World Series

World Series is a 7 game series between two teams (A & B) which terminates as soon as either team wins 4 games. Let X be the random variable that represents the outcome of a World Series between teams A & B. Possible values of X are AAAA, ABABBB, BBBAAAA. Note that for example AAAAB is not a possible outcome ~~of~~. Let Y denote the # games played. ~~Calculate:~~ Assume Both teams are equally good & outcomes of successive games are independent. Calculate:

1. $H(X)$
2. $H(Y)$
3. What is the amount of information you get if you know that A wins
4. What is the amount of information you get with the knowledge that there were 6 games?
5. What is the residual uncertainty (entropy) in X given that there were 6 games?

Solution: first, let us do some preliminary calculations: 2

Event that A wins, & there were 4 games:

Only possibility is $X = AAAA$.

$Pr(\{A \text{ wins, there are 4 games}\}) = \left(\frac{1}{2}\right)^4 = \frac{1}{16}$

Event that A wins, & there are 5 games:

Since A wins, ~~the~~ 5th game should be won by A.
Among the first 4 games, 1 game is won by B

$\# \text{ choices} = \binom{4}{1} = 4.$

These events are:

$BAAAA$
$ABAAA$
$AABAA$
$AAABA$.

Probability of each of these events, ie,

$Pr(\{BAAAA\}) = Pr(\{ABAAA\}) = Pr(\{AABAA\})$
$= Pr(\{AAABA\}) = \left(\frac{1}{2}\right)^5 = \frac{1}{32}$

Thus, $Pr(\{A \text{ wins, there are 5 games}\}) = 4 \cdot \frac{1}{32}$

Event that A wins, there are 6 games :

Since A wins, the 6 th game should be won by A.

Among the first 5, B wins 2.

$\#$ choices $= \binom{5}{2} = 10$.

An example of such an outcome is AABBAA.

$Pr(\{AABBAA\}) = \left(\frac{1}{2}\right)^6 = \frac{1}{64}$

For each of these outcomes, probability is $\frac{1}{64}$.

Thus, $Pr(\{A \text{ wins, there are 6 games}\}) = \frac{10}{64}$

Event the A wins, there are 7 games

Since A wins, the 7 th game should be won by A.

Among the first 6, B wins 3.

$\#$ choices $= \binom{6}{3} = 20$

An example of such an outcome is ABABABA.

$Pr(\{ABABABA\}) = \left(\frac{1}{2}\right)^7 = \frac{1}{128}$

For each of these outcomes, probability is $\frac{1}{128}$

Thus, $Pr(\{A \text{ wins}, \text{ there are 7 games}\}) = \frac{20}{128}$

When B wins, events are exactly symmetric.

$Pr(\{B \text{ wins}, 4 \text{ games}\}) = \frac{1}{16}$

$Pr(\{B \text{ wins}, 5 \text{ games}\}) = \frac{4}{32}$

$Pr(\{B \text{ wins}, 6 \text{ games}\}) = \frac{10}{64}$

$Pr(\{B \text{ wins}, 7 \text{ games}\}) = \frac{20}{128}$

Thus, $Pr(\{4 \text{ games}\}) = Pr(\{4 \text{ games}, A \text{ wins}\}) + Pr(\{4 \text{ games}, B \text{ wins}\})$
$= \frac{1}{16} + \frac{1}{16} = \frac{2}{16}$

$Pr(\{5 \text{ games}\}) = Pr(\{5 \text{ games}, A \text{ wins}\}) + Pr(\{5 \text{ games}, B \text{ wins}\}) = \frac{4}{32} + \frac{4}{32} = \frac{8}{32}$

Similarly, $Pr(\{6 \text{ games}\}) = \cancel{\frac{20}{64}}\ \frac{10}{64} + \frac{10}{64} = \frac{20}{64}$

$Pr(\{7 \text{ games}\}) = \frac{20}{128} + \frac{20}{128} = \frac{40}{128}$

5

1. $\underline{H(X)}$

review

I calculated this in ~~class~~ yesterday. Not going through the steps. Answer is

$$\cancel{\#} \quad 2\left[\frac{1}{16}\log_2 16 + \frac{4}{32}\log_2 32 + \frac{10}{64}\log_2 64 + \frac{20}{128}\log_2 128\right]$$

$$= \cancel{\#} \quad \frac{93}{16}$$

2. $\underline{H(Y)}$. Recall: $Y = \#$ games

As we calculated before,

$Pr(\{4 \text{ games}\}) = \frac{2}{16}$. $Pr(\{5 \text{ games}\}) = \frac{8}{32}$

$Pr(\{6 \text{ games}\}) = \frac{20}{64}$. $Pr(\{7 \text{ games}\}) = \frac{40}{128}$

Thus,
$$H(Y) = \frac{2}{16}\log_2\left(\frac{1}{2/16}\right) + \frac{8}{32}\log_2\left(\frac{1}{8/32}\right) + \frac{20}{64}\log_2\left(\frac{1}{20/64}\right) + \frac{40}{128}\log_2\left(\frac{1}{40/128}\right)$$

$$= \frac{2}{16}\log_2\frac{16}{2} + \frac{8}{32}\log_2\frac{32}{8} + \frac{20}{64}\log_2\frac{64}{20} + \frac{40}{128}\log_2\frac{128}{40}$$

(Please Calculate !!!!!)

(3) What is the amount of information you get given that A wins the ~~...~~ world series: (6)

By symmetry

$$Pr(\{A \text{ wins world series}\}) = Pr(\{B \text{ wins world series}\}) = \frac{1}{2}$$

Thus, in particular, $Pr(\{A \text{ wins world series}\}) = \frac{1}{2}$

$\therefore$ Amount of information you get is $\log_2\left(\frac{1}{1/2}\right) = \underline{\underline{1}}$

(4) What is the amount of information you get with the knowledge that there were 6 games?

As calculated before,

$$Pr(\{6 \text{ games}\}) = \frac{20}{64}.$$

$\therefore$ Amount of information you get $= \log_2\left(\frac{1}{20/64}\right) = \log_2\left(\frac{64}{20}\right)$

<u>Please calculate !!!!!</u>

→ What is the residual uncertainty (entropy) of X given there were 5 games?

5 If there were 5 games, 7

as we calculated before, in total, there are 20 possible outcomes (10 for when A wins, 10 for when B wins)

Each of these outcomes has originally probability $\frac{1}{34}$,

but conditioned on the event that there were 20 games, each of these events has probability $\frac{1}{20}$.

Residual uncertainty $= 20 \log_2 20$ (Please calculate!!!)

This is because if there are m equally likely events, the entropy is $\underbrace{\log \frac{1}{1/m} + \log \frac{1}{1/m} + \cdots}_{m \text{ times}}$

$= m \log m$.

write to

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Whoever pointed out yesterday that I was doing calculations in a complicated manner (even though what I was doing was ~~they were~~ correct), many thanks !!!!! It's best to do things in the easiest possible manner ...

Good luck & peace ...

— Mukul .