cost $> 6$:
   
   $A \overset{5}{\rightarrow} B \rightarrow C > 6 \Rightarrow \text{linkcost}(B, C) > 2$.

3. Look at entry (D, L0, 7).
   
   Three possible paths:
   
   $A \overset{5}{\rightarrow} B \rightarrow D \Rightarrow \text{linkcost}(B, D) = 2$.
   $A \overset{5}{\rightarrow} B \overset{2}{\rightarrow} C \rightarrow D$. Impossible.
   $A \overset{5}{\rightarrow} B \overset{2}{\rightarrow} C \rightarrow E \rightarrow D$. Impossible.

   Besides, the following path cost $> 7$:
   
   $A \overset{6}{\rightarrow} C \rightarrow D \Rightarrow \text{linkcost}(C, D) > 2$.

4. Look at entry (E, L1, 9).
   
   Two possible paths:
   
   $A \overset{6}{\rightarrow} C \overset{2}{\rightarrow} D \rightarrow E \Rightarrow \text{linkcost}(C, E) = 1$.
   However, $\text{cost}(A, B, D, E) = 1$ is the shortest path between $AE$ which is impossible.

   $A \overset{6}{\rightarrow} C \rightarrow E \Rightarrow \text{linkcost}(C, E) = 3$.

   Besides, the following path cost $> 9$:
   
   $A \overset{5}{\rightarrow} B \overset{2}{\rightarrow} D \rightarrow E \Rightarrow \text{linkcost}(D, E) > 3$. 

So, final result: